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Analysis of Image by Using Different Beneficial Technique

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Abstract:

Noise present in the image has remained a fundamental problem in the field of image processing. Removal of such a noise from the original signal is really a challenging for researchers. Many algorithms and methods each having, advantages, disadvantages are there in the market. Noise can be introduced by transmission errors and compression. This paper presents a review of some significant work in the area of image de-noising. In this paper, a new threshold estimation technique has been presented along with the standard thresholding and filtering techniques. And a comparative analysis of different de noising methods has been carried out very efficiently. After a brief introduction, some Methods are classified. An overview of various algorithms and methods is provided. As we know, the de-noising is essential and the step to be taken before the images data is recovered. It is necessary to apply an beneficial de-noising technique to compensate for such data corruption.

Keywords: linear Filters, GN, Median filter

1. Introduction

Data sets collected by image sensors are generally contaminated by noise. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can all degrade the data of interest. Images play an important role both in daily life applications such as satellite television, imaging, computer tomography as well as in areas of research and technology such as geographical information systems and astronomy. Furthermore, noise can be introduced by transmission errors and compression. Thus, de noising is often a necessary and the first step to be taken before the images data is analyzed. It is necessary to apply an efficient de noising technique to compensate for such data corruption.

This paper describes different methodologies for noise reduction giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data given its degraded version. Image de noising still remains a challenge for researchers because noise removal introduces artifacts and causes blurring of the images. These technique is often used in various fields like photography, publishing, medical image processing applications, where an image was somehow degraded but needs to be improved before it can be printed or making observations. For this type of application we need to know something about the degradation process in order to develop a model. It is the process of removing noise in the images. Noise reduction techniques are conceptually very similar regardless of the image being processed; however a prior knowledge of the characteristics of an expected signal can mean the implementations of these techniques varies, greatly depending on the type of signal.

When we have a model for the degradation process, the inverse process can be applied to the image to restore it back to the original form. This type of image restoration is often used in space exploration to help eliminate artifacts generated by mechanical jitter in a spacecraft or to compensate for distortion in the optical system of a telescope. Image de noising finds applications in fields such as astronomy where the resolution limitations are severe, in medical imaging where the physical requirements for two high quality imaging are needed for doing analysis of images in unique events, and in forensic science where potentially useful photographic evidence is sometimes of extremely bad quality. Noise can be random or white noise with no coherence or coherent noise introduced by the devices mechanism or processing algorithm. In the case of photographic film and magnetic tape noise (both visible and audible) is introduced due to the grain structure of the medium. In photographic film, the size of the grains in the film determines the film's sensitivity, more sensitive film having larger sized grains. In magnetic tape, the larger the grains of the magnetic particles, the more prone the medium is to noise.

Noise modeling in images is greatly affected by capturing instruments, data transmission media, image quantization and discrete sources of radiation. Different algorithms are used depending on the noise model. Most of the natural images are assumed to have additive random noise which is modeled as a Gaussian. Speckle noise [1] is observed in ultrasound images whereas Rician noise [2] affects MRI images. The scope of the paper is to focus on noise removal techniques for natural images.

2. Literature Survey

Image De noising has remained a fundamental problem in the field of image processing. Wavelets give a superior performance in image de noising due to properties such as scarcity and multi resolution structure. Image de noising is still a challenging problem for researchers as image de noising causes blurring and introduces artifacts. Different types of images inherit different types of noise and different noise models are used to present different noise types. De noising method tends to be problem specific and depends upon the type of image and noise model. Various de noising methods are discussed in this paper.

