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Agent-based Modeling for the Study of Diffusion Dynamics of Network Technologies

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Abstract:
In the modern world the internet is by all accounts an incredible success, but inspite of this success, its deficiencies have come under increasing scrutiny and triggered calls for new architectures to succeed it. Those architectures however face a formidable incumbent in the internet and likely to depend equally on technical superiority as an economic factors. Recently, economic models have been proposed to study adoption dynamics of entrant and incumbent technologies motivated by the need for new network architectures to complement the current Internet. Diffusion is a process in which the new products and practices into society along with inventions are successfully introduced. Many studies of the diffusion of individual innovations exist and exhibit some similarities such as the famous S-shaped diffusion curve. New ideas, products innovations often take time to diffuse, a fact that is often attributed to some form of heterogeneity among people. The diffusion process enhances an innovation through the feedback of information about its utility across different users that can be use to improve it. This aspect is common to the micro-macro loop which is an essential part of emergent dynamics. The present paper a model for the adoption of computing network technologies by individual users is formulated and solved.

Keywords*: Technology Diffusion, User Heterogeneity, evolutionary game theory, technology adoption, innovations*

1. Introduction

Networking is no exception, and the Internet displaced the traditional phone network as the defact to communication infrastructure. And it also displaced more direct competitors, packet data network technologies such as ATM, Frame Relay, SNA and others. In other words, technical superiority alone is no guarantee for success, especially in the presence of a strong incumbent. But networking technologies exacerbate this phenomenon because of the many factors that affect their value. The Internet itself provides a perfect example of such complex interactions, with its long-standing "migration" from IPv4 to IPv6. We propose a model to study both the dynamics of diffusion of new network technologies in the presence of an incumbent, and their eventual equilibrium adoption levels.

In our model, users can be heterogeneous in the way they value the services deployed on top of any network technology. The drawback of modeling individual level decision-making and user heterogeneity is that it makes the diffusion model complex. The cost of this complexity is out-weighed by two benefits. First, the model allows us to understand both individual-level and system-level dynamics. Second, the model allows us to identify and explain interesting phenomena such as the presence of multiple equilibria and the potential for both network technologies.

We have two main findings to report from our initial analysis:

- Multiple equilibrium adoption levels may exist for the same set of parameters (i.e., price, quality levels, etc). It is possible for both the incumbent and entrant technologies to coexist in equilibrium, even in the absence of gateways or converters.
- Even though the entrant may seem to be diffusing well, it is often doomed to fail if its growth rate is slower than that of the incumbent. When multiple equilibria exist, we observe that small changes in parameters can sometimes have a big impact on the equilibrium adoption levels.

Now-a-days the current Internet has generic deficiencies to provide the security. For a new network architectures that can complement the features lacking in the current Internet architecture. The goal of such type of studies is to develop models assessing the viability of a new network technology and explain the diffusion process of the entrant. Sometimes users may not have the perfect or complete information necessary for optimal decision making, or they may be misinformed. And sometimes, users may not be fully rational, but bounded rational. The diffusion of new technologies consider bounded rationality due to limited information.

Many aspects of bounded rationality, we are interested here in users decision-making behaviors, such as the choice of a satisfying strategy (an inferior strategy) instead of an optimal strategy). The theory of satisfying behavior says that players are more apt to satisfy rather than to optimize. Instead of choosing a strategy giving the highest payoff, a player sets a standard representing the payoff level she wants, called aspiration level, and searches a strategy giving her payoff higher than the aspiration level. The decision-making process itself is called as aspiration-based learning.

In this paper, most of the economic literature studies 2x2 (two users and two strategies). Here each user adjusts her own standard level (aspiration) based on the payoff she has received in the past. Considers N users, where N is sufficiently large and all users have a common aspiration level. Mainly in this paper, we are obtaining two aggregate diffusion models with the bounded rational users: constant aspiration level and time-varying aspiration level. With a fixed common aspiration, the diffusion dynamics of an entrant technology can be modeled as a continuous-time Markov process on finite state space. At that time there is an uncertainty will occur in decision making of bounded rational users. In aspiration-based learning, learning from past experience is incorporated into aspiration level through adaptation. In this way, the users evaluate the technologies and adjust the aspiration level in the direction toward the average payoff. Here we can examine that the dynamics between one incumbent and one entrant and the dynamics among one incumbent and two entrants.

This paper refers to the formulation and analysis of models of aggregate diffusion dynamics. It can describe how our work is related to the existing work. In Markov process model using a fixed common aspiration level and analyze the user's adoption behavior. We formulate mean field dynamics between one entrant and one incumbent with time-varying aspiration. The mean field diffusion dynamics when two entrants are introduced to the market with an incumbent.

The main study on diffusion modeling is based on the Bass model. The Bass diffusion model describes the process how new products get adopted as an interaction between users and potential users. The Bass model also formalizes the aggregate level of penetration of a new product emphasizing two processes: external influence through advertising and mass media, and internal influence through word-of-mouth.

The Bass model assumes all consumers to be homogeneous, and such diffusion models are referred to as aggregate models. Whenever we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters. This stylized fact is often attributed to some form of heterogeneity among people.

We can also derive an individual decision rule from the Bass model: the number of individuals who adopt at a given time is a function of the number of individuals who have already adopted. Our societies consist of individuals and the social systems which largely determine how they behave and interact. One of the cardinal rules of human behavior is "birds of a feather flock together".

As a technological innovation, the Internet provided a new stage for communication and information processing within societies. Many diffusion processes are progressive in the sense that once a node switch from one state to another state. In this class of the diffusion process, we concentrate on the correlations between social interaction patterns and observable transmission rate at the individual level.

2. Related Work

There are four streams of work relevant to our study. The first relates to the literature on adoption of incompatible technologies. The second relates to the literature on new product/technology diffusion. The third relates to the literature on adoption dynamics of technologies and the fourth relates to the literature on aspiration-based modeling in economics.

2.1. Adoption of incompatible technologies

When technologies are incompatible, users of a technology can only reach other users of the same technology and, as a result, the value a user derives from a technology is a function of the size of its installed base (these are referred to as network benefits or network externalities). The main focus of the literature has been on the impact of converters that help make one technology partially compatible with the other.

