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IMC-PID Controller Designing Using Laurent Series and Comparison with Other Methods

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Abstract:
The Internal model control (IMC) method is grounded in an assumed process model andleads to analytic expressions for the controller settings. An advantages of IMC structure are it compensates disturbance and model uncertainty also provide safer performance for large delay time systems. In this paper IMC-PID controller is aimed for the stable SISO system by means of Laurent series expansion and using a Pade approximation and compared with the other PID tuning methods also various process models are tested for set point change and disturbance change. At the end controller performance and robustness also discussed.

1. Introduction

During the 1930s, three mode controllers with proportional, integral and derivative (PID) feedback control action become commercially available. Before it was first used for pneumatic processes, only today its use extended in parallel as well as digital control schemes, despite the advanced search in control engineering. Now PID controller is almost popular in almost all process industries till date, according to Astrom and Hagglund,[1] more than 95% of control loops are of PID type. Thither are many variations of PID control used in practice here, we look at four popular structures. The ideal PID (eq 1) mainly used for academic purpose, it is also known as parallel structure with derivative filter because the derivative mode is usually as derivative filter.

$$G_C^{PID0} = K_C\left(1+\frac{1}{\tau_I s}+\tau_D s\right) \quad (1)$$

Mostly parallel eq (2) and series eq (3) used in process industries.

$$G_C^{PID1} = K_C\left(1+\frac{1}{\tau_I s}+\frac{\tau_D s}{\alpha\tau_D s+1}\right) \quad (2)$$

$$G_C^{PID2} = K_C\left(1+\frac{1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\alpha\tau_D s+1}\right) \quad (3)$$

Despite of this structures, there exists another form that used in IMC design which consist of filter in series with an ideal structure.

$$G_C^{PID3} = K_C\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\left(\frac{1}{\tau_f s+1}\right) \quad (4)$$

The method of tuning controller parameters that are Kc, τ_I and $\tau_D$are very important because the accuracy and performance of controller mainly depend on these parameters.

Parameter settings for controllers such as Zeigler and Nichols,[2] Cohen and Coon,[3]Taurus and Luyben,[4]Astrom and Hagglund,[1] were based on a process reaction curve. The direct synthesis approach is applied to obtain controller parameter in IMC-PID form by Chen and Seaborg,[5]Skogestad,[6] Smith,[7] Chain and Fruehauf,[8]. Panagopoulus et al..,[9]proposed tuning method based on Integral absolute error criteria. Chidambaram and Shree ,[10] obtain controller by desired set point response. Haung et al..,[11] Panda et al..,[16]and Lee et al..,[13]got tuning rules based on desired close loop response.

