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Markov Models in System Reliability with Applications

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Abstract: *over the years, many techniques and methods have been developed to present System Failure. In this paper, we model some mixed series-parallel configuration using Fault-Tree Analysis approach and the System Reliability of some repairable Systems of Mixed Models involving series parallel sub-units are studied with the aid of Markov method. Also, we determine the System Availability for few systems and the same technique can be used to find the Availability of any mixed configuration system involving series and parallel configuration. Numerical examples are given.*

Keywords: *Reliability, Series and Parallel Configuration, General Series-Parallel configuration, Fault-tree Analysis Approach, Markov Method.*

1. Introduction

Reliability Theory is a very matured subject and a lot has been written on this subject. The calculation of the Reliability of system with elements exhibiting dependent failures and involving repairs as stand by operations is in general complicated and several approaches have been suggested to carry out the computations. A technique that has much appeal and works well when failure hazard and repair hazards are constant requires the use of Markov models [10].

The term "Markov model", named after the mathematician Andrei Markov, originally referred exclusively to mathematical models in which the future state of a system depends only on its current state, not on its past history. This "memory less" characteristic, called the "Markovian property" implies that all transitions from one state to another occur at constant rates. Much of the practical importance of Markov models for Reliability analysis is due to the fact that a large class of real world devices (such as electronic components) exhibits essentially constant failure rates, and can therefore be effectively represented and analyzed using Markov models.. One of the notable strengths of Markov models for Reliability Analysis is that they can account for repairs and failures and assessing the Reliability, Availability and Maintainability of devices.

Markov modeling is a modeling technique that is widely useful for the Reliability analysis of complex systems. It is very flexible in the type of systems and system behavior it can model. This modeling technique is very helpful in most of the situations .The model is quite useful to modeling operation system with dependent failure and repair models. In fact it is widely used to perform Reliability and Availability analysis of responsible system with constant failure and repair rates. From time to time the markov method is also used to perform human Reliability Analysis.[3]

System Reliability provides important metrics for evaluating the improvement of new designs and the performance of existing designs and as an aid for maintenance planning [8]. In this paper, we apply markov method to mixed parallel and series configuration systems with constant failure and repair rates and we derive the formulae to obtain the Reliability and Availability of systems. The rest of the paper is organized as follows. In section 2, we present the basics concepts such as Reliability, Series & Parallel Configuration, General Series –Parallel configuration and the Fault tree Analysis approach. In section 3, we discuss the markov model for simple system and we present two different models of mixed parallel and series systems such as Series-Parallel and Series-Parallel-Series Configuration systems. We determine their availabilities with markov approach. Numerical examples are given in section 4. Finally conclusion is drawn in section 5.

2. Basics Concepts

2.1. Reliability

Reliability is defined as the probability of a device (or an item) performing its purpose adequately for the period intended under the given operating condition.

2.2. Fault tree Analysis (FTA) approach

FTA is a top-down approach of System Analysis that is used to determine the possible occurrence of undesirable events or failures. Over the years, the method has gained favor over other reliability analysis approaches because of its versatility in degree of detail of complex systems. There are many symbols used to construct fault trees. The basic four symbols are

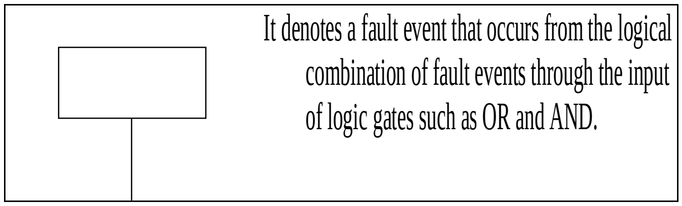

Figure 1: Fault event

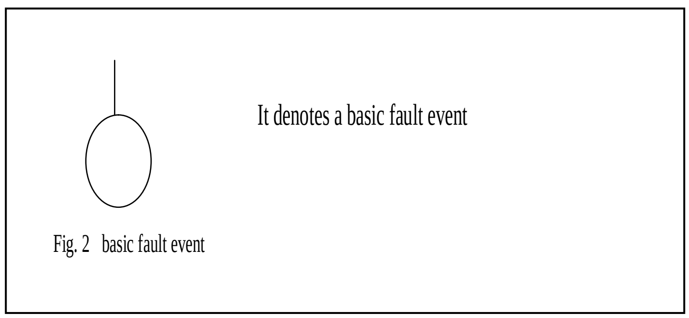

Figure 2: basic fault event

It denotes the output fault event if one or more of input fault events occur.