2.2. New Product Diffusion

These models focus on aggregate adoption dynamics without explicitly modeling individual decision making processes. A few models have focused on individual-level adoption. The adoption of new network technologies and architectures is often influenced by the presence of incumbents.

2.3. Adoption Dynamics of Technologies

Here the diffusion dynamics of a new technology are an active line of research in economics and management science. And also we newly proposed secure BGP protocols are not adopted by ISP inspite of existing security problems of current BGP. we propose a new metric for protocol design, adoptability. Adoptability measures the strength of a protocol's properties in driving the adoption process.

One important factor in user benefit is network externality, i.e., how many users adopt a given technology. The impact of network externalities is studied in various analytic models. We also proposed a "static" economic model in which every user chooses either a new technology or an old one. A user adopts the new technology or adopts a converter that enables the user to use new technology with partial benefit while still using an old technology (if the new technology gives higher benefit to the user than the old one).

2.4. Aspiration-Based Learning

Here each user sets a standard representing the payoff she hopes to get and compares her choice to the standard at each time-step. The standard is called aspiration. If the payoff from her current choice exceeds the standard, then she keeps the choice at the next step. Otherwise, she drops the choice and chooses another alternative with some positive probability at the next time-step.

Research on Networked Agents and Diffusion Dynamics:

Every day, billions of people worldwide make billions of decisions about many things. People constantly interact with each other in different ways and for different purposes. These emergent properties are the result of not only the behavior of individuals but the interactions between them.

Examples include studies on the diffusion of new technologies, herding behavior in stock markets and the diffusion of conventions and social norms. Current methodologies provide insufficient explication of the network externalities concept. The current state-of – the art in agent-based modeling tends to be a mass of agents that have a series of states that they can express as a result of the network structure in which they are embedded.

A basic methodology is to specify how the agents interact, and then observe emergent intelligence that occur at the collective level in order to discover basic principles and key mechanisms for understanding and shaping the resulting intelligent behavior on network dynamics. Agent networks have been used to create systems for supporting real communities that interchange information. Networks are represented as graphs, where nodes represent agents and the edges represent the relationship between them.

3. Problem Formulation

It introduces our model for studying adoption dynamics between two competing network technologies.

3.1. User's Decision Process

Consider two network technologies, labeled 1 and 2, representing the incumbent and entrant respectively: for example, IPv4 versus IPv6 or the current Internet versus a cleanslate alternative. The quality of technology i is denoted by $q_i > 0$ and its price $p_i > 0$, for $i = 1, 2$. We assume a fixed population of N users, with N_i the number of users adopting technology $i = 1, 2$. The proportion of users who adopt technology i is denoted by $x_i = N_{i/N}$, with $\underline{x} = (x_1, x_2) \in S, N$ where

$S = \{(x_1, x_2) | x_1 + x_2 \leq 1, 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ denotes the set of possible adoption levels. An end user's surplus from technology i is modeled as

$$U_i(\theta, \underline{x}) = \theta q_i + v(x_i) - p_i \quad (1)$$

θq_i is the standalone benefit that the user obtains from technology i, with $\theta \in [0, 1]$. The value of θ varies across users capturing their heterogeneity. Individual values of θ are private, but their distribution, denoted by $F(\theta)$, is known. $v(x_i) \geq 0$ denotes the positive network externalities or benefits that the user derives from other adopters of technology i. so that $v(x_i)$ is a non decreasing function with $v(0) = 0$.

In other words, a user chooses

no technology	if	$U_i < 0$ for all i,
technology 1	if	$U_1 > 0$ and $U_1 > U_2$,
technology 2	if	$U_2 > 0$ and $U_2 > U_1$.

$$\Theta_1(\underline{x}) = \{\theta \in [0,1] / U_1(\theta, \underline{x}) \geq U_2(\theta, \underline{x}), U_1(\theta, \underline{x}) > 0\},$$
$$\Theta_2(\underline{x}) = \{\theta \in [0,1] / U_2(\theta, \underline{x}) > U_1(\theta, \underline{x}), U_2(\theta, \underline{x}) > 0\},$$

Under the assumption that the functions $U_i(\theta_i, \underline{x})$, $i = 1, 2$, are continuous, both subsets are easily shown to be connected intervals, so that if we denote as $H_i(\underline{x})$, $i = 1, 2$, the number of users in each subset, and further assume that $F(\theta)$ is continuous, we have the following

expression for $H_i(\underline{x})$

$$H_i(\underline{x}) = F(b_i) - F(a_i)$$

Where $[a_i, b_i]$ = closure of $\Theta_i(\underline{x})$.

Equilibrium adoption levels can then be characterized by:

$$x^* = H_i(\underline{x}^*) \text{ for } i = 1, 2. \quad (2)$$

3.2. Diffusion Dynamics

Suppose that at time't', the "current" technology adoption values, $x_i(t)$, $i = 1, 2$ are announced to all users. Under these assumptions, $(H_i(\underline{x}(t)) - x_i(t))$ would be the proportion of users that proceed to adopt (disadopt) technology i and t. To capture this, we use the diffusion rate of user decisions at time t can be modeled by

$dx_i(t)/dt = (H_i(\underline{x}(t) - x_i t))P(t)$

4. Characterizing Equilibria

The dynamics of technology adoption in our model involve both identifying possible equilibria and characterizing how they are reached.

- θ has a uniform distribution over [0,1]. Although it affects the magnitude of the equilibrium, it does not qualitatively affect the results .
- $v(x_i)$ is linear in x_i, i.e., $v(x_i) = x_i$..This is consistent with Metcalfe's law and commonly used in the literature.
- The entrant technology is of higher quality, i.e., $q_2>q_1$.
- $p_i>0,\ q_i>0,\ i=1,2.$

4.1. Computing Equilibria

C1: $x_1 + (p_2 - p_1) - (q_2 - q_1) < x_2 < x_1 + (p_2 - p_1)$

C2: $x_1 \geq p_1$.

We are not only identifies valid equilibria, but it also indicates that there exist conditions under which both technologies can coexist in equilibrium. When cast in the context of network technologies, this can mean significant inefficiencies if it calls for introducing and maintaining two versions of new and existing service, e.g., an IPv4 and an IPv6 version.