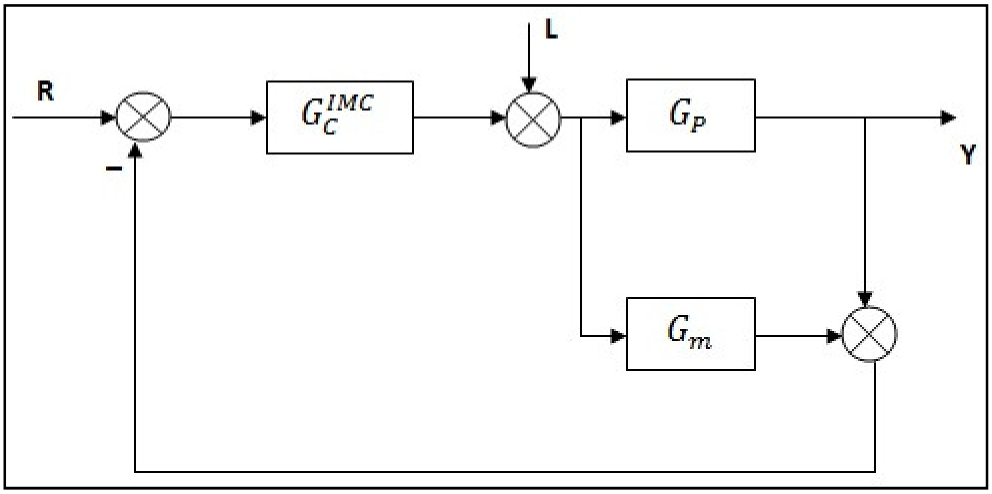


Figure 1: IMC basic structure

A close look into the literature for IMC design (See figure 1) with a time delay process for SISO system direct that Rivera et al.,14 used Pade approximation for exponential term present in denominator of G_C^{true} , in 2002, Chen and Seaborg,[5] used Taylor series and superior performance using Maclaurin series observed by Lee,13;15. They obtained PID parameters for both stable as well as unstable system. But first order plus dead time (FOPDT) system with high D_P/τgives a negative value of D_P . Lee et al..,[15] faced some problem while deriving the analytical solution of controller parameters for IPDT process from G_C^{true} because s term in the denominator disappears at s=0. Then they converted IPDT process in the FOPDT form by considering pole very near to origin of s plane also there tuning formulae are long and need tedious computation.

Based on above facts Panda et al..,[16] suggests IMC- PID parameter synthesis using Laurent series. Laurent series express holomorphic function and having the advantage that it is a generalized form of Maclaurin which generalize form of Taylor series. It can get rid of the singularity problem using Laurent series, which arises due to Taylor or Maclaurin series. So the remaining paper is orchestrate as: In section 2 mathematical development of the true controller using Laurent series. PID controller parameter for various processes are obtained in section 3. Results are discussed in part 4. Simulation results for Level Process using Laurent series controller are discussed in section 5. Grounded on this paper conclusion is passed at the closing.

2. Principle

For SISO stable process, the closed loop transfer function for negative feedback system can be written as,

Servo case:

$$\frac{Y}{R} = \frac{G_P G_C}{1+G_P G_C} \tag{5}$$

Regulatory case:

$$\frac{Y}{L} = \frac{G_L}{1+G_P G_C} \tag{6}$$

The main objective designing controller is to provide robust performance for stable systems. Depending on this object, closed loop transfer function based on IMC design procedure (Morari and Zafiriou, 1989)[17] becomes,

$$\frac{Y}{L} = \frac{G_P G_C^{IMC}}{1+G_C^{IMC}(G_P - G_m)} \tag{7}$$

With Gm as process model and

$$G_C^{IMC} = \frac{1}{G_P^-} \tag{8}$$

Where G_P^- is invertible part of process is transfer function and G_P^+ is non invertible part of system. To make G_C^{IMC}realizable a filter F(s) is introduced so that G_C^{IMC} becomes

$$G_C^{IMC} = \frac{1}{G_P^-(s)} F(s) \tag{9}$$

Where

$$F(s) = \frac{1}{(\lambda s+1)^n} \quad (10)$$

n should be selected such that $G_C^{IMC}$is realizable.
Next, consider desired closed loop response as

$$\left(\frac{Y}{R}\right)_d = \frac{G_P^+}{(\lambda s+1)^n} = \frac{e^{-D_P s}}{(\lambda s+1)^n} \quad (11)$$

This can be compared with the complementary sensitivity function as

$$\frac{Y}{R} = \frac{G_C^{true} G_P}{1+G_C^{true} G_P} \quad (12)$$

By solving eq (11) and (12) true controller can be expressed equally as

$$G_C^{true} = \frac{G_C^{IMC} F(s)}{1-G_P^- G_C^{IMC} F(s)} = \frac{\frac{1}{G_P^-}}{(\lambda s+1)^n - G_P^+} \quad (13)$$

This true controller can be approximated by an approximation series in a complex s plane, by expanding near the vicinity of s≈ 0. Panda,[16] uses Laurent series for approximation instead of Taylor or Maclaurian series. Using Laurent series singularity problems which arise in IPDT process in Taylor or Maclaurian series can be wielded.
Let f(s) be an analytic function in unit circle$|z - z0| < r$, let z be any point inside circle with radius r and z0 is center. With s0 and z0 as poles C is a positively oriented circle of radius r.

$$\frac{1}{s-z} = \frac{1}{s-z0}\left[\frac{1}{1-\frac{z-z0}{s-s0}}\right] = \sum_{j=0}^{\infty} \frac{(z-z0)^j}{(s-z0)^j} \quad (14)$$