With Wavelet Transform gaining popularity in the last two decades various algorithms for denoising in wavelet domain were introduced. The focus was shifted from the Spatial and Fourier domain to the Wavelet transform domain. Ever since Donoho's Wavelet based thresholding approach was published in 1995, there was a surge in the denoising papers being published. Although Donoho's concept was not revolutionary, his methods did not require tracking or correlation of the wavelet maxima and minima across the different scales as proposed by Mallat [3]. Thus, there was a renewed interest in wavelet based denoising techniques since Donoho [4] demonstrated a simple approach to a difficult problem. Researchers published different ways to compute the parameters for the thresholding of wavelet coefficients. Data adaptive thresholds [6] were introduced to achieve optimum value of threshold. Later efforts found that substantial improvements in perceptual quality could be obtained by translation invariant methods based on thresholding of an Undecimated Wavelet Transform [7]. These thresholding techniques were applied to the non-orthogonal wavelet coefficients to reduce artifacts. The noise attenuation capability of a weighted median filter can now be assessed using the L-vector and AI-vector parameters in the new expression. The second major contribution is the development of a new optimality theory for weighted median filters. This theory is based on the new expression for the output moments, and combines the noise attenuation and some structural constraints on the filter's behavior.[8]

A median based filter called relaxed median filter is proposed. The filter is obtained by relaxing the order statistic for pixel substitution. Noise attenuation properties as well as edge and line preservation are analyzed statistically[09].Hidden Markov Models and Gaussian Scale Mixtures have also become popular and more research continues to be published. Tree Structures ordering the wavelet coefficients based on their magnitude, scale and spatial location have been researched. Data adaptive transforms such as Independent Component Analysis (ICA) have been explored for sparse shrinkage. The trend continues to focus on using different statistical models to model the statistical properties of the wavelet coefficients and its neighbors. Future trend will be towards finding more accurate probabilistic models for the distribution of non-orthogonal wavelet coefficients. Various noise removal methods have been proposed so far and their application counts upon the kind of image and noise present in the image. Image noise removal is classified in two categories : spatial domain, Transform domain.

Multiwavelets were also used to achieve similar results. Probabilistic models using the statistical properties of the wavelet coefficient seemed to outperform the thresholding techniques and gained ground. Recently, much effort has been devoted to Bayesian denoising in Wavelet domain[10].

3. Classification of Denoising Algorithms

There are two basic ideas to image denoising, spatial filtering methods and frequency domain processing techniques based on Fourier transform of the image.

3.1 Spatial Domain Filtering

A traditional way to remove noise from image data is to employ spatial filters. Spatial filters can be further classified into non-linear and linear filters.

3.1.1. Non-Linear Filter Method

A non linear filter is the filter whose output is a nonlinear function of the input. By definition, any filter that is not a linear filter is a nonlinear filter. One practical reason to use nonlinear filters instead of linear filters is that linear filters may be too sensitive to a small fraction of anomalously large observations at the input. One of the most commonly used nonlinear filters is the median filter.

With non-linear filters, the noise is removed without any attempts to explicitly identify it. Spatial filters employ a low pass filtering on groups of pixels with the assumption that the noise occupies the higher region of frequency spectrum. Generally spatial filters remove noise to a reasonable extent but at the cost of blurring images which in turn makes the edges in pictures invisible. In recent years, a variety of nonlinear median-type filters such as weighted median [8], rank conditioned rank selection [9], and relaxed median [10] have been developed to overcome this drawback.

- Median Filter

The median filter[3] is normally used to reduce noise in an image, somewhat like the mean filter. However, it often does a better job than the mean filter of preserving useful detail in the image. Like the mean filter, the median filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings. Instead of simply replacing the pixel value with the mean of neighboring pixel values, it replaces it with the median of those values. The median is calculated by first sorting all the pixel values from the surrounding neighborhood into numerical order and then replacing the pixel being considered with the middle pixel value.