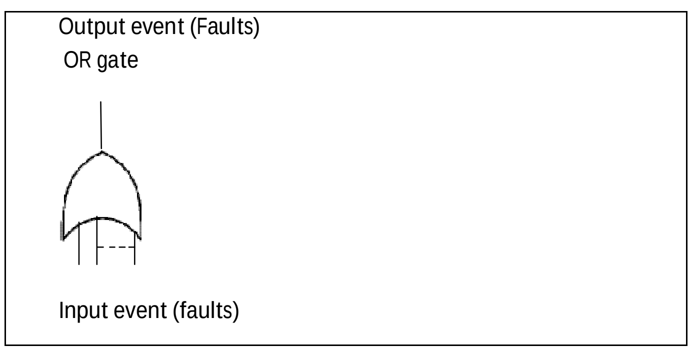

Figure 3: OR Gate

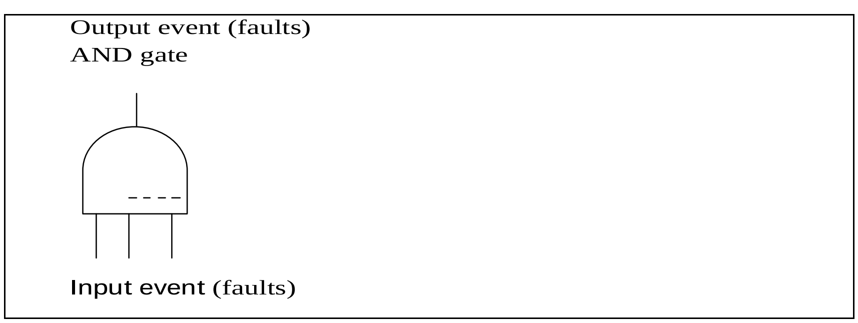

Figure 4: AND Gate

It denotes that an output fault tree event occurs if all the input fault event occur.

2.3. Series Configuration

It is the simplest and probably the most commonly occurring or assumed configuration in reliability evaluation of engineering systems. The success of the systems depends on the success of all its elements. If any one of the elements fails, the system fails. Fig.5 shows the block diagram of a series system.

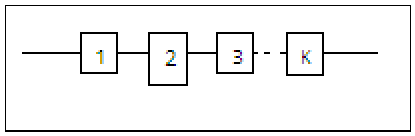

Figure 5: Series configuration

The Reliability of the System is given by

$$R(S) = \prod_{i=1}^{n} P(X_i) \qquad (1)$$

2.4. Parallel Configuration

In this case, all units are active and at least one unit must perform successfully for the system success. Fig.6 shows the block diagram of a parallel system, each block represents a unit.

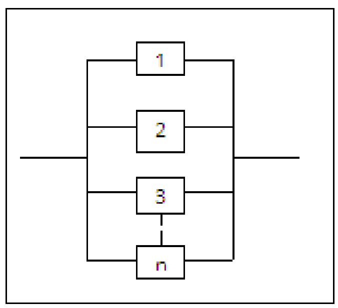

Figure 6: Parallel Configuration

In parallel system reliability is expressed by

$$R(S) = 1 - \prod_{i=1}^{n} \left[1 - P(X_i)\right] \qquad (2)$$

2.5. General Series -Parallel Configuration

The System consists of stage 1, stage 2… stage k connected in series. Each stage contains a number of redundant elements, stage 1 consisting of n_1 redundant elements connected in parallel. The reliability of the system is the product of the reliabilities of each stage. Stage i with n_i elements will have the reliability

