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Curvelet Transform Based EEG Signal Compression

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Abstract:
Biomedical signals need to be digitally stored or transmitted with a large number of samples per second, and with a great number of bits per sample, in order to assure the required fidelity of the waveform for visual inspection. Therefore, the use of signal compression techniques is fundamental for cost reduction and technical feasibility of storage and transmission of biomedical signals. Compressive Sensing is an effective method to make data compressed for EEG signals with high compression ratio and good quality of reconstruction. Experimental results show that the curvelet transform compression method performs much better based on Compression Ratio (CR), Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE).

Keywords: *Compressive Sensing (CS), Curvelet Transform, Compression Ratio (CR), Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE)*

1. Introduction

Electroencephalography (EEG) is the technique of measuring electrical signals generated within the brain by placing electrodes on the scalp. The EEG signal produced provides a non-invasive, high time resolution, interface to the brain, and as such the EEG is a key diagnosis tool for conditions such as epilepsy, and it is frequently used in Brain-Computer Interfaces [1]. EEG compression is achieved by exploiting correlation (redundancy) in the source data. The compressibility of EEG depends on its amplitude distribution and its power spectrum. EEG is not usually considered sufficiently sparse in time or frequency domains for matching the recovery requirements of the clinical practice. However, filtered EEG show an amplitude distribution and a frequency spectrum largely concentrated in suitable ranges [2]. EEG compression schemes have achieved up to 65% data reduction with lossless compression [3], and up to 89% data reduction when lossy compression is employed [4].

Compressive sensing method is to make the signal transform into low dimensional measurement domain with under-sampling and it is also known as compressive sampling in the recent years [5–6]. Just as the bandwidth to the Nyquist-Shannon sampling theory, sparsity of the signal is the essential condition to Compressive Sensing [6]. The relevance of using compressive sensing in these signals is double: On one hand it has been previously reported in [7] that EEG signals meet the necessary requirements to ensure reconstruction after compression when projected in certain basis. Hence compressive sensing appears as a very attractive technique to reduce the power consumption and thus the size of future miniaturized EEG systems, which could be used in a variety of applications ranging from long term medical monitoring [8] to brain computer interfaces [9]. The concept of compressive sensing [10] is based on the fact that there is a difference between the rate of change of a signal and the rate of information in the signal. Traditional Nyquist sampling, putting the signal into the digital domain ready for wireless transmission, is based on the former. A conventional compression algorithm would then be applied to all of these samples taken to remove any redundancy present, giving a reduced number of bits that represent the signal. Compressive sensing [11] is a novel technique which suggests random acquisition of the non adaptive linear projection at lower than the Nyquist rate, which preserves signal structure. By using an optimization problem the signal is reconstructed.

Wavelet Transform (WT) is a powerful time-frequency signal analysis tool and it is used in a wide variety of applications including signal and image coding [12]. Wavelet Transform and Subband Coding (SBC) are closely related to each other. In fact the fast implementation of Wavelet Transforms is carried out using Subband (SB) filter banks. Due to this reason Wavelet Transform based waveform coding methods are essentially similar to the SBC based methods.

Curvelet transform has undergone a major revision since its invention. The first generation curvelet transform is based on the concepts of ridgelet transform [13]. The curve singularities have been handled by smooth partitioning of the bandpass images. In each smooth partitioned block the curve singularities can be approximated to a line singularity. A ridgelet transform is applied on these small blocks, where ridgelets can deal the line singularities effectively. To avoid blocking artifacts, the smooth partitioning is done on overlapping blocks which results in redundancy, and the whole process involves subband decomposition using wavelet transform, smooth partitioning and ridgelet analysis on each block; this process consumes more time. The implementation of second generation curvelet transform is based on the Fourier transform and is faster, less complex, and less redundant.

2. Signal Compression

Signal compression is the process where the redundant information contained in the signal is detected and eliminated. The aim of any biomedical signal compression scheme is to minimize the storage space without losing any clinically significant information, which can be achieved by eliminating redundancies in the signal, in a reasonable manner. The purpose of compression is three-fold:1) to reduce the volume of data to be transmitted, 2) to reduce the bandwidth required for transmission, 3) to reduce the storage requirements. Fig 1 shows the block diagram of data compression.

