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Spectrum Auction Theory and Spectrum Price Model

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Abstract:
Radio spectrum is scarce and invaluable telecommunication resource. Spectrum auction should determine spectrum price as consistent to its actual value. In order to ensure spectrum is efficiently assigned, spectrum is priced to reflect the value it can add to help promote economic and technical efficiency with users who have bid for it. Putting a price on spectrum not only ensures spectrum management efficiency, but can add revenue for the government which will in turn cover the cost of spectrum. This paper presents a radio spectrum valuation method and describes how to calculate reserve price of the spectrum. The contributions in this paper could assist telecom policy makers to gain more understanding in development of radio spectrum valuation.

Keywords: *Spectrum valuation, Spectrum auction, Reserve price.*

1. Introduction

Spectrum auction format has direct effects on market competition. The number of licenses dictates the number of competitors in the market while the license size affects quality of service. Since spectrum is a scarce resource, the license size becomes smaller as the number of licenses increases. Therefore, to determine the number of licenses and the license size, an auction designer must consider both market competition and technical requirements [i].

License sizes and the number of licenses are ones of the most important auction design elements because they dictate the post-auction market structure and competition. An insufficient number of licenses lead to monopolization. If there are too many licenses, operators may not have sufficient spectrum for quality service. Therefore, to choose proper spectrum size and the number of licenses, one must take expected auction outcomes into consideration. There are several choices of license sizes and the number of licenses.

After spectrum packaging has been chosen, the auction designer must choose an auction format that can allocate spectrum licenses efficiently. Hence, the auction format and spectrum packaging must be consistent. There are several auction formats: sealed-bid, ascending-bid and clock auctions [ii].

Even if the auction designer chooses an efficient auction format, the revenue may be small if competition in the auction is weak. Thus, the auction designer usually sets a reserve price to prevent the risk of low auction revenue. On the contrary, firms may not participate if the reserve price is too high. This in turn leads to inefficient allocation. Therefore, the auction designer must set a proper reserve price [iii].

To set a reserve price, the government policy is obligated to take all domestic stakeholders into consideration [iv]. Stakeholders are categorized into three groups. (1) Government: Government should receive reasonable revenue from economic rent that operators gain from acquired spectrum. (2) Consumers: Consumers should be able to receive quality mobile service enhanced by new spectrum at a fair price which can be achieved by competitive auction and mobile market. (3) Mobile operators: the reserve price should be set at the level that allows operators to make reasonable profit and have sufficient incentive to continuously invest in new technologies that benefit telecommunication services in the long run [v].

2. Spectrum Auction Formats

The standard auction formats can be classified into two groups, sealed-bid and open auctions [ii], [vi].

2.1. Sealed-bid Auction

In a sealed-bid auction, each bidder simultaneously submits a price he is willing to pay for the auctioned item in a sealed envelope. There are two main formats: first-price auction and second-price auction.

The first-price auction works as follows. Each bidder simultaneously submits a price he is willing to pay for the auction item in a sealed envelope. The bidder who submits the highest bid wins and pays his sealed bid.

The second-price auction works as follows. Each bidder simultaneously submits a price he is willing to pay for the auction item in a sealed envelope. The bidder who submits the highest bid wins and pays the second-highest bid. For example, in three-bidder case, the first, second and third bidders bid 100, 200 and 300 baht, respectively. The third bidder wins and pays 200 baht which is the second-highest bid.

2.2. Open Auction

The open auction is a format usually seen in series or movies. This format can be classified into two groups: ascending auction and descending auction.

The ascending auction works as follows. The seller sets the starting price. Each bidder expresses his intention to buy or reject the offered price. If there are more than one bidders accepting the offered price, the seller offers a higher price until there is only one bidder accepting the price.

The open descending auction works as follows. The seller sets the starting price. Each bidder expresses his intention to buy or reject the offered price. Since the auction starts at a high price, there is normally no bidder accepting the offered price. Then, the seller lowers the price until there is at least one bidder accepting the offer.

Under some standard assumptions, we can prove that the outcome of the first-price auction is the same as the open descending auction. Also, the second-price auction is the same as the open ascending auction. The revenues from these four auctions are equal.