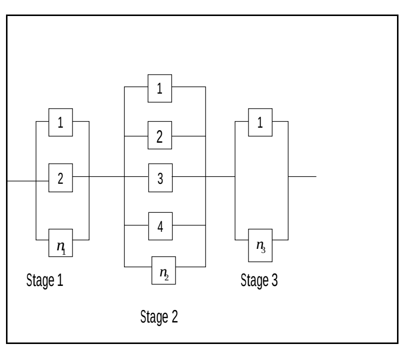

Figure 7: General Series-Parallel Configuration

$$R_i = 1 - \prod_{j=1}^{n_i} \left[1 - P\left(X_{ij} \right) \right]$$

Therefore the System Reliability

$$R(s) = \prod_{i=1}^{k} \left(1 - \prod_{j=1}^{n_i} \left[1 - P\left(X_{ij} \right) \right] \right) \qquad (3)$$

3. Markov Models in Mixed Configuration Systems

3.1. Markov Method

Markov is widely used method to evaluate Reliability of Engineering Systems. The method is particularly useful to handle repairable systems and systems with dependent failure and repair modes. The methods is subject to the following three assumptions

- The probability of the occurrence of transition from one system state to another in the finite time interval Δt is given by $\alpha\Delta t$, where α is the transition rate (i.e., failure rate or repair rate) from one system state to another.
- All occurrences are independent of one another,
- The transitional probability of two or more occurrences in time interval Δt from one system state to another is negligible (e.g. $(\alpha\Delta t)(\alpha\Delta t) \rightarrow 0$).

For any given system, a Markov model consists of list of the possible states of that system, the possible transition paths between those states, and the rate parameters of those transitions. In Reliability analysis the transitions usually consists of failures and repairs. When representing a Markov model graphically, each state is usually depicted as a ”bubble” with arrows denoting the transition paths between states, as depicted in the figure below for a single component that has just two states :normal and failed

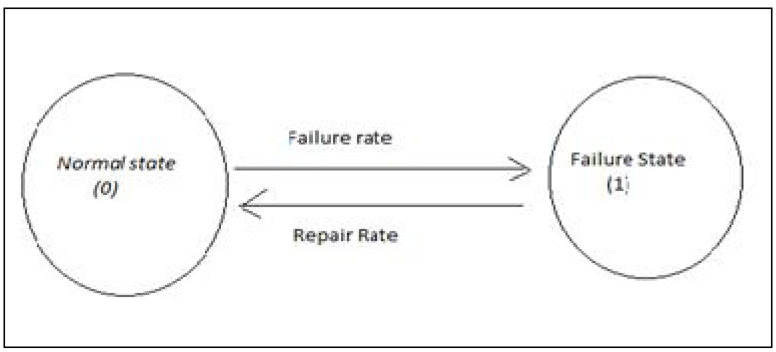

Figure 8: Markov Model of a simple system

$$P_0\left(t+\Delta t\right) = P_0\left(t\right)\left[1-\lambda_x \Delta t\right] + P_1\left(t\right)\mu_x \Delta t \qquad (4)$$

$$P_1\left(t+\Delta t\right) = P_0\left(t\right)\lambda_x \Delta t + P_1\left(t\right)\left[1-\mu_x \Delta t\right] \qquad (5)$$

The above equations can be reformulated as

$$\frac{dP_0(t)}{dt} = -P_0(t)\lambda_x + P_1(t)\mu_x \qquad (6)$$

$$\frac{dP_1(t)}{dt} = -P_1(t)\mu_x + P_0(t)\lambda_x \qquad (7)$$

Taking the limit as Δt approaches to zero, assuming the initial conditions $P_0(0) = 1$ $P_1(0)$ =0 and solving equations (3) and (4) we get

$$P_0(t) = \frac{\mu_x}{(\lambda_x+\mu_x)} + \frac{\lambda_x}{(\lambda_x+\mu_x)} e^{-(\lambda_x+\mu_x)t} \qquad (8)$$

$$P_1(t) = \frac{\lambda_x}{(\lambda_x+\mu_x)} - \frac{\lambda_x}{(\lambda_x+\mu_x)} e^{-(\lambda_x+\mu_x)t} \qquad (9)$$

where $P_0(t)$ and $P_1(t)$ denote the probabilities of the system in the normal and failed state.