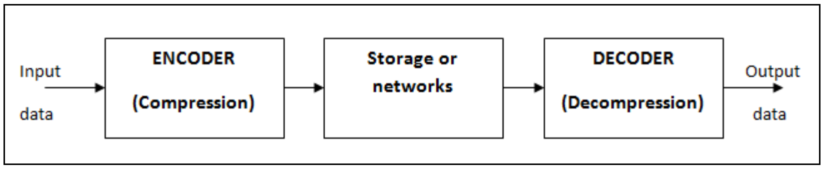

Figure 1: General Data Compression Scheme

The purpose of any signal compression technique is the reduction of the amount of bits used to represent a signal. This must be accomplished while preserving the morphological characteristics of the waveform. Signal compression techniques are commonly classified in two categories: lossless and lossy compression.

The design of data compression schemes involves trade-offs among various factors, including the degree of compression, the amount of distortion introduced, and the computational resources required to compress and uncompress the data.

2.1. Lossless compression

In lossless data compression, the integrity of the data is preserved. The original data and the data after compression and decompression are exactly the same because, in these methods, the compression and decompression algorithms are exact inverses of each other: no part of the data is lost in the process. Redundant data is removed in compression and added during decompression.

2.2. Lossy compression

Our eyes and ears cannot distinguish subtle changes. In such cases, we can use a lossy data compression method. In lossy compression, a controlled amount of distortion is allowed. The reconstructed signal is allowed to differ from the original signal. Lossy signal compression techniques show higher compression gains than lossless ones. Apart from obtaining good compression ratios with imperceptible degradation of signal quality, data reduction techniques should also hold low computational costs; particularly if they are going to be implemented on portable devices. Fig 2 shows the schematic of lossless and lossy compression.

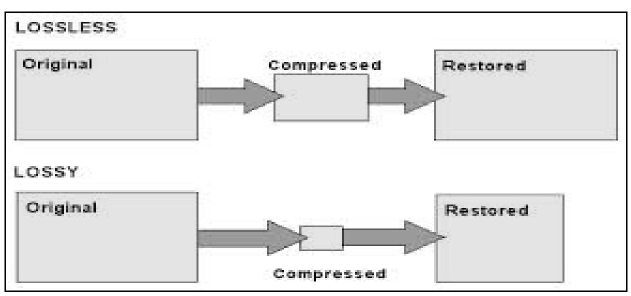

Figure 2: Lossless and Lossy compression

EEG signals have non-stationary behaviour; it means the behaviour through the time is changing every time window. For this reason, the pre-processing, processing, and analysis should be different of the deterministic and stationary signals. EEG signals can be compressed in the following domains: time domain, frequency domain and time-frequency doma

- **Time-domain EEG compression**
  Generally, most of the techniques proposed in the literature devoted to EEG compression are mainly prediction based. This can be explained by the fact that the EEG is a low-frequency signal, which is characterized by a high temporal correlation. Some of these techniques are in fact a direct application of classical digital signal processing methods. These include Linear Prediction Coding (LPC), Markovian Prediction, Adaptive Linear Prediction and Neural Network Prediction based methods. On the other hand, some approaches include the information related to the long-term temporal correlation of the samples. In fact, if we analyze the correlation function of an EEG segment, we will note that spaced samples present a non-neglected correlation that should be taken into account during processing. This information might be integrated into various dedicated

codecs. Finally, we can also evoke the techniques which consist of correcting the errors of the prediction using information intrinsic to the EEG.

- **Frequency-domain EEG compression**
  The compression of the EEG in the frequency domain did not come from classical techniques such as Karhunen-Loève Transform (KLT) or the Discrete Cosine Transform (DCT). The EEG signal is dominated by low frequencies, mainly lower than 20 Hz. In fact, it is considered that the main energy is located around the alpha rhythm (between 8 Hz and 13 Hz).
- **Time-frequency domain EEG compression**
  Among the time-frequency techniques, the wavelet transform has been commonly used to compress the EEG. In this technique, the signal is segmented and decomposed using Wavelet Packets. The coefficients are coded afterwards. Other algorithms such as the well known EZW (Embedded Zerotree Wavelet) have also been successfully applied to compress the EEG signal.