3. Mathematical Model of the First- and Second-price Auctions

3.1. Mathematical Model of the First-price Auction

We can model the first-price auction with game theory and analyze Nash equilibrium as follows. The first-price auction game is the simultaneous game with the following format.

1. Player $i = 1,2,\dots,N$ where N is the number of bidders
2. Player i's strategy is the bid $b_i \geq 0$
3. Utility of bidder i from this game is equal to
    a. $u_i = v_i - b_i$ if b_i is the highest bid and bidder i is the only winner
    b. $u_i = (v_i - b_i)/k$ if b_i is the highest bid, bidder i is one of k joint winners
    c. $u_i = 0$ otherwise

where v_i is the maximum value that bidder i is willing to pay for the auctioned item. Assume that v_i is a private information and v_i follows a uniform distribution on the interval zero to one.

Under this model, we can show that bidder i's strategy in a Nash equilibrium is $b_i = (N-1)/N \cdot v_i$. For example, if the number of bidders is two and the bidder 1's maximum willingness to pay (v_i) is equal to 1/2, he will bid 1/4.

Under the Nash equilibrium, the average revenue is $R = E(\max(v_1, v_2, \dots, v_N))$ where max is the maximum function. In mathematics, we can prove that the average revenue is $(N-1)/(N+1)$. Figure 1 and 2 shows the relationship between bid value as well as the auction revenue and the number of bidders.

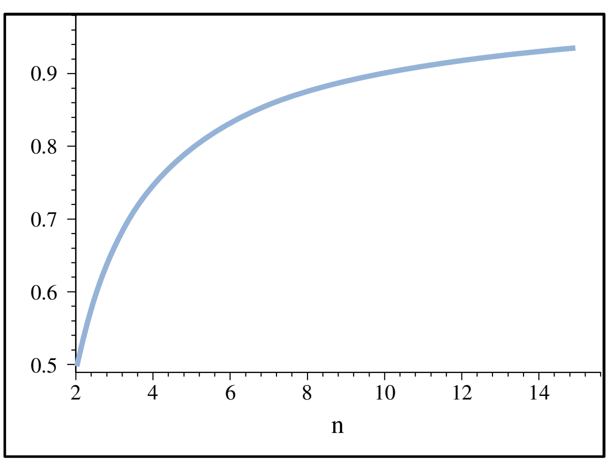

Figure 1: The relationship between bid value and the number of bidders

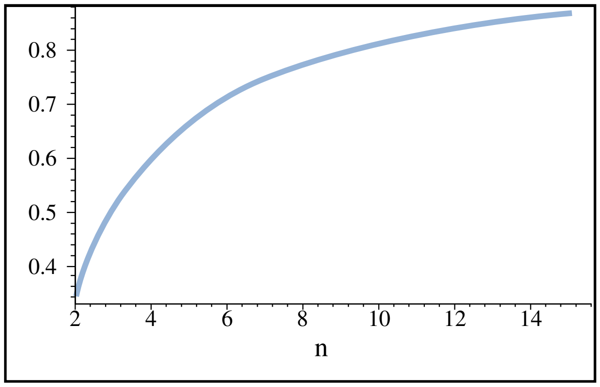

Figure 2: The relationship between the average revenue and the number of bidders

3.2. Mathematical Model of the Second-price Auction

We can model the second-price auction with game theory and analyze Nash equilibrium as follows. The first-price auction game is the simultaneous game with the following format.

1. Player $i = 1,2, \dots, N$ where N is the number of bidders
2. Player i's strategy is the bid $b_i \geq 0$
3. Utility of bidder i from this game is equal to
   a. $u_i = v_i - s_2$ if b_i is the highest bid and bidder i is the only winner
   b. $u_i = (v_i - s_2)/k$ if b_i is the highest bid, bidder i is one of k joint winners.
   c. $u_i = 0$ otherwise

where v_i is the maximum value that bidder i is willing to pay for the auctioned item. Assume that v_i is a private information and v_i follows a uniform distribution on the interval zero to one. s_2 is the second-highest bid.

Under this model, we can show that bidder i's strategy in Nash equilibrium is $b_i = v_i$ and the average revenue is $R = E(\mathrm{second}(v_1, v_2, \dots, v_N))$ where second is the second-highest value function. In mathematics, we can prove that the average revenue is $(N-1)/(N+1)$.

