INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

Buckling Analysis of SSSS Stiffened Rectangular Isotropic Plates using Work Principle Approach

Ibearugbulem, M. O.
Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria.
Ibeabuchi, V. T.
Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria.
Njoku, K. O.
Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria.

Abstract:
The objective of this work is to present Elastic Buckling of Stiffened Rectangular Isotropic Plates using Work principle Approach. Stiffened plates possessing different aspect ratios, varying stiffness properties and varying number of stiffeners were analyzed for critical buckling loads using Work Principle approach. The present analysis was carried out only for uniaxially stiffened plate, where longitudinal stiffeners are presented parallel to inplane load of the plate. The governing differential equation for the stiffened plate system was obtained by super position principle. Polynomial functions were used in this study. Effects of the number of stiffeners, aspect ratios, boundary conditions, stiffener parameters upon the buckling coefficients, K of the stiffened plates were investigated. The results were obtained considering the bending displacements of the plate and the stiffener for all cases of edge conditions presented. Maximum percentage difference in buckling coefficients of SSSS stiffened plate for the case of one stiffener of present work with Reference works is 0.522%. For the case of two stiffeners, the maximum percentage differences in buckling coefficients recorded is 0.5247%, also for the case of three stiffeners maximum percentage difference is 1.239%. These differences revealed a good agreement with previous research works. In the structure modeling, the plate and the stiffeners were treated as a unit member.

Keywords*: Stiffened plate, Buckling, Work principle, Governing differential equation, Shape function.*

1. Introduction

Stiffened plates have wide applications in many civil engineering, aerospace and marine structures. They are used in box girders, plate girders, ship hulls and wing structures. Interest in stiffened plate construction has been widespread in recent years due to its economic and structural benefits. The advantage of stiffening a plate lies in achieving an economical, light weight design. While the stiffening elements add negligible weight to the overall structure, their influence on strength and stability is enormous.

The conventional methods of analysis includes, Energy approach, Orthotropic plate approach, and Numerical approach.

Orthotropic plate is somewhat convenient especially when the stiffeners are of equal spaced. In this method, stiffened plate is converted into an equivalent plate with constant thickness by smearing out the stiffeners. The resulting idealized structure is therefore composed of the original plate layer and additional layer Bedair, (1998). This model is however justified if the stiffeners are closely spaced, also further difficulties could appear if the stiffeners are not identical in both directions or not equally spaced, since the resulting thickness becomes non-uniform. In addition, the method does not take into account the discrete nature of the structure (Timoshenko and Gere, 1961 and Hughes et al., 2004).

Timoshenko and Gere, (1961) used energy approach to solve stability problem of stiffened plates with all the four edges simply supported, they assumed the trigonometric shape function in their work. However, in the analysis of stiffened plates by Energy method, it is a common knowledge that only trigonometric shape functions have been used (Bulson, 1970; Cox, 1954; Szilad 2004).

The accuracy of Numerical approaches such as finite element and finite difference methods depends on the number of finite units created (Mukhopadhyay, 1989; Mukhopadhyay, 1990; Shahabian and Shahasavandi, 2008; Liu et al., 2012).

In recent works Ibearugbulem (2012), Ezeh et al. (2013), Njoku et a.l (2013) have used Polynomial shape function in the analysis of thin plates of various boundary conditions. Practical situations are more likely to involve the consideration of plates incorporating some form of stiffeners.

2. Governing Differential Equation

Consider a plate of length a, width b and thickness t having two longitudinal stiffeners element as shown in Figure 1. According to Wutzow and De Paiva (2008) Stiffeners are as a rule, linear element, nearly always of negligible thickness. In this study, stiffeners are considered as line continuum.

