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## Conditional Equation's of Beal Conjecture

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### Abstract:

This paper introduces two conditional equations which are termed as CE-1 and CE-2 in this paper. The CE-1 and CE-2 equations give a constant result for the variables of beal conjecture and show condition for its occurrence. This paper also includes proof of validity of beal conjecture which is shown by deriving CE-1 and CE-2 from beal conjecture equation.

**Keywords:** conditional equations (CE-1 and CE-2), Beal conjecture, prime numbers, ferman-catalan conjecture (FCC), number theory

### 1. Introduction

Beal conjecture states that if  $a^x + b^y = c^z$  with a, b, c having a common prime factor then  $x, y, z > 2$ . It was formulated by Andrew

beal. Although maudlin [1] also presented it as a generalization of Fermat's last theorem. Beal conjecture can also be concluded as that all Ferman Catalan Conjecture solutions use 2 as an exponent.

Darmon and Granville [2] have also worked in this kind of problem while investigating integer solutions of the super elliptic equation  $z^m = F(x, y)$

Where F is a homogeneous polynomial with integer coefficients of the generalized Fermat equation

$$Ax^p + By^q + Cz^r$$

beal conjecture is concerned with the common prime factor for positive integers and their exponents greater than 2. If  $a^x + b^y = c^z$ , where a, b, c, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor.

Ex:

		Common Prime Factor
$2^3 + 2^3 = 2^4$	=>	2
$2^9 + 8^3 = 4^5$	=>	2
$3^3 + 6^3 = 3^5$	=>	3
$3^9 + 54^3 = 3^{11}$	=>	3
$27^4 + 162^3 = 9^7$	=>	3
$7^6 + 7^7 = 98^3$	=>	7
$33^5 + 66^5 = 33^6$	=>	11
$34^5 + 51^4 = 85^4$	=>	17
$19^4 + 38^3 = 57^3$	=>	19

Table 1

### 2. Conditional Equations (CE-1 and CE-2)

If the equation  $a^x + b^y = c^z$  satisfies beal conjecture then

$$\text{CE-1: } (x-2)\log a + (y-2)\log b - (z-2)\log c = K(\pm \log 2)$$

$$\text{CE-2: } 2\log(c/ab) = k(\pm \log 2)$$

Where  $k=0, 1, 2, 3, \dots$  integers and  $(\pm \log 2)$  is a constant (error correcting constant). Any one or both of the above equations

must be satisfied IF BEAL CONJECTURE IS TRUE.

However the result of CE-1 and CE-2 can be stated as a multiple of  $(\pm \log 2)$ .

From the below table a, b, c, x, y, z are variables of the equation  $a^x + b^y = c^z$  satisfying beal conjecture.

Sl.no,	a	b	c	x	y	z	$k(\pm \log 2)$ for CE-1	$k(\pm \log 2)$ for CE-2	Satisfying equation
1	2	2	2	3	3	4	$0 * \log 2$	$2 * (-\log 2)$	both
2	2	8	4	9	3	5	$4 * \log 2$	$4 * (-\log 2)$	both
3	3	6	3	3	3	5	$1 * \log 2$	$5 * (-\log 2)$	both
4	3	54	3	9	3	11	$9 * \log 2$	$12 * (-\log 2)$	both
5	27	162	9	4	3	7	$1 * \log 2$	$18 * (-\log 2)$	both
6	7	7	98	6	7	3	$19 * \log 2$	$2 * (-\log 2)$	both
7	33	66	33	5	5	6	$13 * \log 2$	$12 * (-\log 2)$	both
8	34	51	85	5	4	4	$14 * \log 2$	$9 * (-\log 2)$	both
9	19	38	57	4	3	3	$8 * \log 2$	$7 * (-\log 2)$	both

Table 2: Table showing solution of beal conjecture using CE-1 and CE-2

**3. Derivation of conditional equations CE-1 and CE-2 using beal conjecture.**

From beal conjecture we have

$$a^x + b^y = c^z \dots\dots\dots (1)$$

Let  $x=k1+2, y=k2+2, z=k3+2$  and  $k1, k2, k3$  are any variables such that  $x, y, z > 2$  and satisfying beal conjecture .

$$\text{Let } \log a^x + \log b^y - \log c^z = K(\pm \log 2) \dots\dots\dots (2)$$

(From above table 3, SL.NO 1 contents the above equation (2) is possible,

Now solution of equation 1 using CE-1 is  $k=0$

Substituting the above in equation 2, we can write equation (2) as

$$\log a^{k1+2} + \log b^{k2+2} - \log c^{k3+2} = 0 \dots\dots\dots (\pm \log 2) \quad [\because k=0]$$

$$\text{Now, } \log a^{k1+2} + \log b^{k2+2} = \log c^{k3+2}$$

$$(k1+2)\log a + (k2+2)\log b = (k3+2)\log c$$

$$k1\log a + 2\log a + k2\log b + 2\log b = k3\log c + 2\log c$$

$$k1\log a + k2\log b - k3\log c = 2(\log c - \log a - \log b)$$

$$k1\log a + k2\log b - k3\log c = 2\log(c/ab) \dots\dots\dots (3)$$

Now solving for  $k1, k2, k3$ , We get  $(x - 2) = k1, (y - 2) = k2, (z - 2) = k3$

Substituting these values in equation (3) we get

$$(x - 2)\log a + (y - 2)\log b - (z - 2)\log c = 2\log(c/ab) \dots\dots\dots (4)$$

Now equation (4) can be resolved into two parts

$$\text{i.e. } (x - 2)\log a + (y - 2)\log b - (z - 2)\log c = K(\pm \log 2) \dots\dots\dots (5)$$

$$\text{And } 2\log(c/ab) = K(\pm \log 2) \dots\dots\dots (6) \quad k=0,1,2,3 \dots n \text{ integers}$$

Where  $(\pm \log 2)$  is a constant, it can be also called as error correcting constant.

Equation 5 and 6 are conditional equations CE-1 and CE-2, any one of them or both of them must be satisfied if beal conjecture is true.

**4. Conclusion**

1. CE-1 and CE-2 results a constant  $K(\pm \log 2)$  only if beal conjecture is true, i.e. it also shows condition for occurrence of beal conjecture.
2. Derivation of CE-1 and CE-2 from beal conjecture is a valid proof for existence of beal conjecture.
3. The result of CE-1 and CE-2 can be stated as a multiple of  $(\pm \log 2)$ .

**5. References**

1. D.R. maudlin="a generalization of flt:beal conjecture and prize problem "in notices of AMS 1997.
2. H. Darmon and A. Granville, "On the equations  $ZS = F(x, y)$  and  $Ax + By = Cz$ .", in Bull. London Math. Soc., 27, 1995, pp. 513-543.