4.2. Markov Process Model with Fixed Aspiration

4.2.1. Model

Consider a fixed population of *N* users and two network technologies, labeled *X* and *Y*, representing a new entrant technology and an incumbent, respectively. We assume that all users adopt technology *Y* at the initial time, and technology *X* has technical features that technology *Y* does not provide for the users. Once the entrant technology is introduced to the market, each user adopts either technology X or technology *Y*.

Let s_i be the user *i's* technology choice and $s_{-i} = (s_1,s_2,\dots,s_{i-1},s_{i+1},\dots,s_N)$ be the other user's choice. Let $-s_i = \{X,Y\} \setminus s_i$, for given s_i: If s_i is X, then $-s_i$ = Y,

	X	**Y**
X	(r, r)	(0,0)
Y	(0,0)	(v,v)

Table 1: Payoff Matrix In A 2x2 Coordination Game: R>V

and vice versa. The payoff of user *i's* choice s_i under other users' choice s_{-i} is

$u_i(s_i, s_{-i}) = 1/N\text{-}1\sum j\#iu(s_i, s_j)$

where $u(s_i, s_j)$ is a payoff in 2x2 coordination game of which payoffs are given by Table I. We assume that r>v in the sense that new technology provides more desirable technical features for users than technology Y.

We further assume that every user repeatedly makes a choice between X and Y. Let X(t) be the number of users adopting X, and Y(t) the number of users choosing Y at time t.

A user i and consider how she makes a decision. The user i does not seek the best strategy, but a strategy satisfying her. If her payoff with the current choice is above or equal to the aspiration level, then she is satisfied with her decision. Hence, she does not want to change her choice at the next opportunity, even though she might get a higher payoff by switching her choice.

Let s_i^t and α_i^t be the strategy and aspiration level of user i at time t, respectively. At time t_{n+1} , a user compares her utility at t_n with the aspiration level α_i^{tn} . She keeps s_i^{tn} if $u_i(s_i^{tn}, s_{-i}^{tn}) \geq \alpha_i^{tn}$. Otherwise, she switches her choice to the other technology with probability h, 0<h<1. More precisely, the strategy of user i at time t_{n+1} is

$s_i^{tn+1}=s_i^{tn}$, if $\alpha_i^{tn} \leq u_i(s_i^{tn},s_{-i}^{tn})$

s_i^{tn} with prob. 1-h, if $\alpha_i^{tn} > u_i(s_i^{tn},s_{-i}^{tn})$

$-s_i^{tn}$ with prob. h, if $\alpha_i^{tn} > u_i(s_i^{tn},s_{-i}^{tn})$

Where hε(0,1). In economics literature, 1-h is called inertia, and h switching probability. If the switching cost is high, users will not tend to switch easily.

We will assume that all users have common aspiration level and that it is constant: $\alpha_i^t=\alpha$ for all i and t. However, this assumption of constant aspiration level explains a widespread (but not always true) belief that a new and better technology will eventually dominate.

With the assumption of common constant aspiration level, X(t) becomes a continuous-time Markov process on the state space {0,1,….,N}as follows. Suppose that X(t) =k at time t. Then, there are k users choosing X and N-k users choosing Y. Therefore, at time t, any user choosing X gets utility $1/N\text{-}1\ \sum j\#iu((s_i^t,s_j^t)=r(k\text{-}1)/N\text{-}1$. Let $u_X(k):=r(k\text{-}1)/N\text{-}1$. If $u_X(k)<\alpha$, then a user adopting X is disappointed by X and tends to switch from technology X to technology Y with probability h. Hence, the system transists from state k to state k-1 with transition rate hk. If $u_X(k)\geq\alpha$, users adopting X are satisfied by X and are not willing to change their technology. Hence, the system stays in state k. The transition rate from state X(t)=k to state k-1 is

qk,k-1=hk, if $k<1+\alpha/r(N-1)$
0, otherwise (3)
Similarly, users choosing Y have $u_Y(k)$=v N-k-1/N-1 and the transition rate from state X(t)=k to state k+1 is
qk,k+1=h(N-k), if$(N-1)(1-\alpha/v)<k$
0, otherwise (4)

4.2.2. Analysis
We examine the limiting behavior of X(t) and the average time to reach state N starting from state 0.
If $\alpha\leq v$, then $q_{0,1}$ =0: There is no transition if all users adopt technology Y at the initial time. Since all users are satisfied with current technology, they do not need to switch to the entrant, even though new technology is available. If users are satisfied with the current incumbent, they do not want to try something new.
Suppose that $\alpha>v$. First consider the case that qk,k-1>0 for all k. From (3), this happens only if $N<1+\alpha/r(N-1)$, which is equivalent to $\alpha>r$. If $\alpha>r$, then qk,k-1=hk for all kε{1,2,…,N)and qk,k+1=h(N-k) for all kε{0,1,…,N-1}(since$(N-1)(1-\alpha/v)<0$). The resulting transition rate matrix, Q=[$_{qij}$]0≤i,j≤N, is irreducible, and the invariant distribution π such that πQ=0,
$\pi_k=\pi_0\prod_{j=1}^{k}N-j+1/j$ for k>0
$\pi_0=(1+\sum_{k=1}^{N}\prod_{j=1}^{k} N-j+1/j)^{-1}$
The invariant distributions are depicted in the below figure.