3.2. Mixed Configuration Models

In this section we present two different models of mixed configurations and analyze its Reliability through combinatorial rules of Probability and Fault Tree Analysis approach. Also we determine the Availability of the system with known failure and repair rates for the base events with the aid of Markov method.

3.2.1. Model 1: SP model

Consider a System with 2 units connected in Series, each unit has 2 subunits connected in parallel. All the units involved in this system are independent and the base event probabilities are known

By equation (3) we get

The Reliability of a Series-Parallel (SP) Configuration System is given by

$$R(S) = \prod_{i=1}^{2}\left(1-\prod_{j=1}^{2}(1-P(X_{ij}))\right) \qquad (10)$$

Considering the base events with known failure and repair rates and using the formula (8) for the probability of success of base events, The Reliability of the system in terms of failure and repair rate is given by

$$R(S) = \left[\begin{array}{l} \left(\frac{\mu_{11}}{\lambda_{11}+\mu_{11}} + \frac{\lambda_{11}}{\lambda_{11}+\mu_{11}} e^{-(\lambda_{11}+\mu_{11})t}\right) \\ + \left(\frac{\mu_{12}}{\lambda_{12}+\mu_{12}} + \frac{\lambda_{12}}{\lambda_{12}+\mu_{12}} e^{-(\lambda_{12}+\mu_{12})t}\right) \\ - \left[\left(\frac{\mu_{11}}{\lambda_{11}+\mu_{11}} + \frac{\lambda_{11}}{\lambda_{11}+\mu_{11}} e^{-(\lambda_{11}+\mu_{11})t}\right)\left(\frac{\mu_{12}}{\lambda_{12}+\mu_{12}} + \frac{\lambda_{12}}{\lambda_{12}+\mu_{12}} e^{-(\lambda_{12}+\mu_{12})t}\right)\right] \end{array}\right] \left[\begin{array}{l} \left(\frac{\mu_{21}}{\lambda_{21}+\mu_{21}} + \frac{\lambda_{21}}{\lambda_{21}+\mu_{21}} e^{-(\lambda_{21}+\mu_{21})t}\right) \\ + \left(\frac{\mu_{22}}{\lambda_{22}+\mu_{22}} + \frac{\lambda_{22}}{\lambda_{22}+\mu_{22}} e^{-(\lambda_{22}+\mu_{22})t}\right) \\ - \left[\left(\frac{\mu_{21}}{\lambda_{21}+\mu_{21}} + \frac{\lambda_{21}}{\lambda_{21}+\mu_{21}} e^{-(\lambda_{21}+\mu_{21})t}\right)\left(\frac{\mu_{22}}{\lambda_{22}+\mu_{22}} + \frac{\lambda_{22}}{\lambda_{22}+\mu_{22}} e^{-(\lambda_{22}+\mu_{22})t}\right)\right] \end{array}\right]$$

which gives

$$\left\{\begin{array}{l}\frac{1}{\lambda_{11}+\mu_{11}}\left[\mu_{11}+\lambda_{11}e^{-(\lambda_{11}+\mu_{11})t}\right]\\+\frac{1}{\lambda_{12}+\mu_{12}}\left[\mu_{12}+\lambda_{12}e^{-(\lambda_{12}+\mu_{12})t}\right]\\-\frac{1}{(\lambda_{11}+\mu_{11})(\lambda_{12}+\mu_{12})}\left[\begin{array}{l}\mu_{11}\mu_{12}+\lambda_{11}\mu_{12}e^{-(\lambda_{11}+\mu_{11})t}\\+\lambda_{12}\mu_{11}e^{-(\lambda_{12}+\mu_{12})t}\\+\lambda_{11}\lambda_{12}e^{-(\lambda_{11}+\mu_{11}+\lambda_{12}+\mu_{12})t}\end{array}\right]\end{array}\right\}$$