3. Compressive Sensing

Compressive sensing is a useful tool for eliminating the inefficiencies caused by traditional signal processing algorithms, because 1) it offers simpler hardware implementation for encoder, as it transforms its computational burden from encoder to decoder, 2) no need to encode the location of the largest coefficients in the wavelet domain, 3) its ability to reconstruct the signal from significantly fewer data samples compared to conventional Nyquist sampling theory.

Compressed Sensing is about acquiring and recovering a sparse signal in the most efficient way possible with the help of an incoherent projecting basis.

- The signal needs to be sparse
- The technique acquires as few samples as possible
- Later, the original sparse signal can be recovered
- This done with the help of an incoherent projecting basis

4. Curvelet Transform Based Compression

The Curvelet transform is a higher dimensional generalization of the wavelet transform designed to represent images at different scales and different angles. Curvelet transform is a special member of the multi scale geometric transforms. Basis functions of wavelet transform are isotropic and thus it requires large number of coefficients to represent the curve singularities. Curvelets obey parabolic scaling. Because of these properties, curvelet transform allows almost optimal sparse representation of curve singularities. It is a transform with a multi scale pyramid with many directions at each length scale.

4.1. Curvelet Transform

The overview of the curvelet transform is shown below:

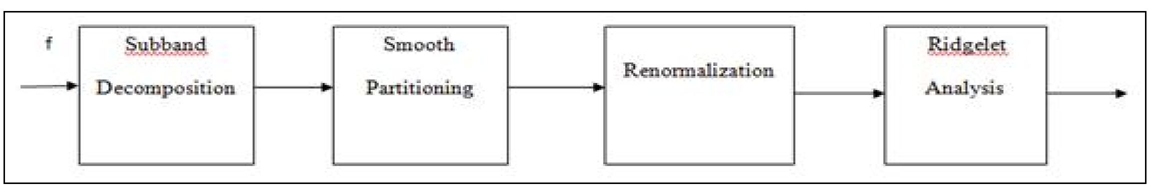

Figure 3: Curvelet Transform

Curvelet coefficients can be obtained from scaling and windowing function. Window frame's width and length is selected by the relation:

$\text{Width} = \text{Length}^2$ (1)

This is also called as curvelet scaling law. This law gives better advantage of curvelet transform over wavelet transform because wavelet coefficient window has directly proportional relation between width and length. Also there are other advantages of curvelet over wavelet transform like PSNR and edge representation.

4.1.1. Sub-band decomposition

We define a bank of subband filter P_0, (Δ_s, s ≧0). The object f is filter into subbands:

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \ldots)$$

This step divides the image into sevel resolution layers. Each layer contains details of different frequencies:

- P_0 → Lowpass filter
- $\Delta_1, \Delta_2, \ldots$ – Band-pass (high-pass) filters.

Φ_0 : A lowpass filter. The filter deal with low frequency near $|\xi| \le 1$

Ψ_{2s} : The band pass filters. The filter deal with frequencies near the domain $|\xi| \in [2^{2s}, 2^{2s+2}]$.

Besides, there is Recursive construction: $\Psi_{2s}(x) = 2^{4s}\,\Psi(2^{2s}x)$.
The sub-band decomposition is simply applying a convolution operator:

$$P_0 f = \Phi_0 * f \qquad \Delta_s f = \Psi_{2s} * f \tag{20}$$

4.1.2. Smooth Partitioning

It is defined as a collection of smooth window $\mathbf{w_Q(x_1 , x_2)}$ localized around dyadic squares:

$$Q_{(s,k_1,k_2)} = \left[\frac{k_1}{2^s}, \frac{k_1+1}{2^s}\right] \times \left[\frac{k_2}{2^s}, \frac{k_2+1}{2^s}\right] \in \mathbf{Q}_s \tag{2}$$

Let **w** be a smooth windowing function with 'main' support of size $2^{-s} \times 2^{-s}$. Multiplying a funtion by the corresponding window function $\mathbf{w_Q}$ produces a result localized near Q ($\forall\, Q \in Q_s$). Doing this for all Q at a certain scale, i.e. all Q=Q(s , k_1 , k_2) with k_1 and k_2 varying but s fixed, procedure, we apply this windowing dissection to each of the subbands isolated in the previous stage of the algorithm. This step produces a smooth dissection of the function into 'squares'.

$$h_Q = w_Q \cdot \Delta_s f \tag{3}$$

The image become smooth after multiplying w_Q function. The partitioning make us more easier to analyze local line or curve singularities.