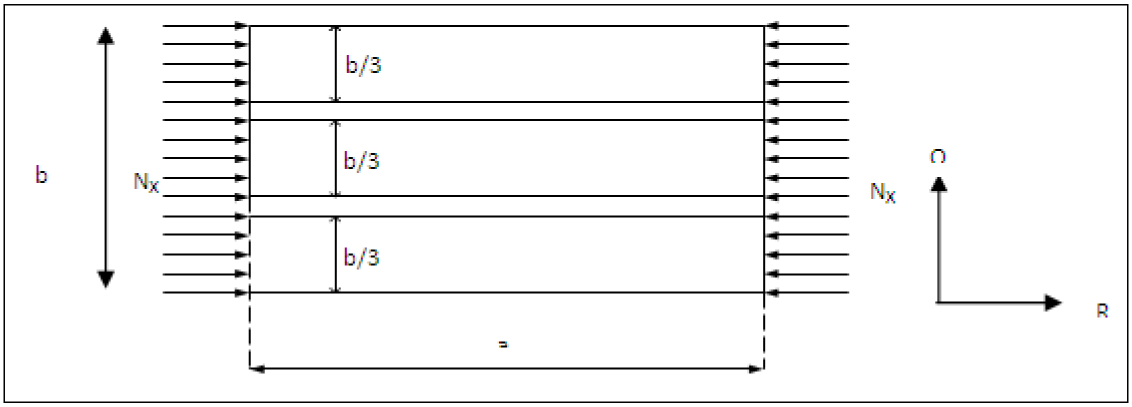

Figure 1: A Stiffened System under in plane load

From the principles of the theory of elasticity, the governing equation for stiffened rectangular isotropic plates was given by Ibeabuchi (2014) as;

$$\frac{1}{P^4}\frac{\partial^4 w}{\partial R^4} + \frac{2}{P^2}\cdot\frac{\partial^4 w}{\partial R^2 \partial Q^2} + \frac{\partial^4 w}{\partial Q^4} + \frac{1}{P^4}\cdot\sum_{i=1}^{n}\gamma_i\left(\frac{\partial^4 w}{\partial R^4}\right)_{Q=ci} + \frac{b^2}{P^2}\cdot\frac{N_x}{D}\cdot\frac{\partial^2 w}{\partial R^2} + \frac{b^2}{P^2}\cdot\frac{N_x}{D}\cdot\sum_{i=1}^{n}\delta_i\left(\frac{\partial^2 w}{\partial R^2}\right)_{Q=ci} = 0 \quad (1.0)$$

where; Q and R are non dimensional parameter

$Q = \frac{y}{b}$ that is $y = bQ$; $R = \frac{x}{a}$ that is $x = aR$

$\gamma_i = \frac{EI_i}{Db}$ = Ratio of bending stiffness rigidity of stiffeners to the plate

$\delta_i = \frac{A_i}{bh}$ = Ratio of cross-sectional area of the stiffeners to the plate

2.1. Work Principle

Work is defined mathematically as the product of average force and distance travelled by the force. Hence, for the combined action of work done by the compressive and resistive force on the stiffened system through a distance w, equation (1.0) becomes;

$$\frac{1}{2}\left[\frac{A^2}{P^4}.H\frac{\partial^4 H}{\partial R^4} + \frac{2A^2}{P^2}.H\frac{\partial^4 H}{\partial R^2 \partial Q^2} + A^2\frac{H.\partial^4 H}{\partial Q^4} + \frac{A^2}{P^4}\cdot\sum_{i=1}^{n}\gamma_i\left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci} + \frac{b^2}{P^2}\cdot\frac{N_x A^2}{D}.H\frac{\partial^2 H}{\partial R^2} + \frac{b^2}{P^2}\cdot\frac{N_x}{D}.A^2\sum_{i=1}^{n}\delta_i\left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}\right] = \text{ei} \quad (2.0)$$

Where; Deflection function, W = AH; A = coefficient; H is the shape function: ei is the introduced error, "i" is the number of points on the continuum.

Integrating equation (3.100) twice with respect to R and Q gave;

$$\Pi = \frac{A^2}{2}\int_0^1\int_0^1\left[\frac{H}{P^4}\cdot\frac{\partial^4 H}{\partial R^4} + \frac{2H}{P^2}\cdot\frac{\partial^4 H}{\partial R^2 \partial Q^2} + \frac{H.\partial^4 H}{\partial Q^4} + \frac{1}{P^4}\cdot\sum_{i=1}^{n}\gamma_i\left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci}\right]\partial R\partial Q + \frac{A^2 b^2}{2P^2}\cdot\frac{N_x}{D}\int_0^1\int_0^1\left[\frac{H.\partial^2 H}{\partial R^2} + \sum_{i=1}^{n}\delta_i\left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}\right]\partial R\partial Q \quad (3.0)$$

