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Relation between P.D.F and Wavelets

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Abstract:
This paper proposes a relation between continuous probability density function and wavelets. Here we proved that, derivatives of probability density functions of continuous distribution are continuous wavelets satisfying conditions of wavelets. This proof is given completely in mathematical formation, and here we took basic concepts of probability distribution and Holders inequality to prove this theorem. And we took two examples to show the proof, 1)p.d.f of t-distribution for degrees of freedom 2 and 2).p.d.f of logistic distribution. These derivatives form new continuous wavelet family. Here we took p.d.f function and those successive derivatives, which are again continuous by fundamental theorem of calculus will become continuous wavelets by satisfying the conditions of the wavelets without FIR filters and without scaling function like Mexican and Morlet. Wavelet analysis has attracted attention for its ability to analyze rapidly changing transient signals.

Keywords: *Admissibility condition, Continuous function, Probability density function, Holders inequality, Wavelets.*

1. Introduction

Wavelets have created much excitement in the mathematics community (perhaps more so than in engineering) because the mathematical development has followed a very interesting path. The recent developments can be viewed as resolving some of the difficulties inherent in Fourier analysis [7]. For example, a typical question is how to relate the Fourier coefficients to the global or local behavior of a function. The development of wavelet analysis can be considered an outgrowth of the Littlewood-Paley theory (first published in 1931), which sought a new approach to answer some of these difficulties. Again, it is unifying framework made possible by recent results in wavelet theory related to problems of harmonic analysis (also to similar problems in operator theory called the Calderon- Zygmud theory) that has generated much of the excitement. In signal processing Continuous Wavelet Transform (CWT) is very efficient in determining the damping ratio of oscillating signals (e.g. identification of damping in dynamical systems). CWT is also very resistant to the noise in the signal [7]. Here we will give a relation between Probability density function and Wavelets with two examples.

1.1. Definition of Probability density function (p.d.f):

Suppose that X has continuous distribution on $S \subseteq R^n$. A real valued function f defined on S is said to be Probability density function for X, if f satisfies the following conditions [4].

A. $f(x) \geq 0 \ \forall \ x \in S \subseteq R$
B. $\int_s f(x)\, dx = 1$

1.2. Threorem1: suppose that g is non-negative function on $S \subseteq R$.

Let

$$c = \int_s g(x)dx$$

If $0 < c < \infty$.Then $f(x) = (1/c)g(x)$ for $x \in S$, defines a probability density function on S.

- Proof: clearly $f(x) \geq 0$ for $x \in S$. Also

$\int_s f(x)dx = (1/c) \int_s g(x)\, dx$
$= c/c = 1$
$\Rightarrow f(x)$ is p.d.f.

- Note: That f is just a scaled version of *g*. Thus this result can be used to construct p.d.f with desired properties (domain, shape, symmetry and so on).The constant c is sometimes called the Normalizing constant.

From above Theorem
$f(x)/g(x) = (1/c)$ if $0 < c < \infty$
$\Rightarrow \infty > 1/c > 0$
$\Rightarrow 0 < f(x)/g(x) < \infty$
For our convenience we replace $1/c = C$, then
$f(x) = C\, g(x)$ (1)

- Hölder Inequality

If S is a measurable subset of $\boldsymbol{R}^n$ with the Lebesgue measure, and F and G are measurable real- or complex-valued functions on S, then Hölder inequality [3],[6] is

$$\int_{-\infty}^{\infty} |F(x)G(x)|dx \le \left(\int_{-\infty}^{\infty} |F(x)|^p\right)^{1/p} \left(\int_{-\infty}^{\infty} |G(x)|^q\right)^{1/q}$$

1.3. Theorem 2: The derivative of Probability density function which is even, positive and differentiable of continuous distribution is continuous wavelet (or mother wavelet).

- Proof: Let suppose $F(x) = f''(x)$ where $f(x)$ is p.d.f of continuous distribution which is differentiable.