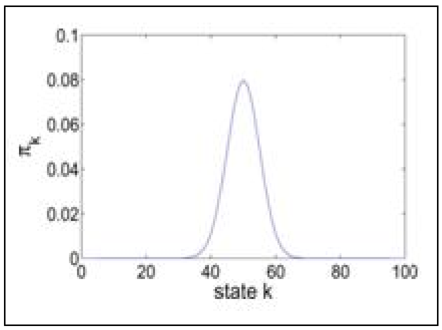


Figure 1: Invariant distribution π_k when N=100

Since πk>0 for all k, changing states continually occur from one state k to either k-1 or k+1 without absorbing into a particular state. If users are fully rational (i.e., they choose the optimal strategy), they will choose the entrant giving them the higher utility when neither technology satisfies them, under the circumstance when they should choose one.
Suppose that $v<\alpha\leq r$. Let K=max{k/k<1+α/r(N-1)}. Note that this K is a unique integer such that $(K-1)/N-1<\alpha/r\leq K/N-1$. The transition rates are now
qk,k-1=hk, for k≤K
0, for k>K
qk,k+1=h(N-k) for 0≤k≤N
It can be easily checked that the invariant distributions are π_N=1 and π_i=0 for 0≤i≤N-1. Thus, we have the following result.
Theorem 1: If $v<\alpha\leq r$, then lim t→αX(t)=N.
That α>v can be interpreted as follows. α<r means that the users do not want a new technology better than technology X. Hence, technology X is the one the market seeks, and ultimately all users adopt it.
Our next point of interest is the average time to reach the state N from state 0. The hitting time to the state N starting from state i for a given sample path ω is defined as
$H_i^N(\omega)=\inf\{t\geq0:X_\omega(t)=N|X_\omega(0)=i\}$ (5)
Where $X_\omega(t)$ is the number of users adopting X at t and the average hitting time is
$k_i^N=E(H_i^N(\omega))$
The average hitting time can be obtained by solving the system of linear equations
$k_N^N=0$
$hNk_0^N-hNk_1^N=1$ (6)
$-hnk_{n-1}^N+hNk_n^N-h(N-n)k_{n+1}^N=1$, for 1≤n≤K (7)
$h(N-n)k_n^N-h(N-n)k_{n+1}^N=1$, for K<n<N (8)
By solving the system of linear equations (6)-(8), we have the following result that tells us how much time is taken for the entrant to take full adoption of it.
Theorem 2: The average hitting time to state N starting from state 0 is
$k_0^N(K)=\sum_{j=K+1}^{N-1}1/(N-j)h+1/h\sum_{i=0}^{K}2^i/(N-i)(1+\sum_{k=i+1}^{K}(-1)^{k-i}k!/i!(k-i)!)$
(9)
and $k_n^N(K)$, the average hitting time to state N from state n, for 1≤n≤N-1, is given by
$k_n^N(K)=k_0^N(K)+1/h\sum_{j=0}^{n-1}a_{n,j/N-j}$, for 1≤n≤K

$\sum_{j=n}^{N-1} 1/h(N-j)$, for $K+1 \leq n \leq N-1$

Theorem 2 tells us that it takes an exponentially proportional amount of time from state 0 to state K and a linearly proportional amount of time from state K to state N. Here, the average hitting time to state N from state 0 depends on the value of K that in turn depends on the aspiration level α(or α/r, where r is fixed). Since K increases as the common aspiration level increases, high aspiration level results that it takes longer to reach to the state where all users adopt X.

$K_0^N(K)=k_0^K(K)+k_K^N(K)$

Where

$k_0^K(k)=1/h\sum_{i=0}^{K} 2^i/(N-i)(1+\sum_{k=i+1}^{K}(-1)^{k-i}k!/i!(k-i)!)$

$k_K^N(K)=\sum_{j=K+1}^{N-1} 1/(N-j)h$

once the system reaches state K, it stochastically moves toward the absorbing state N without returning to state 0. However, the

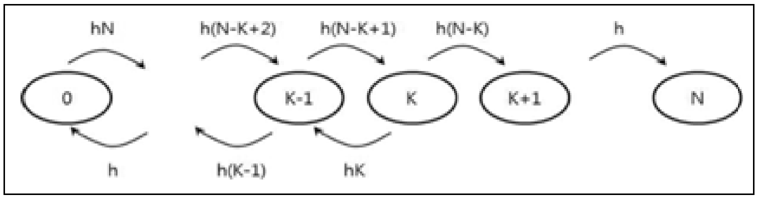


Figure 2: Transition rate diagram for v<α≤r

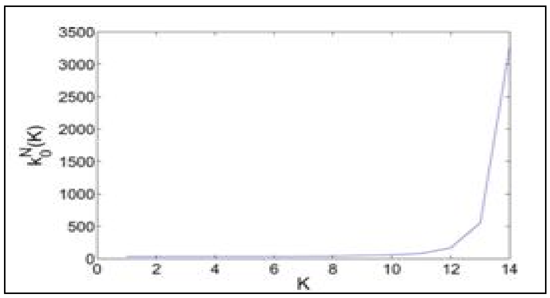


Figure 3: Average hitting time with N=15, h=1/10

System moves back and forth to reach state K, which results in exponential increase (note the 2^i term in $k_0^K(K)$) in time.

4.3. Mean Field Model

It may be set too low or too high when users have no experience using a new technology. As the aspiration adapts with time, the degree of disappointment, expresses by the difference between the aspiration level and the received utility, also becomes time-varying. We consider a switching probability "function" depending on the degree of disappointment rather than a constant switching probability: the bigger disappointed the higher switching probability.

4.4. Time-Varying Aspiration and Switching Probability Function

To incorporate prior experience of an individual into her decision making, a user sets a new standard, i.e., new aspiration level, taking into account the payoff of her own and (or) payoff of others in evolutionary game theory.

$a_i(t_{n+1})=a_i(t_n)+\lambda(t_n)(u_i(t_n)-a_i(t_n))(t_{n+1}-t_n)$

$=a_i(t_0)+\sum_{k=0}^{n}\lambda(t_k)(u_i(t_k)-a_i(t_k))(t_{k+1}-t_k)$ (10)

Where $u_i(t_n)$ and $a_i(t_n)$ denote her payoff and aspiration level at time t_n, respectively, and $\lambda(t_n)\varepsilon(0,1)$ a weight function. Note that current aspiration level $a_i(t_{n+1})$ depends on the past aspiration level $a_i(t_n)$ and the past utility $u_i(t_n)$in the Eqn. (10). The aspiration level $a_i(t_{n+1})$ represents a payoff level that a player hopes to get based on her past history of utility up to time t_n.

Users observe experiences of other users and share their aspiration levels. We assume that the average aspiration level is delivered to every user as soon as there is a change on the value of the current aspiration level, without delay. To consider the evolution of common aspiration level, we assume that is constant and every user simultaneously adapts her aspiration level at the time t_{n+1}(in discrete time unit) with the adaptation rule. Then, by summing over i and dividing the summation by N, the average of updated new aspiration level at time t_{n+1}. Where $A_N(t)=\sum_{i=1}N\alpha_i(t)/N$ and $\varphi(t)=\sum_{i=1}^{N}u_i(t)$, the total sum of payoffs, and $\lambda(t)$ is a constant λ. We assume that the change on the common aspiration level in a short time interval is proportional to the quantity $\varphi(t)/N-A_N(t)$.