$\prod$ is the Total Work error Functional. Minimizing and making N_x the subject of equation (3.0) gave;

$$N_{x(cri)} = \frac{D\int_0^1\int_0^1\left[\frac{H}{P^2}\cdot\frac{\partial^4 H}{\partial R^4} + 2\text{H}.\frac{\partial^4 H}{\partial R^2\partial Q^2} + \text{P}^2\frac{H.\partial^4 H}{\partial Q^4} + \frac{1}{P^2}.\Sigma_{i=1}^{n}\gamma_i\left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci}\right]\partial R\partial Q}{-b^2\int_0^1\int_0^1\left[\frac{H.\partial^2 H}{\partial R^2} + \Sigma_{i=1}^{n}\delta_i\left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}\right]\partial R\partial Q} \quad (4.0)$$

Equation (4.0) is the Buckling equation for rectangular plate stiffened longitudinally under in-plane loading.

2.2. Boundary Conditions for SSSS Stiffened Plates

A Stiffened plate with simply supported edges and dimensionless R-Q axes is shown in Figure 2. The boundary conditions in dimensionless coordinate system are given below as;

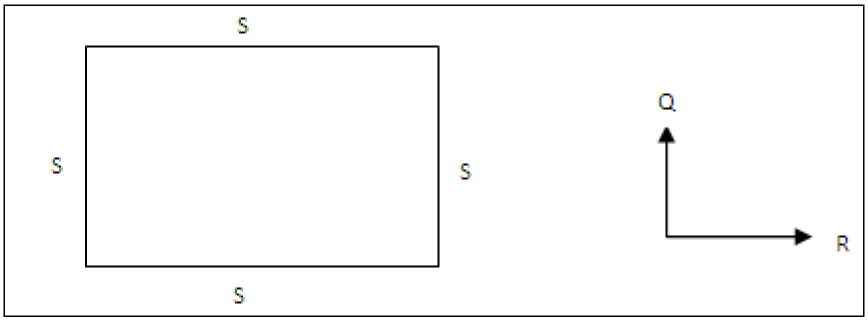

Figure 2: Longitudinally stiffened rectangular plate with edge numbers

$$\left.\begin{aligned} w(R=0) &= \frac{\partial^2 w}{\partial R^2}(R=0) = 0\,;\, w(R=1) = \frac{\partial^2 w}{\partial R^2}(R=1) = 0 \\ w(Q=0) &= \frac{\partial^2 w}{\partial Q^2}(Q=0) = 0\,;\, w(Q=1) = \frac{\partial^2 w}{\partial^2 Q}(Q=1) = 0 \end{aligned}\right\}$$

[redacted]

2.3. Polynomial Shape Function

Ibearugbulem in 2012 formulated the general shape function for rectangular plates from Taylor-McLaurin's series as;

$$w = \sum_{m=o}^{4}\sum_{n=o}^{4} a_m b_n R^m Q^n \quad (7.0)$$

For M = N = 4

Substituting $w(R=0) = 0$ and $\frac{\partial^2 w}{\partial^2 R}(R=0) = 0$ into equation (7.0) gave;

$$a_0 = 0;\ a_2 = 0 \quad (8.0)$$

Similarly, substituting $w(Q=0) = 0\ and\ \frac{\partial^2 w}{\partial^2 Q}(Q=0) = 0$ into equation (7.0) respectively gave;

$$b_0 = 0;\ b_2 = 0 \quad (9.0)$$

Substituting $w(R=1) = 0\ and\ \frac{\partial^2 w}{\partial R^2}(R=1) = 0$ into equation (7.0) respectively gave;

$$a_1 + a_3 + a_4 = 0\ and\ 6a_3 + 12a_4 = 0 \quad (10.0)$$

Solving the two equations simultaneously gave;

$$a_1 = a_4;\ a_3 = -2a_4 \quad (11.0)$$

Similarly, substituting $w(Q=1) = 0\ and\ \frac{\partial^2 w}{\partial Q^2}(Q=1) = 0$ into equation (7.0) gave;

$$b_1 = b_4;\ b_3 = -2b_4 \quad (12.0)$$

Substituting the values of a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3 and b_4 into (7.0) gave;

$$w = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)a_4 b_4$$

$$w = A(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \quad (13.0)$$

$$where\ A = a_4 b_4$$

3. Formulation of Stability Equation for SSSS Stiffened Plates

Three cases of SSSS stiffened plates are considered as shown in figure 3.