To prove $F(x)$ is mother wavelet it must satisfy below conditions
I) condition for finite energy
$0 \le \int_{-\infty}^{\infty} |F(x)|^2\, dx < \infty$
II) The admissibility condition
$C_f = \int_0^{\infty} \frac{|\boldsymbol{F}(\omega)|^2}{|\omega|} d\omega < \infty$
Where $\boldsymbol{F}(\omega)$ is Fourier transformation of $F(x)$. The admissibility condition (II) implies
$0 = F(0) = \int F(x)dx$
Also, if
(II.1) $\int F(x)dx = 0$ and
(II.2) $\int_{-\infty}^{\infty} (1 + |x|^{\alpha})\, F(x)dx < \infty$ for some $\alpha > 0$, then $C_f < \infty$ [1].
So, to prove this theorem it is enough to prove (I), (II.1) and (II.2).
From (1)
$f(x) = C\, g(x)$ where $g(x) > 0$.
differentiating on both sides w. r. t x, we get
$f''(x) = C\, g'(x)$ taking mod on both sides [3], we get
$|f''(x)| = C\, |g'(x)|$ since $0 < C < \infty$ (2)
Let us take $F(x) = f''(x)$ and $G(x) = g'(x)$ then from (2)
$0 < |F(x)| / |G(x)| < \infty$
⇨ $0 < |F(x)| < \infty$
Squaring and integrating on both sides

⇨ $0 < \int_{-\infty}^{\infty} |F(x)|^2\, dx < \infty$ (3)
and from Holder's inequality

$$\int_{-\infty}^{\infty} |F(x)G(x)|dx \le \left(\int_{-\infty}^{\infty} |F(x)|^p\right)^{1/p} . \left(\int_{-\infty}^{\infty} |G(x)|^q\right)^{1/q}$$

in above put $p = q = 2$, then

$(\int_s |F(x)|dx)^2 \le \infty . (\int_s |F(x)|^2 dx)$

$\rightarrow (1/\infty) . (\int_s |F(x)|dx)^2 \le (\int_s |F(x)|^2 dx)$

$\rightarrow 0 . (\int_s |F(x)|\, dx)^2 \le (\int_s |F(x)|^2 dx)$

$\rightarrow 0 \le \int_s |F(x)|^2 dx$ where $S = R$ (4)
From (3) and (4)
$0 \le \int_{-\infty}^{\infty} |F(x)|^2\, dx < \infty$ (5)
Now we will prove (II.1)

By the definition of p.d.f of continuous distribution we have

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

Differentiating w.r.t x on both sides

d($\int_{-\infty}^{\infty} f(x)\,dx$) = 0 where *f(x)* is even function

$\rightarrow \int_{-\infty}^{\infty} F(x)\,dx = 0$

Now we will prove (II.2)

Since by definition of p.d.f $f(x) > 0$.

$\rightarrow F(x) = f''(x) < \infty$ but not equal to ∞ ,since from (II.1)

$\rightarrow F(x) < \infty$

$\rightarrow \int_{-\infty}^{\infty} (1 + |x|^{\alpha})\, F(x)dx < \infty$

Hence the Theorem.

For example, derivatives of p.d.f of Logistic distribution and t-distribution satisfies all the above conditions to become a wavelets, those plots and C_k, C_f values are given below.

Where $C_k = \int_{-\infty}^{\infty} |F(x)|^2\, dx$, $C_f = \int_{-\infty}^{\infty} (1 + |x|^{\alpha})\, F(x)dx$.

Derivatives of p.d.f of t- distribution figures are:

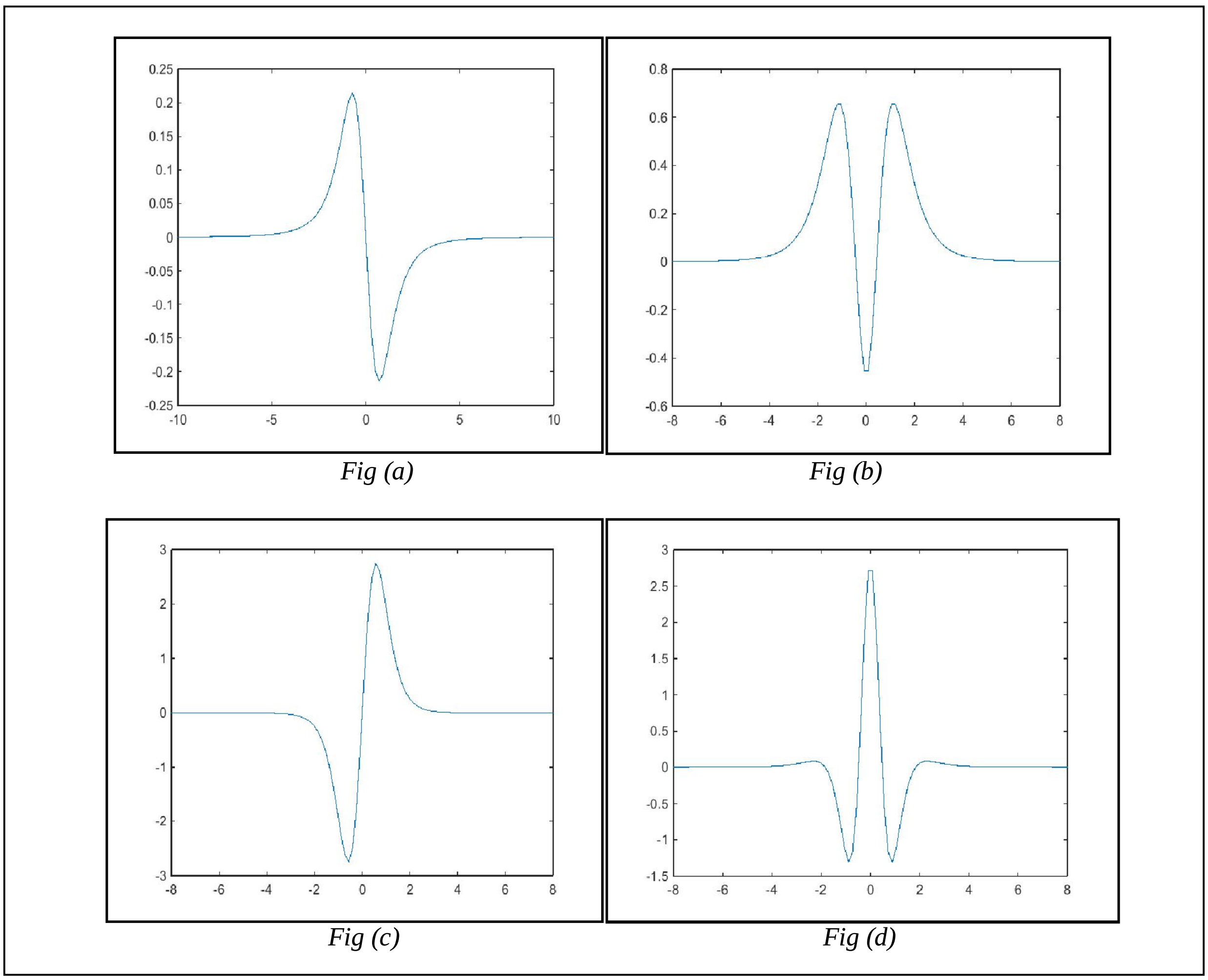

Figure 1:Fig (a) is 1st derivative, Fig (b) is 2nd derivative, Fig (c) is 3rd derivative and Fig (d) is 4rth derivative of p.d.f of t-distribution for v=2.

Derivatives of p.d.f of logistic distribution figures are:

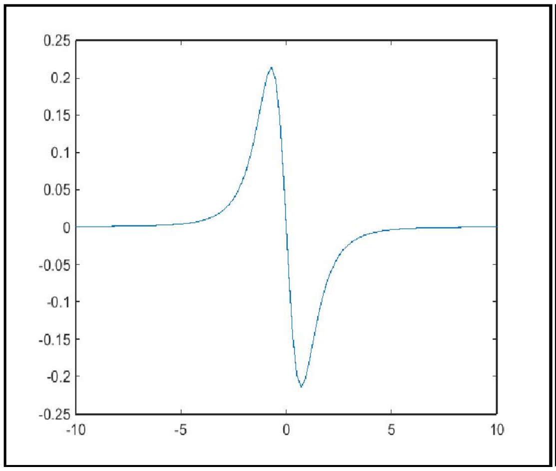

Fig (a.1)