$$\left\{\begin{array}{l}\frac{1}{\lambda_{21}+\mu_{21}}\left[\mu_{21}+\lambda_{21}e^{-(\lambda_{21}+\mu_{21})t}\right]\\+\frac{1}{\lambda_{22}+\mu_{22}}\left[\mu_{22}+\lambda_{22}e^{-(\lambda_{22}+\mu_{22})t}\right]\\-\frac{1}{(\lambda_{21}+\mu_{21})(\lambda_{22}+\mu_{22})}\left[\begin{array}{l}\mu_{21}\mu_{22}+\lambda_{21}\mu_{22}e^{-(\lambda_{21}+\mu_{21})t}\\+\lambda_{22}\mu_{21}e^{-(\lambda_{22}+\mu_{22})t}\\+\lambda_{21}\lambda_{22}e^{-(\lambda_{21}+\mu_{21}+\lambda_{22}+\mu_{22})t}\end{array}\right]\end{array}\right\} \quad (11)$$

Availability of the system as $t \to \infty$

$$A(S)_{t\to\infty} = \left[\left(\frac{\mu_{11}}{\lambda_{11}+\mu_{11}}+\frac{\mu_{12}}{\lambda_{12}+\mu_{12}}\right)-\frac{\mu_{11}\mu_{12}}{(\lambda_{11}+\mu_{11})(\lambda_{12}+\mu_{12})}\right]$$
$$\left[\left(\frac{\mu_{21}}{\lambda_{21}+\mu_{21}}+\frac{\mu_{22}}{\lambda_{22}+\mu_{22}}\right)-\frac{\mu_{21}\mu_{22}}{(\lambda_{21}+\mu_{21})(\lambda_{22}+\mu_{22})}\right]$$
$$=\left[\frac{\mu_{11}(\mu_{12}+\lambda_{12})+\mu_{12}\lambda_{11}}{(\lambda_{11}+\mu_{11})(\lambda_{12}+\mu_{12})}\right]\left[\frac{\mu_{22}(\mu_{21}+\lambda_{21})+\mu_{21}\lambda_{22}}{(\lambda_{21}+\mu_{21})(\lambda_{22}+\mu_{22})}\right]$$

3.2.2. Model 2: SPS model

Consider a series-parallel-series configuration model where the system has 2 units connected in series, each of the 2 units has 2 subunits connected in parallel. Each of the parallel subunits has 2 units connected in series. All the events involved in this system are independent and the base events' failure and repair rates are known.

The Reliability of the series-parallel-series configuration system is given by

$$R(S) = \prod_{i=1}^{2}\left(1-\prod_{j=1}^{2}\left(1-\prod_{k=1}^{2}P(X_{ijk})\right)\right) \quad (13)$$

Substituting

$$P_{ijk} = \left(\frac{\mu_{ijk}}{\lambda_{ijk}+\mu_{ijk}}+\frac{\lambda_{ijk}}{\lambda_{ijk}+\mu_{ijk}}e^{-(\lambda_{ijk}+\mu_{ijk})t}\right)$$

in the above formula we get

$$R(S)=\left\{\frac{1}{(\lambda_{111}+\mu_{111})(\lambda_{112}+\mu_{112})(\lambda_{121}+\mu_{121})(\lambda_{122}+\mu_{122})}\left[\begin{array}{l}\left(\mu_{111}+\lambda_{111}e^{-(\lambda_{111}+\mu_{111})t}\right)\\ \left(\mu_{112}+\lambda_{112}e^{-(\lambda_{112}+\mu_{112})t}\right)\\ (\lambda_{121}+\mu_{121})(\lambda_{122}+\mu_{122})\\ +\left(\mu_{121}+\lambda_{121}e^{-(\lambda_{121}+\mu_{221})t}\right)\\ \left(\mu_{122}+\lambda_{122}e^{-(\lambda_{122}+\mu_{122})t}\right)\\ (\lambda_{111}+\mu_{111})(\lambda_{112}+\mu_{112})\\ -\left(\begin{array}{l}\left(\mu_{111}+\lambda_{111}e^{-(\lambda_{111}+\mu_{111})t}\right)\\ \left(\mu_{112}+\lambda_{112}e^{-(\lambda_{112}+\mu_{112})t}\right)\\ \left(\mu_{121}+\lambda_{121}e^{-(\lambda_{121}+\mu_{221})t}\right)\\ \left(\mu_{122}+\lambda_{122}e^{-(\lambda_{122}+\mu_{122})t}\right)\end{array}\right)\end{array}\right]\right\}$$