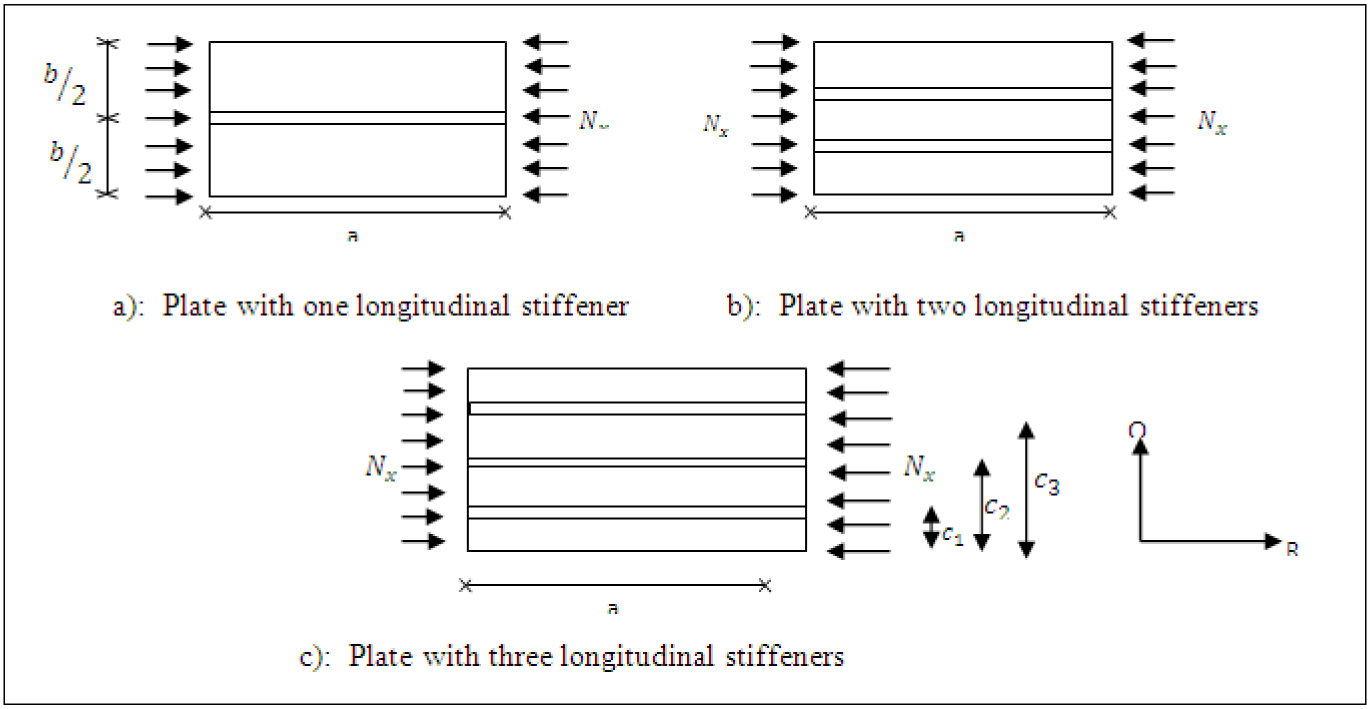

Figure 3: Stiffeners Arrangement for SSSS Stiffened Plates

3.1. Case of One Stiffener
Consider figure 3.0(a), the stiffener divides the plate into two equal widths.
From equation (13.0) deflection function, H is given as;

$$H = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$$

FOR THE STIFFENER /RIB
For $1 \leq Q \leq 1$, we have;

$$(H)_{Q=c1} = (H)_{Q=1/_2} = (R - 2R^3 + R^4)\left(1/_2 - 2/_8 + 1/_{16}\right)$$

$$= 0.3125\,(R - 2R^3 + R^4) \qquad (14.0)$$

$$\int_0^1\int_0^1 \left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=1/_2} = -0.047433 \qquad (15.0)$$

$$\int_0^1\int_0^1 \left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=1/_2} = 0.46875 \qquad (16.0)$$

FOR THE PLATE ELEMENT

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial R^4}\partial R\partial Q = 0.23619 \qquad (17.0)$$

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q = 0.23619 \qquad (18.0)$$

$$\int_0^1\int_0^1 H.\frac{\partial^4 H}{\partial R^2\partial Q^2}\partial R\partial Q = 0.23592 \qquad (19.0)$$

$$\int_0^1\int_0^1 H.\frac{\partial^2 H}{\partial R^2}\partial R\partial Q = -0.023900 \qquad (20.0)$$