3.1.2. Linear Filters

A mean filter is the optimal linear filter for Gaussian noise in the sense of mean square error. Linear filters too tend to blur sharp edges, destroy lines and other fine image details, and perform poorly in the presence of signal-dependent noise. The wiener filtering [1] method requires the information about the spectra of the noise and the original signal and it works well only if the underlying signal is smooth. Wiener method implements spatial smoothing and its model complexity control correspond to choosing the window size. To overcome the weakness of the Wiener filtering, Donoho and Johnstone proposed the wavelet based denoising scheme in [2,3]. A mean filter is the optimal linear for Gaussian noise in the sense of mean square error. Linear filters tend to blur sharp edges, destroy lines and other fine details of image. It includes Mean filter and Wiener filter Median filter [7] follows the moving window principle and uses 3×3 , 5×5 or 7×7 window. The median of window is calculated and the center pixel value of the window is replaced with that value.

- Mean Filter

Mean filtering is a simple, intuitive and easy to implement method of smoothing images i.e. reducing the amount of intensity variation between one pixel and the next. It is often used to reduce noise in images.

The idea of mean filtering is simply to replace each pixel value in an image with the mean ('average') value of its neighbors, including itself. This has the effect of eliminating pixel values which are unrepresentative of their surroundings. Mean filtering is usually thought of as a convolution filter. Like other convolutions it is based around a kernel, which represents the shape and size of the neighborhood to be sampled when calculating the mean. Often a 3×3 square kernel is used, as although larger kernels (e.g. 5×5 squares) can be used for more severe smoothing. (Note that a small kernel can be applied more than once in order to produce a similar but not identical effect as a single pass with a large kernel.) averaging kernel often used in mean filtering

Mean filtering is a simple, intuitive and easy to implement method of smoothing images, i.e. reducing the This filter acts on an image by smoothing it. It reduces the intensity variations between the adjacent pixels. Mean filter is nothing just a simple sliding window spatial filter that replaces the centre value of the window with the original signal and it works well only if the underlying signal is smooth. Wiener method implements the spatial smoothing and its model complexity control corresponds to the choosing the window size. Wiener Filter assumes noise and power spectra of object a priori. An image with the mean ('average') value of its neighbors, including itself. This has the effect of eliminating pixel values which are unrepresentative of their surroundings. next. It is often used to reduce noise in images. The idea of mean filtering is simply to replace each pixel value in amount of intensity variation between one pixel and the The two main problems with mean filtering, which are:

- 1) A single pixel with a very unrepresentative value can significantly affect the mean value of all the pixels in its neighborhood.
- 2) When the filter neighborhood straddles an edge, the filter will interpolate new values for pixels on the edge and so will blur that edge. This may be a problem if sharp edges are required in the output.

- Wiener Filter

Weiner filter incorporate both the degradation function and statistical characteristics of noise in to the restoration process . The scientist winer proposed this the concept in the year 1942. The filer , which consists of the terms inside the brackets, also is commonly referred as the minimum mean square error filter or least square error filter The Wiener filter is used to signal estimation problem for stationary signals. The Wiener filter is the MSE-optimal stationary linear filter for images degraded by additive noise and blurring. In analysis of the Wiener filter requires the assumption that the signal and noise processes are second-order stationary (in the random process sense). Wiener filters are also applied in the frequency domain.

3.2. Transform Domain Filtering

The transform domain filtering methods can be subdivided according to the choice of the basis functions. The basis functions can be further classified as data adaptive and non-adaptive. Non-adaptive transforms are discussed first since they are more popular. Transform domain mainly includes wavelet based filtering techniques [6].

3.2.1. Spatial-Frequency Filtering

Spatial-frequency filtering refers use of low pass filters using Fast Fourier Transform (FFT). In frequency smoothing methods [3] the removal of the noise is achieved by designing a frequency domain filter and adapting a cut-off frequency when the noise components are de correlated from the useful signal in the frequency domain. These methods are time consuming and depend on the cut-off frequency and the filter function behavior. Furthermore, they may produce artificial frequencies in the processed image. This is the traditional way to remove the noise from the digital images to employ the spatial filters. Spatial domain filtering is further classified into linear filters and non- linear filters

3.2.2. Wavelet domain

Filtering operations in the wavelet domain can be subdivided into linear and nonlinear methods.