Because $\frac{z-z0}{s-s0} < 1$, this series results in a uniform convergence. Then,

$$\int_c \frac{f(s)}{s-z} ds = \sum_{j=0}^{\infty} \int_c \left(\frac{f(s)}{(s-z0)^{j+1}} ds\right)(z - z0)^j \quad (15)$$

Or

$$f(z) = \frac{1}{2\pi j}\int_c \frac{f(s)}{s-z} ds$$

$$= \sum_{j=0}^{\infty} \frac{1}{2\pi j}\int_c \left(\frac{f(s)}{(s-z0)^{j+1}} ds\right)(z - z0)^j + \sum_{j=1}^{\infty} \frac{1}{2\pi j}\int_c \left(\frac{f(s)}{(s-z0)^{-j+1}} ds\right)\frac{1}{(z-z0)^j}$$

$$= \sum_{j=0}^{\infty}(z - z0)^j + \sum_{j=1}^{\infty} \frac{b_j}{(z-z0)^j} \quad (16)$$

where$0 < |z - z0| < r$ and

$$a_j = \frac{1}{2\pi j}\int_c \frac{f(s)}{(s-z0)^{j+1}} ds \text{(for j= 0,1,2,…)} \quad (17)$$

and

$$b_j = \frac{1}{2\pi j}\int_c \frac{f(s)}{(s-z0)^{-j+1}} ds \qquad \text{(for j= 1,2,…..)} \quad (18)$$

These two series can be sum and written as

$$f(z) = \sum_{j=-\infty}^{\infty} c_j (z - z0)^j \quad (19)$$

where

$$c_j = \frac{1}{2\pi j}\int_c \frac{f(s)}{(s-s0)^{j+1}} ds \qquad \text{(for j= 0,±1,±2,…)} \quad (20)$$

Here only b_1, a_0 and a_1 are determined as

$$b_1 = \underset{s=z0}{Res} f(z) = \frac{1}{(m-1)!}\lim_{z\to z0}\left\{\frac{d^{m-1}}{dz^{m-1}}[(z - z0)^m f(z)]\right\} \quad (21)$$

The $b_1$coefficient can be determined using residue theorem, as for f(s) having an m^th order pole and a_j can be determined as

$$f^n(z0) = \frac{n!}{2\pi j}\int_c \frac{f(s)}{(s-z0)^{n+1}} ds \quad (22)$$

where$f^n$is nth derivative of f(z). As G_C^{true}should be in the realizable form for implementation purpose, so comparing it with eq(2) gives

$$G_C^{true} = \frac{f(s)}{s} = \frac{\frac{(1+\beta s)f(s)}{(1+\beta s)}}{s} = \frac{\emptyset(s)}{s(1+\beta s)} \quad (23)$$

where

$$\beta = \alpha\tau_D \quad (24)$$

This true controller can be expanded using Laurent series as

$$G_C^{true} = \frac{1}{s(\beta s+1)}\left[\sum_{j=-\infty}^{\infty} c_j(s)^j\right]$$

$$= \frac{1}{s(\beta s+1)}\left[\ldots + \emptyset(0) = \emptyset'(0)s + \frac{\emptyset''(0)s^2}{2!} + \cdots\right] \quad (25)$$

Comparing the coefficient of the s terms of eqs (25) and (2) gives

$$K_C = a_0 = \emptyset''(0) = f'(0) + \beta f(0) \quad (26)$$

$$\frac{K_C}{\tau_I} = b_1 = \emptyset(0) = f(0) \quad (27)$$

$$K_C\tau_D = a_1 = \frac{\emptyset''}{2!} = \frac{f''(0)+2\beta f'(0)}{2} \quad (28)$$

where

$$G_C(s) = \frac{\emptyset(s)}{s(\beta s+1)} \quad (29)$$

$$\emptyset(s) = (\beta s+1)f(s) \quad (30)$$

fromeqs (26)-(28) Kc, $\tau_I$and τ_D can be computed.

3. PID Tuning Parameter

Panda,[16] gives analytical expressions for PID parameters for various process as shown in Table1.