Applying equations (15.0), (16.0), (17.0), (18.0), (19.0), (20.0) into (5.0), gave;

$$\therefore N_x = \frac{D\left[\frac{0.23619}{P^2} + 2(0.23592) + P^2(0.23619) + \frac{(0.46875)}{P^2}\gamma\right]}{-b^2[-0.02390 - 0.047433\delta]}$$

$$N_x = \frac{\pi^2 D}{b^2P^2}\frac{[(1+P^2)^2 + 1.987\,\gamma]}{[1+1.985\delta]} \qquad (21.0)$$

Where,

$$K = \frac{[(1+P^2)^2 + 1.987\,\gamma]}{P^2[1+1.985\delta]} \qquad (22.0)$$

4. Stiffeners Case of Two

Consider figure 3.0(b), the stiffeners divides the plates into three equal parts.

$$C_1 = \frac{1}{3},\quad C_2 = \frac{2}{3}$$

Where C_1 *and* C_2 are the distances of the stiffeners from the edge y = 0.

Assuming stiffeners are symmetrical, hence;

$\delta_1 = \delta_2$ *and* $\gamma_1 = \gamma_2$

Following the same procedure in the case of one stiffener, we obtain the equations as follows;

$$\int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C1}\partial R\partial Q = \int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C2}\partial R\partial Q = -0.0358308 \qquad (23.0)$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C1}\partial R\partial Q = \int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C2}\partial R\partial Q = 0.3540923 \qquad (24.0)$$

Substituting equations (17.0), (18.0), (19.0), (20.0), (23.0), and (24.0) into (5.0), gave;

$$\therefore N_x = \frac{D\left[\frac{0.23619}{P^2} + 2(0.23592) + P^2(0.23619) + (2*0.3540923)^{\gamma}/_{P^2}\right]}{-b^2[-0.0239 - 0.3540923*2\delta]}$$

$$\therefore N_x = \frac{\pi^2 D}{b^2P^2}\frac{[(1+P^2)^2 + 3\gamma]}{[1+2.9630\delta]} \qquad (25.0)$$

$$K = \frac{[(1+P^2)^2 + 3\gamma]}{P^2[1+2.9630]} \qquad (26.0)$$

5. Case of Three Stiffeners

Consider figure 3.0(c), the stiffeners divides the plate into four equal parts, thus;

$$C_1 = \frac{1}{4},\quad C_2 = \frac{1}{2}, C_3 = \frac{3}{4}$$

$$\delta_1 = \delta_2 = \delta_3 = \delta;\quad \gamma_1 = \gamma_2 = \gamma_3 = \gamma$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C1}\partial R\partial Q = \int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C3}\partial R\partial Q = -0.022225 \qquad (27.0)$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C1}\partial R\partial Q = \int_0^1\int_0^1\left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=C3}\partial R\partial Q = 0.237964 \qquad (28.0)$$

$$\int_0^1\int_0^1\left(H.\frac{\partial^2 H}{\partial R^2}\right)_{Q=C1}\partial R\partial Q = -0.047433 \qquad (29.0)$$

$$\int_0^1\int_0^1 \left(H.\frac{\partial^4 H}{\partial R^4}\right)_{Q=c3} \partial R \partial Q = 0.46875 \quad (30.0)$$

$$\sum_{i=1}^{3}\int_0^1\int_0^1 \delta_i \left(\frac{H.\partial^2 H}{\partial R^2}\right)_{Q=ci}^{\partial R\partial Q} 2(-0.022225)\delta + (-0.047433)\delta = -0.091883\delta \quad (31.0)$$

$$\sum_{i=1}^{3}\int_0^1\int_0^1 \gamma_i \left(\frac{H.\partial^4 H}{\partial R^4}\right)_{Q=ci}^{\partial R\partial Q} 2(-0.237964)\gamma + (0.046875)\gamma = -0.944678\gamma \quad (32.0)$$

Substituting equations (17.0), (18.0), (19.0), (20.0), (31.0), and (32.0) into (5.0), gave;

$$\therefore N_x = \frac{D\left[\frac{0.23619}{P^2} + 2(0.23592) + P^2(0.23619) + (0.944678)\gamma\right]}{-b^2[-0.02390 - 0.091883\delta]}$$

$$= \frac{\pi^2 D}{b^2 P^2}.\frac{[(1+P^2)^2 + 4.0048\gamma]}{[1+3.84445\delta]} \quad (33.0)$$

$$K = \frac{[(1+P^2)^2 + 4.0048\gamma]}{P^2[1+3.84445\delta]} \quad (34.0)$$

6. Results and Discussion

The buckling coefficient, K for SSSS stiffened plate for the three cases considered is presented, as well as comparism with other published works.