4.1.3. Renormalization

For a dyadic square Q, let

$$T_Q f(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \tag{4}$$

denote the operator which transports and renormalizes f so that the part of the input supported near Q becomes the part of the output supported near [0,1]×[0,1]. In this stage of the procedure, each 'square' resulting in the previous stage is renormalized to unit scale:

$$g_Q = T_Q^{-1} h_Q \tag{5}$$

4.1.4. Ridgelet Analysis

Each 'square' is analyzed in the orthonormal ridgelet system. This is a system of basis element ρ_λ making an orthonomal basis for $L^2(\Re^2)$. There are some frequency domain analysis for Ridgelet. The ridgelet construction divides the frequency domain to dyadic coronae $|\xi| \in [2^s, 2^{s+1}]$. In the angular direction, it samples the *s*-th corona at least 2^s times. In the radial direction, it samples using local wavelets.
The ridgelet element has a formula in the frequency domain:

$$\hat{\rho}_\lambda(\xi) = \tfrac{1}{2}|\xi|^{-\frac{1}{2}}\left(\hat{\psi}_{j,k}(|\xi|)\cdot \omega_{i,l}(\theta) + \hat{\psi}_{j,k}(-|\xi|)\cdot \omega_{i,l}(\theta+\pi)\right) \tag{6}$$

- $\omega_{i,l}$: periodic wavelets for $[-\pi, \pi)$.
- i : the angular scale , $l \in [0, 2^{i-1}-1]$: the angular location.
- $\psi_{j,k}$: Meyer wavelets for $\Re$.
- j : the ridgelet scale , k : the ridgelet location.

Each normalized square is analyzed in the ridgelet system:

$$\alpha_{(Q,\lambda)} = \langle g_Q, \rho_\lambda \rangle \tag{7}$$

- The ridge fragment has an aspect ratio of $2^{-2s} \times 2^{-s}$.
- After the renormalization, it has localized frequency in band $|\xi| \in [2^s, 2^{s+1}]$.
- A ridge fragment needs only a very few ridgelet coefficients to represent it.

4.2. Inverse Curvelet Transform

Inverse the procedure of curvelet transform with some mathematic revising:

4.2.1. Ridgelet Synthesis

Each ‘square’ is reconstructed from the orthonormal ridgelet system. Summation all the Ridgelet coefficinets with basis:

$$g_Q = \sum_{\lambda} \alpha_{(Q\lambda)} \cdot \rho_\lambda \qquad (8)$$

4.2.2. Renormalization

Each ’square’ resulting in the previous stage is renormalized to its own proper square.

$$h_Q = T_Q g_Q \quad , \quad Q \in Q_s \qquad (9)$$

4.2.3. Smooth Integration

We reverse the windowing dissection to each of the windows reconstructed in the previous stage of the algorithm.

$$\Delta_s f = \sum_{Q \in \mathbf{Q}_s} w_Q \cdot h_Q \qquad (10)$$

4.2.4. Subband Recomposition

We undo the bank of subband filters, using the reproducing formula to summation all the subbands:

$$f = P_0(P_0 f) + \sum_{s} \Delta_s(\Delta_s f) \qquad (11)$$

5. Performance Evaluation

Depending on the nature of the application there are various criteria to measure the performance of a compression algorithm.

- Mean Square Error (MSE)
- Peak Signal to Noise Ratio (PSNR)
- Compression Ratio (CR)

5.1. Mean Square Error

Mean Square Error is defined as follows:

$$\sigma^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - y_n)^2 \qquad (12)$$

Where x_n, y_n and N are the input data sequence, reconstructed data sequence and length of the data sequence respectively.

5.2. Peak Signal to Noise Ratio

Peak Signal to Noise Ratio is defined as follows:

$$PSNR = 10\log_{10}\frac{x_{peak}^2}{\sigma_d^2} \quad (13)$$

where σ_d^2 is the MSE.