	Transfer Functions	
1	$\frac{K_P e^{-D_P s}}{\tau_P s+1}$	$K_C = \frac{\tau_I}{K_P(\lambda+D_P)}, \tau_I = (\tau_P+\beta) + \frac{D_P^2}{2(\lambda+D_P)}$, $\tau_D = \frac{D_P^2}{2(\lambda+D_P)\tau_I}\left[(\tau_I-\beta) - \frac{D_P}{3}\right] + \frac{\beta\tau_P}{\tau_I}$
2	$\frac{K_P e^{-D_P s}}{(\tau_{P1}s+1)(\tau_{P2}s+1)}$	$K_C = \frac{\tau_I}{K_P(2\lambda+D_P)}, \tau_I = (\tau_{P1}+\tau_{P2}+\beta) + \frac{2\lambda^2-D_P^2}{2(\lambda+D_P)}$, $\tau_D = \frac{\left[(\tau_{P1}\tau_{P2}+(\tau_{P1}+\tau_{P2})\beta) - \frac{D_P^3}{6(2\lambda+D_P)}\right](2\lambda+D_P) - \tau_I(\lambda^2 - \frac{D_P^2}{2})}{(2\lambda+D_P)\tau_I}$
3	$\frac{K_P e^{-D_P s}}{\tau_P s(as+1)}$	$K_C = \frac{\tau_P\beta(\tau_I+\beta)}{K_P(\lambda+D_P)}, \beta\tau_I = (a+\beta) + \frac{D_P^2}{2(\lambda+D_P)}$, $\lambda\tau_D = \frac{\tau_I[2a\beta(\lambda+D_P) + D_P^2\beta\tau_I]}{2(\lambda+D_P)(a+\beta+\beta^2) + 2D_P^2} + (a+\beta+\beta^2) + \frac{D_P^2}{2(\lambda+D_P)}$

Table 1: Analytical expression for PID controller parameters for standard transfer functions

3.1. Selection of λ

Here section of λ is based upon suggestion of Luyben,[18]to make the closed loop response faster.

- For FOPDT system: λ=max(0.2τ_P, 1.7D_P)
- For SOPDT system: λ=max(0.2τ_P, 0.25D_P)

3.2. Selection of β

Normally β is equal to τ_D , where α is a constant (α = 0:1) and τ_D is derivative time. Thus, the appearance of τ_D term in ϕ(s) makes it difficult to get exclusive solution of Kc, $\tau_I$and τ_D . To make the response faster and stable Panda, [16] consider β as

$$\beta = \alpha(0.25)\max(\tau_P, D_P)$$

with the help of these parameters λ, β and eqs(26) to (30), the PID parameters can be evaluated

4. Result and Analysis

Here for implementation G_C^{PID1} structure is used from eq.(2). The performance of Panda,[16] controller is compared with Lee.et al.,[13;15],Chein and Fruehauf,[8] and Chen and Seaborg,[5] controller using the following examples:

Example 1	$\frac{\mathbf{e^{-0.25s}}}{\mathbf{(s+1)}}$
Example 2	$\frac{2e^{-s}}{(10s+1)(5s+1)}$
Example 3	$\frac{0.2e^{-7.4s}}{s}$
Example 4	$\frac{0.0506e^{-6s}}{s}$
Example 5	$\frac{(s^2+2s+0.25)}{(s^4+0.5s^3+15s^2+14s+4)}$

Table 2: Examples

4.1. PID Parameters

For the above process we calculate PID parameters for Lee.et.al[13;15] controller(i.e IMC-MAC) which used Maclaurin series, Chein and Fruehauf [8] controller (i.e IMC-CF), Chen and Seaborg [5] controller (i.e IMC-SEB) and Panda [16] controller (i.e. IMC-LAU) with proper selection of standard controller and λ.

For IMC-MAC, IMC-CF and IMC-SEB we used standard controller (eq.1) and for IMC-LAU we used parallel structure (eq.2). The closed loop controller parameters for selected examples are given in Table (3).