Szilad (2004) used single fourier series to obtain buckling coefficient for a plate with one central longitudinal stiffener as;

$$\lambda = \frac{(1+\beta^2)^2 + 2\gamma}{\beta^2(1+2\delta)} \quad (35.0)$$

Where; $\boldsymbol{\lambda}$ = buckling coefficient

Timoshenko and Gere (1961) applied Energy Principle in Ritz method to obtain buckling coefficients for the three cases considered as follows;

For the cases of one, two and three longitudinal stiffeners, K was given as;

$$K = \frac{(1+\beta^2)^2 + 2\gamma}{\beta^2(1+2\delta)}\ ;\quad K = \frac{(1+\beta^2)^2 + 3\gamma}{\beta^2(1+3\delta)}\ ;\quad K = \frac{(1+\beta^2)^2 + 4\gamma}{\beta^2(1+4\delta)} \quad (36.0)$$

Where; β represents aspect ratio, γ represents the ratio of flexural rigidity of stiffener to that of plate, **δ** represents the cross-sectional area of stiffener to that of plate.

- Comparison of K values obtained using work principle approach and those from Timoshenko and Gere (1961) for the case of one longitudinal stiffener at $\boldsymbol{\gamma}$ = 5.

P	**δ = 0.05**		**Percentage difference**	**δ = 0.10**		**Percentage difference**
	Present	**Reference**		**Present**	**Reference**	
0.1	996.60	1001.83	0.522	914.07	918.34	0.46
0.2	250.55	251.85	0.516	229.80	230.87	0.46
0.3	112.43	113.01	0.513	103.12	103.60	0.45
0.4	64.14	64.46	0.496	58.83	59.09	0.44
0.5	41.84	42.05	0.499	38.37	38.54	0.44
0.6	29.78	29.92	0.467	27.31	27.43	0.44
0.7	22.57	22.67	0.411	20.70	20.78	0.38
0.8	17.94	18.03	0.499	16.46	16.52	0.36
0.9	14.84	14.90	0.403	13.61	13.66	0.37
1.0	12.68	12.73	0.393	11.63	11.67	0.34

Table 1

- Comparison of Buckling Coefficient, K for SSSS stiffened plates obtained in equation (26.0) with Szilad (2004), a case of one central longitudinal stiffener dividing the plate into two equal parts having, γ = 5.

P	δ = 0.05		Percentage difference	δ = 0.10		Percentage difference
	Present Work	Szilad (2004)		Present Work	Szilad (2004)	
0.1	996.598	1001.827	0.5247	914.068	918.342	0.4676
0.2	250.548	251.855	0.5214	229.800	230.867	0.4642
0.3	112.431	113.011	0.5158	103.121	103.593	0.4586
0.4	64.138	64.464	0.5076	58.827	59.092	0.4505
0.5	41.838	42.045	0.4968	38.373	38.542	0.4396
0.6	29.779	29.923	0.4830	27.313	27.430	0.4259
0.7	22.567	22.672	0.4662	20.698	20.782	0.4091
0.8	17.945	18.025	0.4463	16.459	16.523	0.3892
0.9	14.837	14.900	0.4235	13.609	13.659	0.3664
1.0	12.677	12.727	0.3980	11.627	11.667	0.3409

Table 2

- Comparison of Buckling Coefficient, K for SSSS stiffened plates obtained in equation (34.0) with Timoshenko and Gere (1961), a case of three longitudinal stiffeners dividing the plate into four equal parts having, $\gamma = 5$.