$$\left\{\frac{1}{(\lambda_{211}+\mu_{211})(\lambda_{212}+\mu_{212})(\lambda_{221}+\mu_{221})(\lambda_{222}+\mu_{222})}\left[\begin{array}{l}\left(\mu_{211}+\lambda_{211}e^{-(\lambda_{211}+\mu_{211})t}\right)\\ \left(\mu_{212}+\lambda_{212}e^{-(\lambda_{212}+\mu_{212})t}\right)\\ (\lambda_{221}+\mu_{221})(\lambda_{222}+\mu_{222})\\ +\left(\mu_{221}+\lambda_{221}e^{-(\lambda_{221}+\mu_{221})t}\right)\\ \left(\mu_{222}+\lambda_{222}e^{-(\lambda_{222}+\mu_{222})t}\right)\\ (\lambda_{211}+\mu_{211})(\lambda_{212}+\mu_{212})\\ -\left(\begin{array}{l}\left(\mu_{211}+\lambda_{211}e^{-(\lambda_{211}+\mu_{211})t}\right)\\ \left(\mu_{212}+\lambda_{212}e^{-(\lambda_{212}+\mu_{212})t}\right)\\ \left(\mu_{221}+\lambda_{221}e^{-(\lambda_{221}+\mu_{221})t}\right)\\ \left(\mu_{222}+\lambda_{222}e^{-(\lambda_{222}+\mu_{222})t}\right)\end{array}\right)\end{array}\right]\right\}$$

(14)

Availability of the system as $t\rightarrow\infty$

$$A(S)_{t\to\infty} = \left\{ \frac{1}{(\lambda_{111}+\mu_{111})(\lambda_{112}+\mu_{112})(\lambda_{121}+\mu_{121})(\lambda_{122}+\mu_{122})} \begin{bmatrix} \mu_{111}\mu_{112}(\lambda_{121}+\mu_{121}) \\ (\lambda_{122}+\mu_{122}) \\ +\mu_{121}\mu_{122}(\lambda_{111}+\mu_{111}) \\ (\lambda_{112}+\mu_{112}) \\ -(\mu_{111}\mu_{112}\mu_{121}\mu_{122}) \end{bmatrix} \right\}$$

$$\left\{ \frac{1}{(\lambda_{211}+\mu_{211})(\lambda_{212}+\mu_{212})(\lambda_{221}+\mu_{221})(\lambda_{222}+\mu_{222})} \begin{bmatrix} \mu_{211}\mu_{212}(\lambda_{221}+\mu_{221}) \\ (\lambda_{222}+\mu_{222}) \\ +\mu_{221}\mu_{222}(\lambda_{211}+\mu_{211}) \\ (\lambda_{212}+\mu_{212}) \\ -(\mu_{211}\mu_{212}\mu_{221}\mu_{222}) \end{bmatrix} \right\}$$

(15)

The above method of determining the Reliability and Availability of the System is applicable to any system with mixed configurations having any number of subsystems.

4. Numerical Examples

- Example 1: SP Model

Consider a Medical Operator performing tasks A & B independently. To finish task A he prefers either of the methods 1 or 2. Similarly to finish task B he can prefer either of the methods 1 or 2. We evaluate the availability of the Operator.