Example	**IMC-LAU**				**IMC-MAC**			
	K_C	τ_I	τ_D	λ	K_C	τ_I	τ_D	λ
Ex 1	1.5871	1.0713	0.0650	0.425	1.5501	1.0463	0.0423	0.425
Ex 2	1.9407	14.8583	3.2229	1.414	1.9081	14.6083	3.0280	1.414
Ex 3	0.3551	30.5976	1.8057	12.58	0.4952	38.7959	2.1675	12.58
Ex 4	1.4014	31.6091	1.8609	10.2	2.3883	30.8141	1.7404	10.2
Ex 5	63.9848	2.7865	0.5727	0.2	62.8367	2.7365	0.5332	0.2

Example	**IMC-CF**				**IMC-SEB**			
	K_C	τ_I	τ_D	λ	K_C	τ_I	τ_D	λ
Ex 1	2.0455	1.1250	0.1111	0.425	1.3666	0.8084	-0.028	0.425
Ex 2	6.2138	15	3.3333	1.414	11.7970	5.02889	1.5570	1.414
Ex 3	0.1229	32.56	3.2795	12.58	0.3554	41.44	1.0862	12.58
Ex 4	0.1515	26.4	2.6591	10.2	1.7323	33.6000	0.8807	10.2
Ex 5	6.1111	2.75	0.5455	0.2	298.3232	0.8095	0.2607	0.2

Table 3: Closed loop controller parameters

4.2. Closed Loop Response

Depending upon the closed loop parameters the response for selected examples are drawn here.

1. FOPDT: For the FOPDT process Panda [16] uses first order filter to design a controller. The presence of β term make the response little bit faster compared to IMC-MAC. If the λ increases, then τ_I approaches τ_P and K_C and τ_D almost vanishes and controller reduces to integral controller.

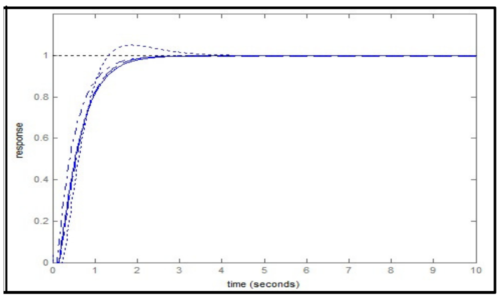
2. SOPDT: For computation of controller for SOPDT process both IMC-LAU and IMC-MAC uses second order filter. Given SOPDT example is overdamped process. For the SOPDT process, Chen and Seaborg [5] uses Taylor series expansion (i.e $e^{-D_P s} \approx 1 - D_P s$).
3. IPDT: According to Chein and Freuhauf [8] and Luyben[18] many chemical processes can be converted into integrator plus dead time processes. For IPDT process Lee et al..[15] faces the problem due to s=0, the denominator vanishes. So they modeled IPDT process to FOPDT form by choosing unstable pole near zero. But for IMC-LAU no such arrangement is required.
4. Higher Order Process: The given example of Higher Order Process results into large overshoot due to strong lead term. Thus to compensate it the actual process transfer function is reduced to SOPDT system using Skogestad's rule [6] as

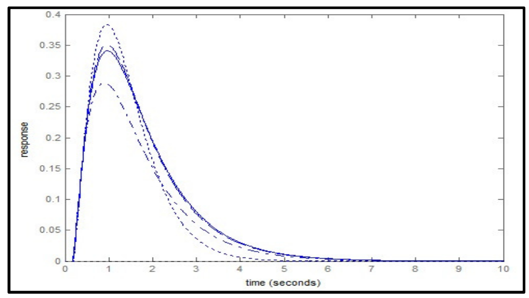
$$G_P = \frac{0.0670(7.46s+1)e^{-0.25s}}{(2s+1)(0.75s+1)} \quad (31)$$

Based on this reduced model (31) and neglecting the numerator zero controller parameters are obtained.
The closed loop response for above process are shown in fig.(2) also the performance and robustness are shown in Table(4) and Table(5).