4.5. Modeling

Let X(t) be the number of users who adopt the entrant and $A_N(t)$ be the common aspiration level at time t, when there are total N users. i.e., $0\leq A_N(t)\leq\alpha_{max}$ for all t. Consider how the number of users adopting X changes. A user i choosing the entrant (X) gets the utility

$u_X(t)=1/N-1\sum_{j:sjt=X}u_i(X,s_j^t)=X(t)-1/N-1r$

A user choosing the incumbent (Y) has the utility

$u_Y(t)=(1-X(t)/N-1)v$

at time t. At time t, a user choosing X changes her technology choice with probability $h(A_N(t)-u_X(X(t)))$ and a user choosing Y changes her technology choice with probability $h(A_N(t)-u_Y(X)(t)))$. Therefore, the change in X(t) in a short time interval is

$-h(A_N(t)-u_X(t))X(t)+h(A_N(t)-u_Y(t))(N-X(t))$ (11)

The first term is the amount of decrease in the number of users choosing the entrant, i.e., the number of users who switch from technology X to technology Y, and the second term is the number of users who switch from the incumbent to the entrant. That the change on the common aspiration level in a short time interval is proportional to $\varphi(X(t))/N-A_N(t)$ where $\varphi(X(t))$ is the total sum of payoffs at time t

$\Phi(X(t))=X(t)(X(t)-1)/N-1r+(N-X(t))N-1-X(t)/N-1v$

Note that $(X(t),A_N(t))$ is a Markov process defined on $\{0,1,\ldots,N\}x[0,\alpha_{max}]$. We will see the dynamics of $(X(t)/N,A_N(t))$can be approximated as a set of deterministic differential equations when N is sufficiently large.

First, note that if we let $\alpha(t)=A_N(t)$,x(t)=X(t)/N, and y(t)=Y(t)/N and assume that N is sufficiently large, then by Eqn. (11)

dx/dt=-h(a-rx)x+h(a-v(1-x))(1-x)

Since the change of the common aspiration level in a short time period is proportional to $\varphi(t)-a(t):=\varphi(X(t))/N-A_N(t)$, the evolution of the common aspiration level can be modeled as

$da(t)/dt=\lambda(\varphi(t)-a(t))$

The average payoff $\varphi(t)=\varphi(t)/N$ is

$\Phi(t)=rx^2(t)+v(1-x(t))^2$

Let $u_X(k)$ be the utility of a user adopting the entrant, $u_Y(k)$ be the utility of a user adopting the incumbent, and $\varphi(k)$ be total sum of the payoffs when k users choose the entrant; that is

$u_X(k)=\sum_{j\neq i}u(X,s_j)=k-1/N-1r$

$u_Y(k)=\sum_{j\neq i}u(Y,s_j)=(1-k/N-1)v$

$\varphi(k)=ku_X(k)+(N-k)u_Y(k)$

Let $X_N(t)=X(t)/N$. Then, $(X_N(t),A_N(t))$ is a Markov process (since $(X(t),A_N(t))$is a Markov process) defined on the state space $\{(i/N,j/N)|0\leq i\leq N,0\leq j\leq j_{max}$ and $i,j\varepsilon Z\}$.

Since a user choosing Y changes her technology choice (to the entrant)with probability $h(A_N(t)-u_Y(X(t)))$

$(i/N,j/N)\rightarrow(i+1/N,j/N)$ with rate $h(j/N-u_Y(i))(N-i)$

Since a user choosing X changes her technology choice (to the incumbent) with probability $h(A_N(t)-u_X(X(t)))$

$(i/N,j/N)\rightarrow(i-1/N,j/N)$ with rate $h(j/N-u_X(i))i$

Theorem 3: For a fixed time interval [0,T] and tε[0,T], the Markov chain $(X_N(t),A_N(t))$ converges almost surely to $(x(t),\alpha(t))$, which is the solution of the (deterministic) differential.

Note that a(t) is a stochastic process, but $\alpha(t)$ is the deterministic approximation of common aspiration $A_N(t)=a(t)$ in the above theorem.

Theorem 4: By the above theorem, the dynamical system has three equilibrium points, (0,v),(1,r),(v/r+v,rv/r+v): full market penetration of the incumbent, full market penetration of the entrant, and coexistence of both technologies. When the two technologies share the market, the portion of the market share taken by the entrant is v/r+v<1/2 since r is bigger than v. Note that rv/r+v<v, the aspiration level at the nontrivial equilibrium, is less than the payoff of the incumbent.

Theorem 5: The equilibrium point (v/r+v,rv/r+v) is unstable while(0,v) and (1,r) are locally stable. When the security protection is not strong, there are three deployment levels as equilibria: $0,d_1,d_2$ with $0<d_1<d_2<1$, where 1 means full deployment. No deployment and deployment level d_2 are stable, but d_1 is unstable.

For example, the N-person coordination game model in exhibits a different long-run equilibrium. In that model, at a discrete time unit t+1, a user chooses the best response strategy with probability (1-ε), and chooses the other strategy with probability ε>0.

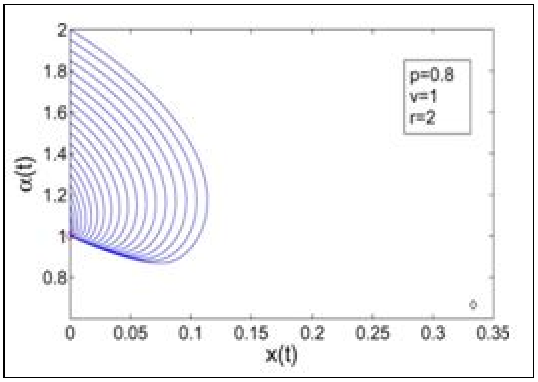
Eventually all the users adopt the entrant technology X. However, in the mean field model, the asymptotic behavior does not always converge to the state of full adoption of technology X, even α(0)>V. When α>r, the Markov process model exhibits a cyclic behavior without absorbing into one state, but repeatedly moving from one state to another. In the Markov process model with fixed aspiration level, when v<α<r, the hitting time to full adoption of the entrant is increasing with the fixed aspiration level.