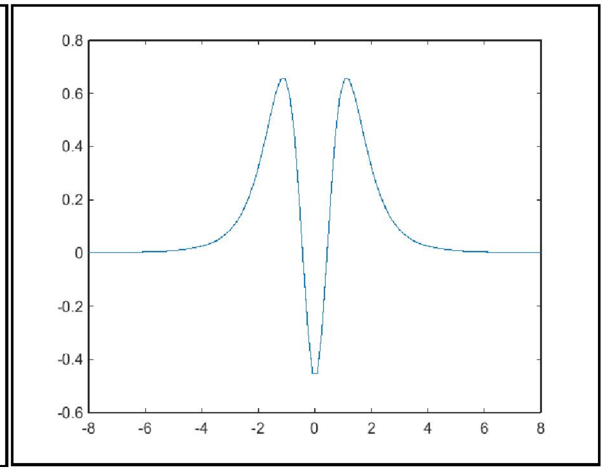

Fig (b.1)

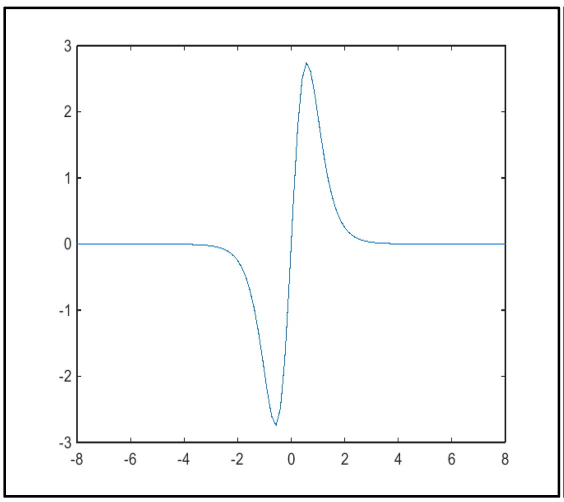

Fig (c.1)

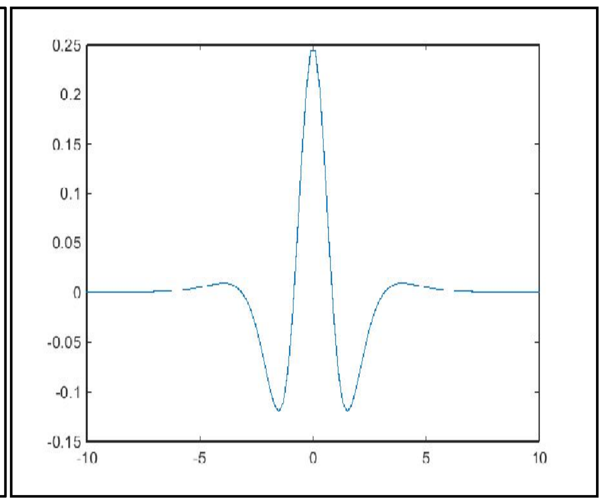

Fig (d.1)

Figure 2: Fig (a.1) is 1st derivative, Fig (b.1) is 2nd derivative, Fig (c.1) is 3rd derivative and Fig (d.1) is 4rth derivative of p.d.f of Logistic distribution.

	C_k		C_f			
Order of derivative of p.d.f	**t-dis.n for v=2**	**Logistic dis.n**	**α=1 of t-dis.n**	**α=2 of t dis.n**	**α=1 of logistic dis.n**	**α=2 of logistic dis.n**
1	0.0976	0.0333	0	0	0	0
2	0.9863	0.238	5.0514	8.9587	0.5	2
3	11.6612	0.0333	0	0	0	0
4	5.3745	0.0758	-0.9430	0	-0.2500	0

Table 1: C_k and C_f values of wavelets

2. Conclusion

There is a relationship between Wavelets and Probability density function. The above Theorem gives the proof of the same and the values mentioned in the Table shows practical results of derivatives of p.d.f of t-distribution and derivatives of p.d.f of logistic distribution. From these we conclude these are continuous wavelets.

3. References

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