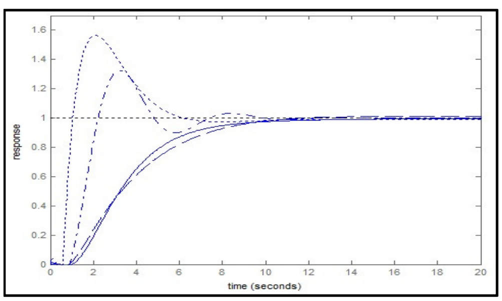
Example	**Set Point**	**Load disturbance**

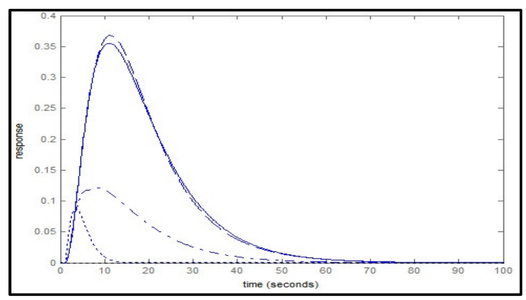
Ex 1



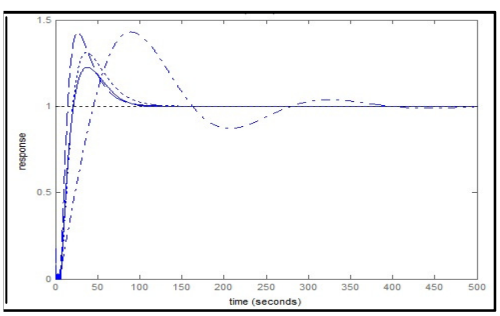


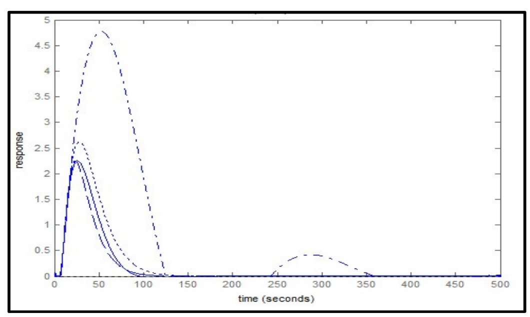
Ex 2





Ex 3





Ex 4

Ex 5

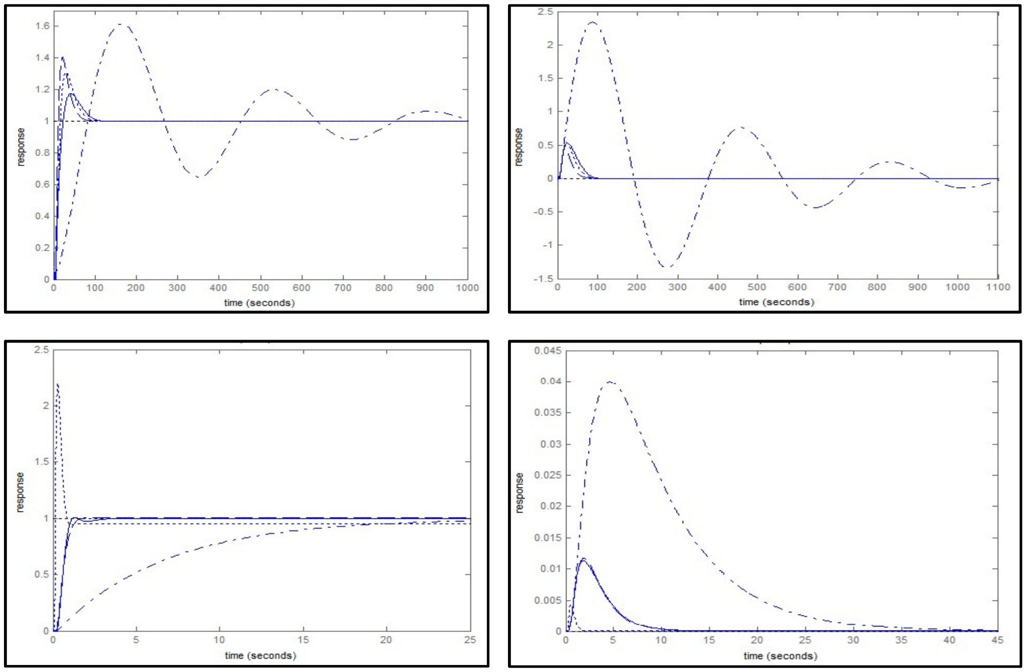


Figure 2: Closed loop response of an example processes using a PID controller (solid line (-) represents response of Panda[16] controller, dashed line (--) interprets response of Lee et al..13 controller, dashed dot line (-.) corresponding to Chein and Fruehauf 8 controller and dotted line(..) belongs to Chen and Seaborg5 controller)