- Linear Filters

Linear filters such as Wiener filter in the wavelet domain yield optimal results when the signal corruption can be modeled as a Gaussian process and the accuracy criterion is the mean square error (MSE) [4,5]. However, designing a filter based on this assumption frequently results in a filtered image that is more visually displeasing than the original noisy signal, even though the filtering operation successfully reduces the MSE. In [6] a wavelet-domain spatially-adaptive FIR Wiener filtering for image denoising is proposed where wiener filtering is performed only within each scale and intrascale filtering is

not allowed.

- Non-Linear Threshold Filtering

The most investigated domain in denoising using Wavelet Transform is the non-linear coefficient thresholding based methods. The procedure exploits sparsity property of the wavelet transform and the fact that the Wavelet Transform maps white noise in the signal domain to white noise in the transform domain. Thus, while signal energy becomes more concentrated into fewer coefficients in the transform domain, noise energy does not. It is this important principle that enables the separation of signal from noise. average values of its all neighboring pixels values including itself. It is implemented with the convolution mask, which provides the results that is weighted sum of vales of a pixel and its neighbors.

The procedure in which small coefficients are removed while others are left untouched is called Hard Thresholding [5]. But the method generates spurious blips, better known as artifacts, in the images as a result of unsuccessful attempts of removing moderately large noise coefficients. To overcome the demerits of hard thresholding, wavelet transform using soft thresholding was also introduced in [5]. In this scheme, coefficients above the threshold are shrunk by the absolute value of the threshold itself. Similar to soft thresholding, other techniques of applying thresholds are semi-soft thresholding and Garrote thresholding [6]. Most of the wavelet shrinkage literature is based on methods for choosing the optimal threshold which can be adaptive or non-adaptive to the image. With the non-linear filter, noise is removed without any attempts to explicitly identify it. Spatial filters employ a low pass filtering on the group of pixels with the assumption that noise occupies the higher region of frequency spectrum. Generally spatial filters remove the noise to reasonable extent but at the cost of blurring the images which in turn makes the edges in the picture invisible.

- Non-Adaptive thresholds

VISU Shrink [7] is non-adaptive universal threshold, which depends only on number of data points. It has asymptotic equivalence suggesting best performance in terms of MSE when the number of pixels reaches infinity. VISU Shrink is known to yield overly smoothed images because its threshold choice can be unwarrantedly large due to its dependence on the number of pixels in the image.

- Adaptive Thresholds

SURE Shrink [8] uses a hybrid of the universal threshold and the SURE [Stein's Unbiased Risk Estimator] threshold and performs better than Bayes Shrink [7,8] minimizes the Bayes' Risk Estimator function assuming Generalized Gaussian prior and thus yielding data adaptive threshold. Bayes Shrink outperforms SURE Shrink most of the times. Cross Validation [9] replaces wavelet coefficient with the weighted average of neighborhood coefficients to minimize generalized cross validation (GCV) function providing optimum threshold for every coefficient.

The assumption that one can distinguish noise from the signal solely based on coefficient magnitudes is violated when noise levels are higher than signal magnitudes. Under this high noise circumstance, the spatial configuration of neighboring wavelet coefficients can play an important role in noise-signal classifications. Signals tend to form meaningful features (e.g. straight lines, curves), while noisy coefficients often scatter randomly.

4. Non-orthogonal Wavelet Transforms

UN decimated Wavelet Transform (UDWT) has also been used for decomposing the signal to provide visually better solution. Since UDWT is shift invariant it avoids visual artifacts such as pseudo-Gibbs phenomenon. Though the improvement in results is much higher, use of UDWT adds a large overhead of computations thus making it less feasible. In normal hard/soft thresholding was extended to Shift Invariant Discrete Wavelet Transform. In Shift Invariant Wavelet Packet Decomposition (SIWPD) is exploited to obtain number of basis functions. Then using Minimum Description Length principle the Best Basis Function was found out which yielded smallest code length required for description of the given data. Then, thresholding was applied to denoise the data.