P	δ = 0.05		Percentage difference	δ = 0.10		Percentage difference
	Present	Timoshenko and Gere (1961)		Present	Timoshenko and Gere (1961)	
0.1	1765.111	1751.675	0.7671	1520.033	1501.436	1.239
0.2	442.568	439.200	0.767	381.119	376.457	1.238
0.3	197.689	196.186	0.766	170.241	168.160	1.238
0.4	112.026	111.175	0.765	96.472	95.293	1.237
0.5	72.424	71.875	0.764	62.368	61.607	1.236
0.6	50.964	50.578	0.763	43.887	43.352	1.234
0.7	38.077	37.789	0.761	32.790	32.392	1.232
0.8	29.768	29.544	0.759	25.635	25.323	1.230
0.9	24.128	23.947	0.756	20.536	20.778	1.227
1.0	20.151	20.000	0.753	17.353	17.143	1.224

Table 3

For the case of one central longitudinal stiffener, Table 1.0 shows a comparison between present study and reference. At constant value of $\gamma = 5$, and **δ** = 0.05, 0.10, it is observed that maximum and minimum percentage difference values are 0.5247 and 0.393 which occurs at P = 0.1 and 1.0.
Table 2.0 shows a comparison between present study and Szilad (2004). The values obtained shows good agreement. Percentage difference is between 0.5247 to 0.3980 which shows good agreement.
Similarly, for the case of three longitudinal stiffeners dividing the plates into four equal parts, Table 3.0 shows the comparison between present study and reference. For $\gamma = 5$ and **δ** = 0.10, the percentage difference is between 1.239 to 1.224.
Hence, there is a good agreement.

7. References

1. Bedair, O.K. (1998). A contribution to the stability of stiffened plates under uniform compression. Computers and Structures, 66 (5): 535-570.
2. Brown, C. J. and Yettram, A. L. (1986). The Elastic Stability of Stiffened Plates Using The Conjugate Load/Displacement Method. Computers and Structures, 23 (3): 385-391.
3. Bulson, P. S. (1970). The Stability of Flat Plates. London: Chatto and Windus.
4. Chakraborty, S. and Mukhophyay, M. (2000). Estimation of In-plane elastic parameters and stiffener geometry of stiffened plates. Journal of Sound and Vibration, 231 (1): 99-124.
5. Ezeh, J. C., Ibearugbulem, O. M., Njoku, K. O., and Ettu, L. O. (2013). Dynamic Analysis of Isotropic SSSS Plate Using Taylor Series Shape Function in Galerkin's Functional. International Journal of Emerging Technology and Advanced Engineering, 3 (5): 372-375.
6. Hughes, O. F., Ghosh, B. and Chen, Y. (2004). Improved Prediction of Simultaneous Local and Overall Buckling of Stiffened Panels. Thin-Walled Structures, Vol. 42, Pp. 827–856.

7. Ibeabuchi, V. T. (2014). Analysis of Elastic Buckling of Stiffened Rectangular Isotropic Plates Using Virtual Work Principles. A Master's Thesis submitted to the Post graduate School, Federal University of Technology, Owerri, Nigeria.
8. Ibearugbulem, O.M. (2012). Application of a Direct Variational Principle in Elastic Stability Analysis of Thin Rectangular Flat Plates. Ph.D Thesis Submitted to the School of Postgraduate Studies, Federal University of Technology, Owerri.
9. Mukhopadhyay, M. (1978). A semi analytic solution for rectangular plate bending. Computers and Structures, 9: 81-87.
10. Mukhopadhyay, M. (1989). Vibration and stability of stiffened plates by semi-analytic finite difference method, Part I Consideration of bending displacement only. Journal of Sound Vibrations, 130: 27-39.
11. Mukhopadhyay, M. (1989). Vibration and stability of stiffened plates by semi-analytic finite difference method, Part II Consideration of bending and axial displacements. Journal of Sound Vibrations, 130: 41-53.
12. Mukhopadhyay, M. (1990). Finite Element Buckling Analysis of Stiffened Plates. Computers and Structures, 34 (6): 795-803.
13. Njoku, K. O., Ezeh, J. C., Ibearugbulem, O. M., Ettu, L. O., and Anyaogu, L. (2013). Free Vibration of Thin Rectangular Isotropic CCCC Plate Using Taylor Series Formulated Shape Function in Galerkin's Method. Academic Research International, 4 (4): 126-132.
14. Szilard, R. (2004). Theories and Applications of plate analysis (Classical, Numerical and Engineering Methods). New Jersey: John Wiley & Sons.
15. Timoshenko, S.P. and Gere, J. M. (1961). Theory of Elastic Stability. Newyork: McGraw-Hill.
16. Ventsel, E. And Krauthammer, T. (2001). Thin Plates and Shells: Theory, Analysis and Applications. New York: Marcel Dekker.