Example	**IMC-LAU**		**IMC-MAC**		**IMC-CF**		**IMC-SEB**	
	IAE_S	IAE_R	IAE_S	IAE_R	IAE_S	IAE_R	IAE_S	IAE_R
Ex 1	7.1971	6.8336	7.2811	6.8325	6.1724	5.5855	7.7858	5.9969
Ex 2	38.7295	76.553	40.0707	76.5515	24.011	24.1416	31.9328	4.7255
Ex 3	33.6673	88.0762	21.3268	78.9229	72.640	96.3561	36.0177	90.796
Ex 4	29.4615	22.7762	23.5392	12.9788	90.868	55.0401	19.9327	19.409
Ex 5	7.0599	0.4355	7.1066	0.4356	53.475	4.1558	13.4184	0.0347

Table 4: Performance parameters of close loop controller

Example	**IMC-LAU**		**IMC-MAC**		**IMC-CF**		**IMC-SEB**	
	G_M	P_M	G_M	P_M	G_M	P_M	G_M	P_M
Ex 1	7.7759	74.8540	8.3300	72.7531	4.4	76.8634	4.5219	61.4801
Ex 2	4.7314	71.2618	6.7415	72.1484	1.9102	42.5602	1.7371	20.2172
Ex 3	3.4146	49.2533	2.5261	51.2453	9.4207	53.1287	3.3264	52.8216
Ex 4	4.1612	56.1481	2.5456	49.8856	37.1769	49.6126	3.3084	51.3139
Ex 5	3.0926	64.9861	4.2061	67.4103	42.5169	87.8673	1.2022	5.2781

Table 5: Robustness parameters of close loop controller

5. Simulation

From the above discussion we conclude that IMC- LAU gives better response, performance and robustness than other three controllers. We further extend this discussion to real time simulation results of Level process using IMC-LAU controller. The experimental setup of level process is shown in fig. (3)