In addition to UDWT, use of Multiwavelets is explored which further enhances the performance but further increases the computation complexity. The Multiwavelets are obtained by applying more than one mother function (scaling function) to given dataset. Multiwavelets possess properties such as short support, symmetry, and the most importantly higher order of vanishing moments. This combination of shift invariance & Multiwavelets is implemented in which give superior results for the Lena image in context of MSE.

5. Wavelet Coefficient Model

This approach focuses on exploiting the multiresolution properties of Wavelet Transform. This technique identifies close correlation of signal at different resolutions by observing the signal across multiple resolutions. This method produces excellent output but is computationally much more complex and expensive. The modeling of the wavelet coefficients can either be deterministic or statistical.

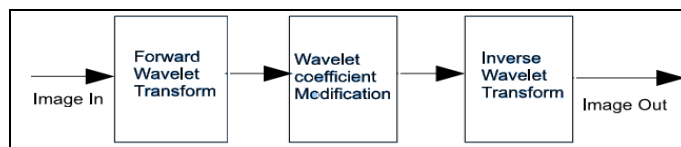


Figure 1: Noise removal using Wavelet Transform Filtering

wavelet transforms fall into transform domain filtering. Transform domain filters tend to cause Gibbs oscillations in the denoised image. Transform domain filtering can be further divided into three broad classes based on the type of transform used:

- Fourier transform filters
- Wavelet transform filters
- Miscellaneous transform filters such as curve lets, ridge lets etc. [10]

We are focusing on the wavelet transform filtering method. This method is chosen because of all the benefits associated with that. All wavelet transform noise removal algorithms involve the following three steps in general (with ref to Figure 1):

- Forward Wavelet Transform: Wavelet coefficients are obtained by applying the wavelet transform.
- Estimation: Clean coefficients are estimated from the noisy ones.
- Inverse Wavelet Transform: A clean image is obtained by applying the inverse wavelet transform.

Wavelet transform is a mathematical function that analyzes the data according to scale or resolution. Noise reduction using wavelets is performed by first decomposing the noisy image into wavelet coefficients i.e. approximation and detail coefficients. Then, by selecting a proper thresholding value the detail coefficients are modified based on the thresholding function. Finally, the reconstructed image is obtained by applying the inverse wavelet transform on modified coefficients.

Basic procedure for all thresholding method is

- Calculate DWT of the Image.
- Threshold the wavelet components.
- Compute IDWT to obtain denoised estimate.

There are two thresholding functions frequently used i.e. Hard Threshold, Pan et al. [9], Soft threshold. Hard-Thresholding function keeps the input if it is larger than the threshold; otherwise, it is set to zero. Soft-thresholding function takes the argument and shrinks it toward zero by the threshold. Soft-thresholding rule is chosen over hard-thresholding, for the soft-thresholding method yields more visually pleasant images over hard thresholding. A result may still be noisy. Large threshold alternatively, produces signal with large number of zero coefficients. This leads to a smooth signal. So much attention must be paid to select optimal threshold

5.1. Deterministic

The Deterministic method of modeling involves creating tree structure of wavelet coefficients with every level in the tree representing each scale of transformation and nodes representing the wavelet coefficients. This approach is adopted in [23]. The optimal tree approximation displays a hierarchical interpretation of wavelet decomposition. Wavelet coefficients of singularities have large wavelet coefficients that persist along the branches of tree. Thus if a wavelet coefficient has strong presence at particular node then in case of it being signal, its presence should be more pronounced at its parent nodes. If it is noisy coefficient, for instance spuriouslip, then such consistent presence will be missing. Lu et al. [9], tracked wavelet local maxima in scale-space, by using a tree structure. Other denoising method based on wavelet coefficient trees is proposed by Donoho [10].

5.2. Statistical Modeling of Wavelet Coefficients

This approach focuses on some more interesting and appealing properties of the Wavelet Transform such as multiscale correlation between the wavelet coefficients, local correlation between neighborhood coefficients etc. This approach has an inherent goal of perfecting the exact modeling of image data with use of Wavelet Transform. A good review of statistical properties of wavelet coefficients can be found in the following two techniques exploit the statistical properties of the wavelet coefficients based on a probabilistic model.