4.6. Numerical Examples

We conducted numerical experiments to investigate how the values of r,p,and α(0) affect the adoption dynamics. Here, all users adopt the incumbent technology at the initial time, x(0)=0, and the entrant technology is introduced with α(0)>v: when α(0)≤v, clearly no one has the need to try the new technology. We used v=1 and the switching probability function in all experiments.

Show the impact of r, the payoff from the entrant, on the limiting behavior of diffusion dynamics. We need p=0.8 and N=1000. Whenever we use, where r=2, the entrant does not survive with any initial aspiration level v<α(0)≤r.Here, when we recall that in the Markov process model, α>v guarantees the elimination of the incumbent. Whenever the entrant technology dies out for α(0)<2.6, but it defeats the incumbent if α(0)≥2.6.

when r=10, the trajectories with α(0)<2.5 converge to the state of full adoption of the incumbent Y. Note that most of trajectories converge to the state of full adoption of the entrant X if p=0.8 .



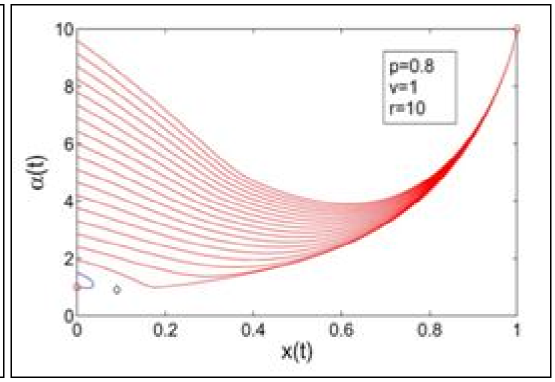
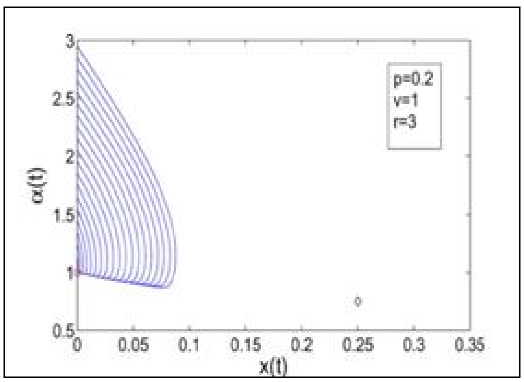


Figure 4: Diffusion dynamics: r=2 and p=0.8. Circle: limit of a trajectory. Diamond: interior equilibrium.
Figure 5: Diffusion dynamics: r=10 and p=0.8. Circle: limit of a trajectory. Diamond: interior equilibrium.

4.7. Two Competing Emerging Technologies and One Incumbent

It studies the adoption dynamics of two entrant technologies and one incumbent one among N users. Let us denote the entrants X_1 and X_2 with $r_1=u(X_1,X_1)$ and $r_2=u(X_2,X_2)$. The incumbent is denoted by $X_0=Y$ with $r_0=v=u(Y,Y)$. We assume that $u(X_i,X_j)=0$ for i#j and $r_2>r_1>v$. X_i is the number of users choosing technology X_i and $x_i(t)=X_i(t)/N$ at time t and N is sufficiently large.



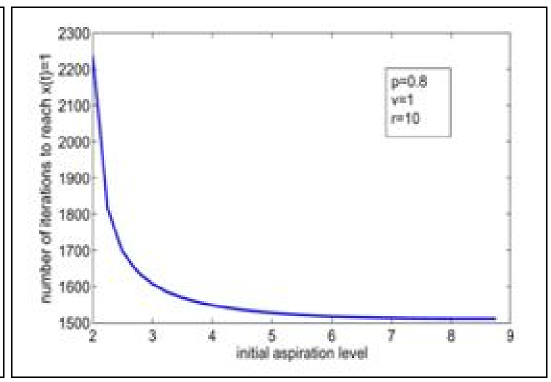
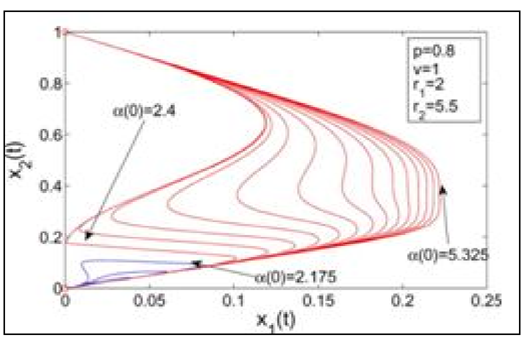


Figure 6: Diffusion dynamics: r=3 and p=0.2. Circle: limit of a trajectory. Diamond: interior equilibrium.
Figure 7: Hitting time (number of iterations) to reach the state x(t)=1.



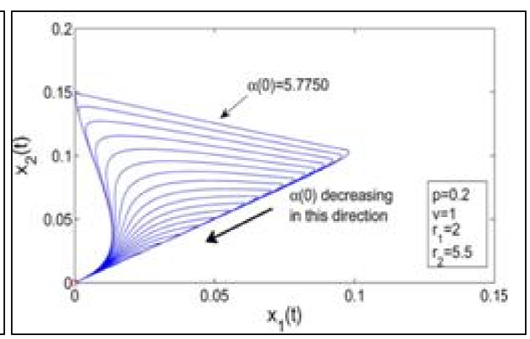
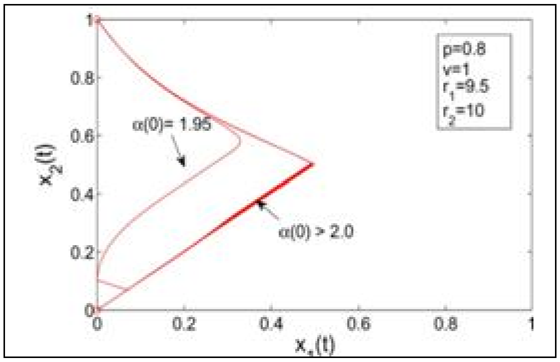


Figure 8: Diffusion dynamics with two entrants r_1=2 and r_2=5.5 with low switching cost (circles are limits of trajectories).
Figure 9: Diffusion dynamics with two entrants r_1=2 and r_2=5.5 with high switching cost (circles are limits of trajectories).