5.3. Compression Ratio

Compression Ratio (CR) is the most important parameter in data compression algorithms. The amount of compression is measured by Compression Ratio. High Compression Ratio leads to a better response. It is the ratio between the numbers of bits before compression to that after compression.

$$CR = \frac{\text{number of bits of original signal}}{\text{number of bits of compressed signal}} \quad (14)$$

6. Results

The simulation results obtained using MATLAB show that compressive sensing is an effective method to make data compressed for EEG signals with high compression ratio and good quality of reconstruction. First of all, the EEG signal is compressed using Compressive Sensing. The sparse signal is thus obtained. The Discrete Curvelet Transform of the sparse signal is obtained. Afterwards, Inverse Discrete Curvelet Transform is used to reconstruct the coefficients. The simulation results are shown in Fig 4. Fig. 4(a) is the original EEG signal, and Fig. 4(b) shows the compressive sensed signal. Fig. 4(c) shows the reconstructed EEG signal.

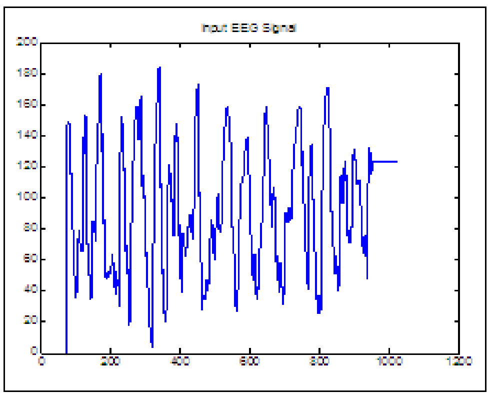

Figure 4(a) Original EEG signal

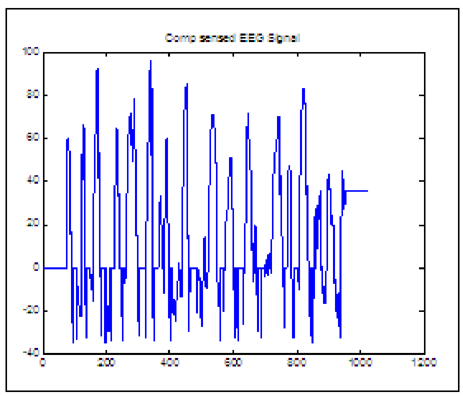

Figure 4(b) Compressive sensed signal

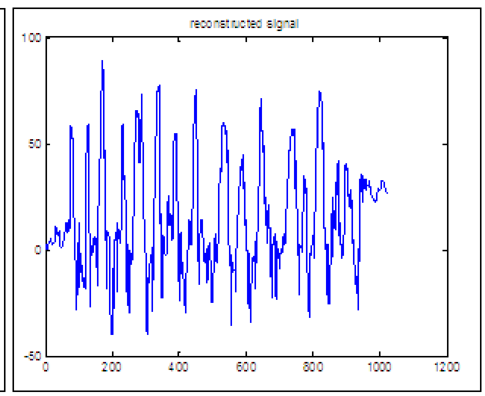

Figure 4(c) Reconstructed signal

Table 1 shows the performance of curvelet transform compression method based on Compression Ratio (CR), Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE).

Input signal	Compression Ratio	PSNR in decibels	MSE
eeg_signal1 (awake)	17.16	40.24	6.16
eeg_signal2 (drowsy)	17.05	41.04	5.12
eeg_signal3 (sleep stage-1)	16.95	42.17	3.94
eeg_signal4 (sleep stage-2)	17.25	35.67	17.61
eeg_signal5 (deep sleep)	17.5	31.7	43.93
eeg_signal6 (REM sleep)	16.58	44.61	2.25

Table 1: Performance of curvelet transforms method based on CR, PSNR and MSE

7. Conclusion and Future Work

EEG is not only a key diagnostic tool for neurologists, but it is growingly used in Brain-Computer Interfaces (BCI) applications. The traditional approach to EEG signal processing is to perform Nyquist sampling on the band-limited version of the signal. Compressive Sensing is a useful tool for eliminating the inefficiencies caused by traditional signal processing algorithms. Compressive Sensing is an effective method to make data compressed for EEG signals with high compression ratio and good quality of reconstruction. Experimental results show that the curvelet transform compression method performs much better based on Compression Ratio (CR), Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE).

The compressive sensing (CS) methodology can be used in EEG analysis for discriminating among different, possibly pathological, brain states. It can be used for diagnosing and controlling Alzheimer's disease patients. An interesting alternative to use the CS derived compression coefficients can be to use a set of connection weights extracted from a trained Spiking Neural Network. This approach can certainly be the object of further studies.

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