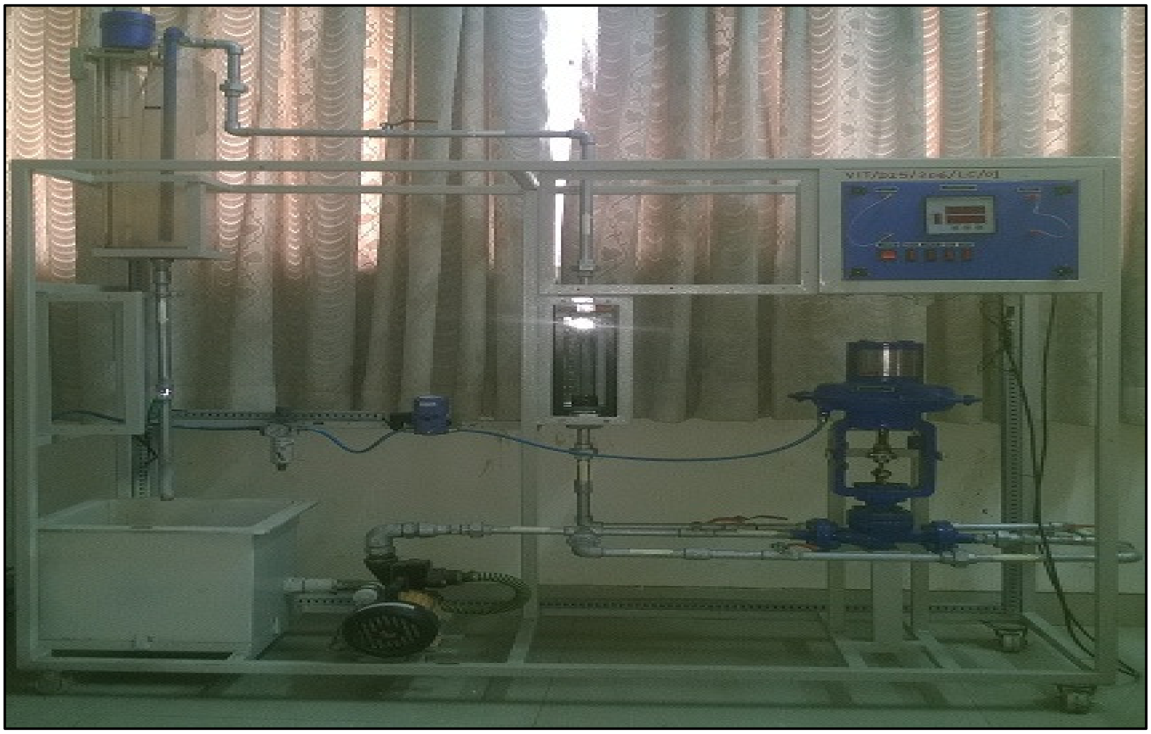


Figure 3: Experimental setup of level loop

The empirical modeling of the above process is done using LabVIEW and System Identification Toolbox. We get the transfer function of above process as

$$G_P = \frac{43.33e^{-0.012s}}{s+0.2024} \quad (32)$$

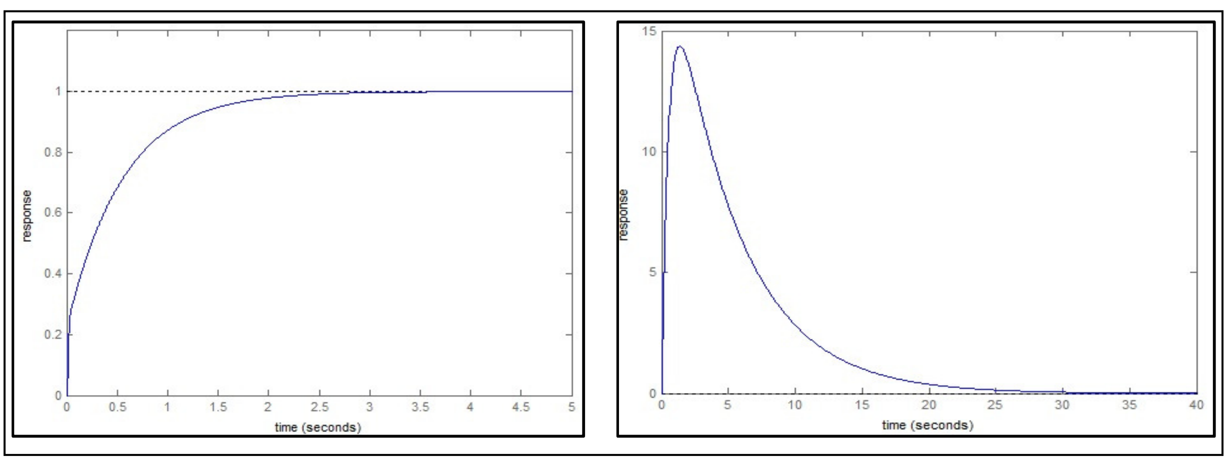


Figure 4

The closed loop response of above process is shown in fig.(4) From the closed loop response we get T_R=1.242 sec, T_S=2.0092 sec which shows close loop response of system is faster also by varying λ we can get more faster response. Due to IMC-LAU controller GM is 9.4523 and P_M is 104.8135 also IAE_S is 4.9483 is achieved.

6. Conclusion

IMC-LAU controller consists of zero in numerator of G_C^{true} due to which (1) controller become faster and (2) required order of filter for proper realization of G_C^{true} gets reduced. Also IMC-LAU has extra parameter i.e. β than IMC-MAC controller which helps to select proper PID algorithm form and make the response faster. Singularity problem which persist in IMC-MAC is effectively handle by Laurent series specially in integrating process. IMC-LAU gives much better responses in case of IPDT processes than IMC-MAC, IMC-CF and IMC-SEB. One can choose faster or sluggish response by selecting proper value of λ . IMC-LAU controller is stable, robust and can be implemented easily on real time processes.

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