Whenever we are using the three equilibrium points, $(0,0,v)$,$(1,0,r_1)$and $(0,1,r_2)$ for the dynamical system.



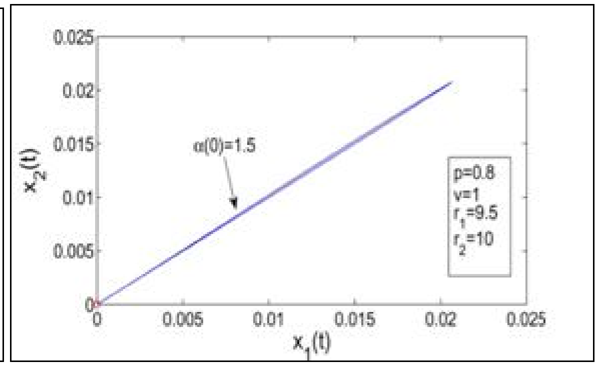


Figure 10: X_2 dominates for α(0)≥1.95 (circles are limits of trajectories).
Figure 11: X_0 dominates for α(0)≤1.5 (circles are limits of trajectories).

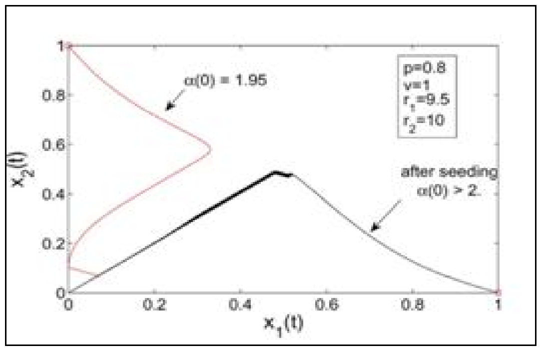


Figure 12: X_1dominates for α(0)>2.0 (circles are limits of trajectories)

Users depending on the initial aspiration level: If the initial aspiration is high, then the superior entrant takes full adoption. If the initial aspiration is low, then the incumbent takes full adoption and the entrants. Whenever when there are two entrants, no new technology defeats the incumbent even for the case $\alpha(0)>r_2$ with $\alpha(0)=5.775$ and $r_2 = 5.5$.
If we illustrate the adoption dynamics when r_2-$r_1$is small, with sufficiently high initial aspiration, $x_1$and x_2 increase together until $x_2>0.5$. Once the superior entrant gets bigger adoption than 0.5, then the inferior entrant loses its market and technology X_2 eventually becomes the winner of the market. In our experiment, we increased x_1 to 0.51when $x_1 \varepsilon (0.48,0.5)$.

5. A Rational Decision Model without Social Influence: A Logic Model
The Logic model is based on the stochastic utility theory in individual decision-making. In the stochastic utility theory, an agent is assumed to behave rationally by selecting the option that brings a high utility.
Let the utilities associated with the choices of A and B are given by adding some random terms as follows:
$V_i=U_i+\varepsilon_i$: the utility of choosing i, i=A,B
Where U_i, i=A,B are deterministic utilities of the choices of A and B and ε_i, i=A,B, are random variables. The probability of choosing A is then given
$\mu=Pr(V_A>V_B)=Pr(U_A+\varepsilon_A>U_B+\varepsilon_B)$
$=Pr(U_A-U_B>\varepsilon_B-\varepsilon_A)$ (12)
Assuming that random variables are independent and follow the gumbel density function
$F(x)=\exp\{-\exp(-x/\lambda)\}$ (13)
We can obtain the next expression by the substitution integration.
$\mu=1/1+\exp\{-(U_A-U_B)/\lambda\}$ (14)
The choice probability of A in the Figure given below as the function of the difference of the utility between the choices of A and B.

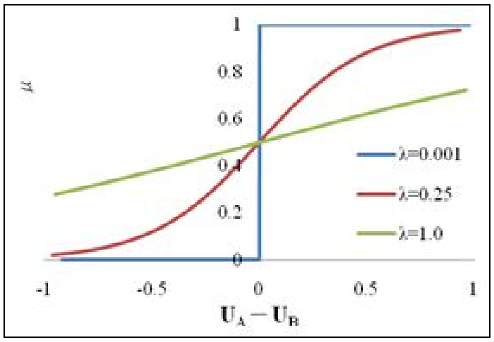


Figure 13: Individual choice probability

5.1. An Individual Decision Model with Social Influence

We assume that there are two factors to individual decisions. The first is based on individual judgment (preference) and the second on social influence. The first factor, we assume that an agent isolated from social influence would choose the objectively the option A with probability μ given in the above figure. Thus P reflects the quality of the evidence about the relative merits of A versus B of each agent. If an agent's preference over A is strong enough, then μ will be close to 1, and if the two alternatives are nearly interchangeable then P will be close to 0.5

Thus, if A_t denotes the number of agents who have chosen A by period t and B_t denotes the number who have chosen B, then the social pressure at time t+1 to an agent who chooses is A is simply the ratio of,

$F_t=A_t/(A_t+B_t)$

We assume that an agent's choice is simply a weighted average of the two factors, individual preference and social influence. Then the probability to choose A is in period t+1 is

P[agent in period t+1 chooses A]=$P_{t+1}=(1-\alpha)\mu+\alpha F_t$ (15)

Where μ is the choice probability of an agent without social influence as given in the above equation, and $\alpha(0<\alpha<1)$ is the strength of social influence.

5.2. Diffusion Dynamics as Sequential Decisions

Here we can investigate properties of the collective decision process when a large number of agents (N=1,000)

First let us examine how it behaves over time by considering first the two extreme cases: $\alpha=0$ and $\alpha=1$.

(1)$\alpha=0$(independent decisions without social influence) In this case, the sequential decision process becomes simply an accumulation of independent decisions.