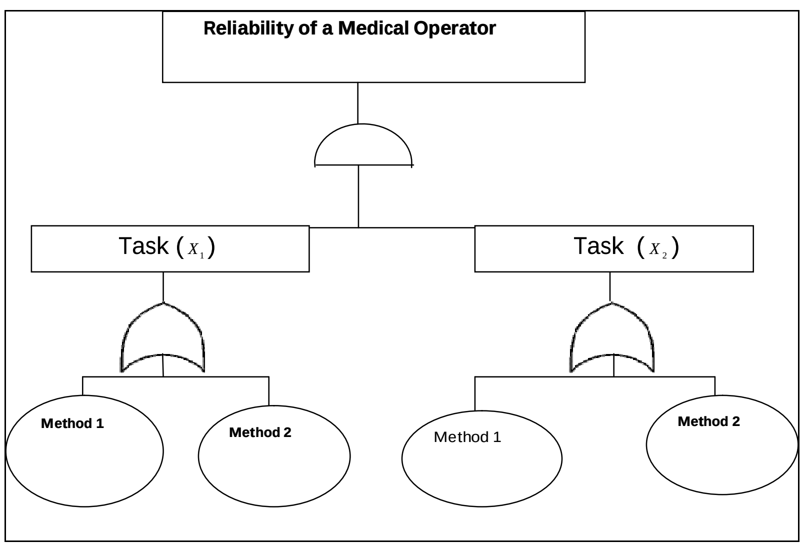

Figure 9

Considering the values of the failure and repair rates for the base events as

$\lambda_{11} = 0.2, \lambda_{12} = 0.4, \lambda_{21} = 0.3, \lambda_{22} = 0.1$

$\mu_{11} = 0.3 \;\; \mu_{12} = 0.5 \;\; \mu_{21} = 0.4 \;\; \mu_{22} = 0.2$

By equation (12), we get the Availability of the Operator as

$A(S)_{t\to\infty}$ =0.7047

- Example 2: SPS Model

Consider a Medical System with 2 main departments to carry out its function. Each department has two wings. The work to be done by the departments can depend either of the wings according to the need. Each of the wings has 2 branches to carry out its operation. We determine the Availability of the Medical System with known failure and repair (correction) rates of the branches

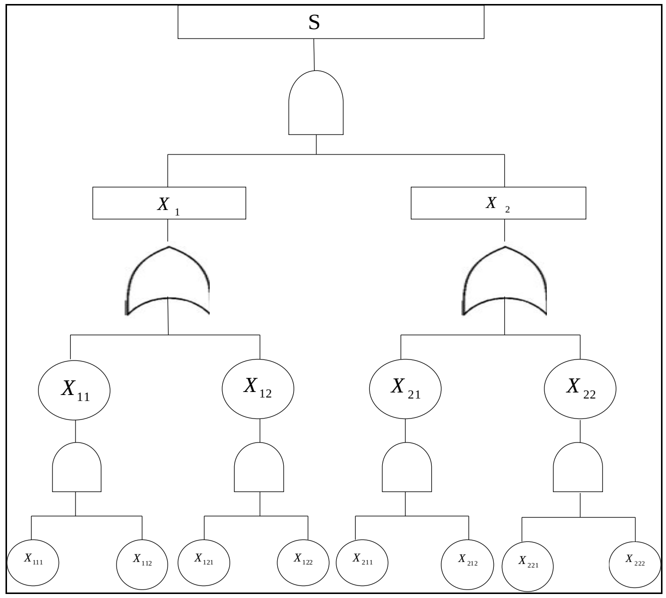

Figure 10

Considering the values of the failure and repair rates for the base events

$$\lambda_{111}=0.1,\ \lambda_{112}=0.2,\ \lambda_{121}=0.1,\ \lambda_{122}=0.1,$$
$$\lambda_{221}=0.8,\ \lambda_{222}=0.6,\ \lambda_{211}=0.1,\ \lambda_{212}=0.1$$

$$\mu_{111}=0.6,\ \mu_{112}=0.7,\ \mu_{121}=0.5,\ \mu_{122}=0.6,$$
$$\mu_{221}=0.9,\ \mu_{222}=0.7,\ \mu_{211}=0.8,\ \mu_{212}=0.5.$$

By equation (15), the Availability of the System is given by

$A(S)_{t\to\infty}=0.9042$

5. Conclusion

Markov modeling is a widely used technique in the study of Reliability analysis of Systems. In this paper we proposed the Markov approach to determine the System Reliability and Availability of models of mixed series and parallel configurations involving subunits with known failure and repair rates. Fault Tree Analysis Approach helps to determine the formulae to evaluate the Reliability and Availability of Systems. Numerical examples are provided.

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