5.2.1. Marginal Probabilistic Model

A number of researchers have developed homogeneous local probability models for images in the wavelet domain. Specifically, the marginal distributions of wavelet coefficients are highly kurtosis, and usually have a marked peak at zero and heavy tails. The Gaussian mixture model (GMM) and the generalized Gaussian distribution (GGD) are commonly used to model the wavelet coefficients distribution. Although GGD is more accurate, GMM is simpler to use. methodology in which the wavelet coefficients are assumed to be conditionally independent zero-mean Gaussian random variables, with variances modeled as identically distributed, highly correlated random variables. An approximate Maximum A Posteriori (MAP) Probability rule is used to estimate marginal prior distribution of wavelet coefficient variances. All these methods mentioned above require a noise estimate, which may be difficult to obtain in practical applications. Simon celli and Adel son used a two-parameter generalized Laplacian distribution for the wavelet coefficients of the image, which is estimated from the noisy observations. Chang et al. proposed the use of adaptive wavelet thresholding for image denoising, by modeling the wavelet coefficients as a generalized Gaussian random variable, whose parameters are estimated locally (i.e., within a given neighborhood).

5.2.2. Joint Probabilistic Model

Hidden Markov Models (HMM) [13] models are efficient in capturing inter-scale dependencies, whereas Random Markov Field [13] models are more efficient to capture intra scale correlations. The complexity of local structures is not well described by Random Markov Gaussian densities whereas Hidden Markov Models can be used to capture higher order statistics. The correlation between coefficients at same scale but residing in a close neighborhood are modeled by Hidden Markov Chain Model where as the correlation between coefficients across the chain is modeled by Hidden Markov Trees. Once the correlation is captured by HMM, Expectation Maximization is used to estimate the required parameters and from those, denoised signal is estimated from noisy observation using well-known MAP estimator. In a model is described in which each neighborhood of wavelet coefficients is described as a Gaussian scale mixture (GSM) which is a product of a Gaussian random vector, and an independent hidden random scalar multiplier. Strela et al. described the joint densities of clusters of wavelet coefficients as a Gaussian scale mixture, and developed a maximum likelihood solution for estimating relevant wavelet coefficients from the noisy observations. Another approach that uses a Markov random field model for wavelet coefficients was proposed by Jansen and Bulthel. A disadvantage of HMT is the computational burden of the training stage. In order to overcome this computational problem, a simplified HMT, named as UHMT, was proposed.

5.2.3 Data-Adaptive Transforms

Recently a new method called Independent Component Analysis (ICA) has gained wide spread attention. The ICA method was successfully implemented in [8,9] in denoising Non-Gaussian data. One exceptional merit of using ICA is its assumption of signal to be Non-Gaussian which helps to denoise images with Non-Gaussian as well as Gaussian distribution. Drawbacks of ICA based methods as compared to wavelet based methods are the computational cost because it uses a sliding window and it requires sample of noise free data or at least two image frames of the same scene. In some applications, it might be difficult to obtain the noise free training data.

6. Conclusions

Performance of denoising algorithms is measured using quantitative performance measures such as peak signal-to-noise ratio (PSNR), signal-to-noise ratio (SNR) as well as in terms of visual quality of the images. Many of the current techniques assume the noise model to be Gaussian. In reality, this assumption may not always hold true due to the varied nature and sources of noise. An ideal de-noising procedure requires a priori knowledge of the noise, whereas a practical procedure may not have the required information about the variance of the noise or the noise model. Thus, most of the algorithms assume known variance of the noise and the noise model to compare the performance with different algorithms. Gaussian Noise with different variance values is added in the natural images to test the performance of the algorithm. Not all researchers use high value of variance to test the performance of the algorithm when the noise is comparable to the signal strength.

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