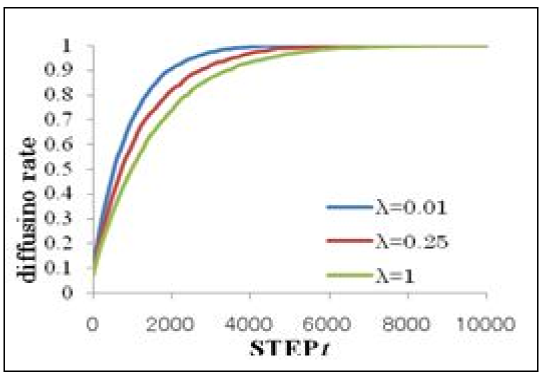


Figure 14

(2)$\alpha=1$(pure social influence) In this case, the sequential diffusion process looks like the well-known the Polya's urn process. That is the expected proportion exactly equals the current proportion with pure social influence ($\alpha=1$).

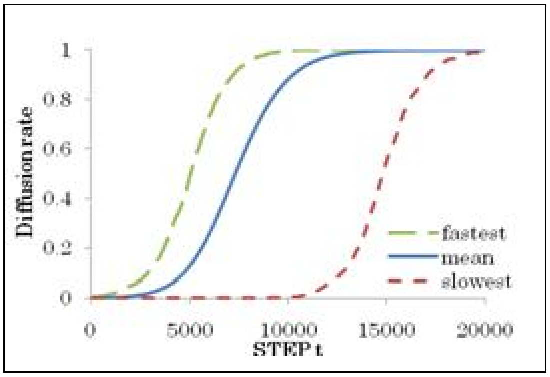


Figure 15

Simulation of the diffusion process for 1,000 rounds. In this figure three sample paths are shown: fastest case, slowest case and the average

If we compare the two processes, the pure social process is less predictable as measured by the penetration rate of F_t. In one case, the diffusion speed becomes relatively high, but in the other case it becomes to be extremely slow.
(3)0<α<1 (mixed decisions with a partial social influence): In this case, the sequential diffusion process becomes a mixed between two extreme cases with α=0 and α=1. If we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters.
We can derive an individual decision rule from the Bass model: the number of individuals who adopt at a given time is a function of the number of individuals who have already adopted.

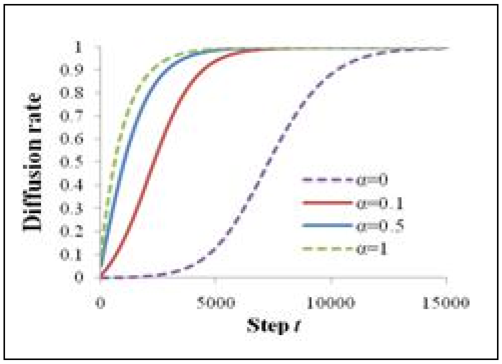


Figure 16

Simulations of the complex process after 1000 rounds. In both cases (α=0.1 and α=0.5) we choose μ=1. As can be seen, the diffusion process becomes for large α, i.e., each agent puts high weight on social trend.

5.3. Diffusion Dynamics on Social Networks

One of the major focuses of this research is the on networks. Here, each node of the network represents a dynamical system. Individual systems are coupled according to the network topology. Thus, the topology of the network remains static while the states of the nodes change dynamically.
We investigate properties of the collective decision process when a large number of agents (N=10,000) sequentially make decisions with the decision rule described. We consider how a new alternative technology A diffuses through a population of agents while all agents enjoy an old technology B.
P[agent in period t+1 chooses A]=$(1-\alpha)\mu+\alpha N_b$
As examples we consider two cases (a) regular networks where all agents are connected to 10 neighbors, (b) complete network where all agents connected to all other agents.
Comparing these results, the fastest diffusion is possible in the case when agents are locally connected. On the other hand, when agents are fully connected, the diffusion process is the slowest and the process is also unstable: in one case it is relatively fast, but in another case it is extremely slow.

Regular Network

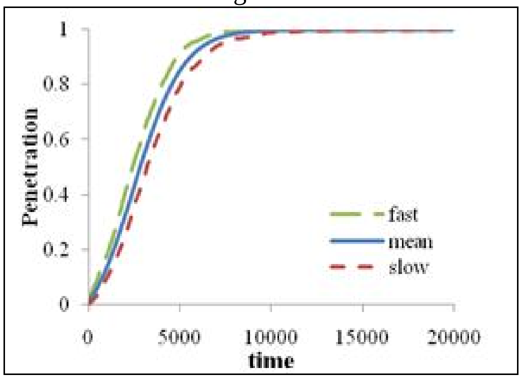


Figure 17

Complete Network

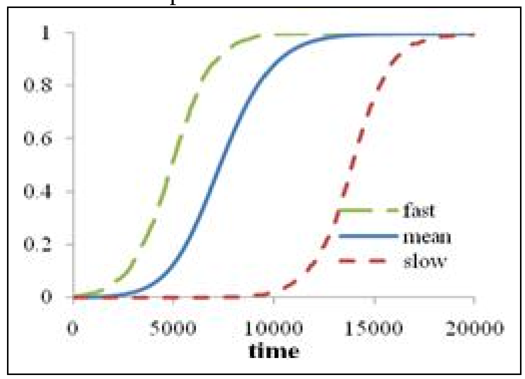


Figure 18

Simulations of the complex process after 1000 rounds. Under the different agent network topologies,: (a) regular network, (b) complete network. In all cases, we set α=1 and μ=1 in the above figure. In each figure three sample paths are shown: fastest case, slowest case and the average.

6. Conclusion

The model developed in this paper is a first step towards better understanding what affects the outcome of competition between an incumbent network technology and an entrant.

We proposed models of diffusion dynamics among users on a fully connected network graph. Our Markov process model, with assumed constant aspiration, explained how the incremental deployment guarantees the success of an entrant technology. In the mean field model, where common aspiration level adapts with time toward averaged payoff, the diffusion dynamics show various behaviors depending on switching cost and initial aspiration level as well as r and v. We also numerically examined the diffusion dynamics with one incumbent and two entrants. If there is little difference between the payoffs from the two entrants, then the competition between two entrants can be intensive.

When agents make choices under strong social influence by referring to social trend, the diffusion process settles down to a value that is not predetermined by the distribution of the agents preferences. The main study on diffusion modeling is based on the Bass model. The Bass diffusion model describes the process how new products get adopted as an interaction between users and potential users. The Bass model assumes all consumers to be homogeneous, and such diffusion models are referred to as aggregate models. If we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters. The number of individuals who adopt at a given time is a function of the number of individuals who have already adopted.

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