4. Optimal Reserve Price in Complete Information Setting

In the previous section, we study an auction without reserve price. The optimal reserve price is a part of auction design study, which is a subfield of game theory. The renowned work is that of William Vickrey, who received the Nobel Prize in 1994.

We use a simple game theoretical model to study and gain understanding into the effect of reserve price on auction outcome and determinants of an optimal reserve price. Under this model,

- The government has one spectrum license. The government auctions this license with the reserve price of R.
- There is one firm participating in the auction. Its maximum willingness to pay or valuation is V where V is the marginal benefit if it wins (the determinants of V is discussed in the Appendix).

Under the complete information setting, assume that the government and firm know the actual value of V.

4.1. Effect of Reserve Price on Firm's Payoff, Government's Revenue and Benefit to Consumers

Under this model, since there is only bidder, the strategy is to bid R if $V > R$ and not to participate if $V < R$. The firm's payoff is written as follows.

$$\text{Payoff} = V - R \text{ if } V > R$$
$$\text{Payoff} = 0 \text{ if } V < R \textit{ and does not participate}$$

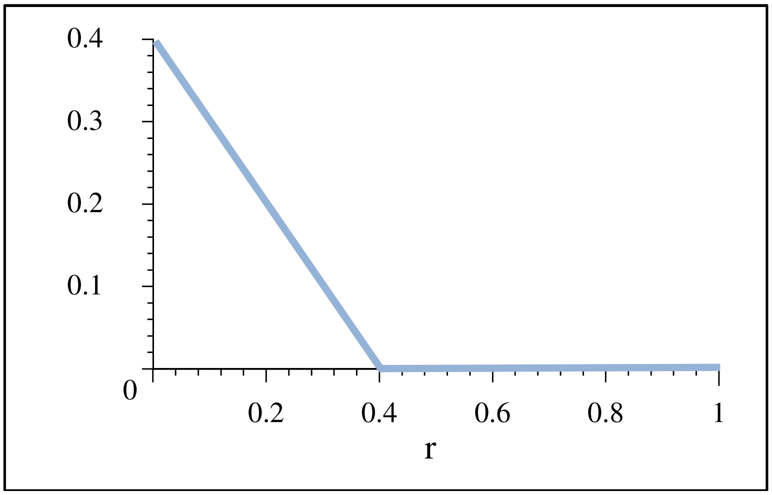

Figure 3: The relationship between reserve price and firm's payoff

Fig. 3, shows the relationship between the reserve price R and firm's payoff. The firm's payoff is decreasing in the reserve price and is equal to zero if R is greater than V.

The government's revenue from setting the reserve price R is the following.

$$\text{Revenue} = \text{R if V} > \text{R}$$
$$\text{Revenue} = 0 \text{ if V} < \text{R}$$

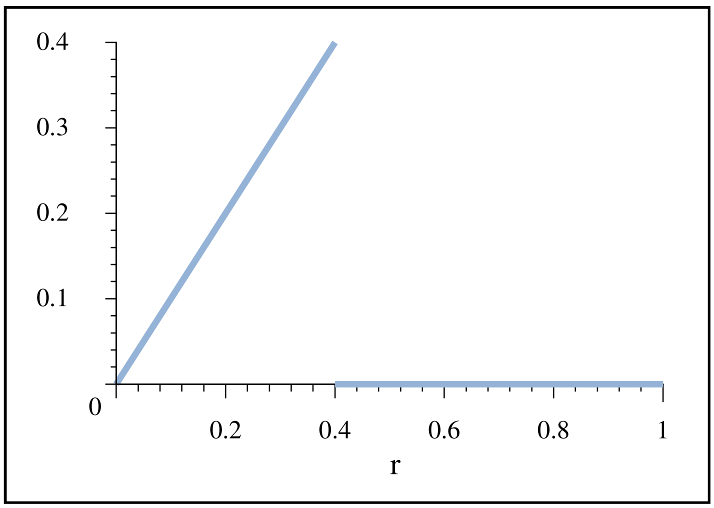

Figure 4: Reserve price R and the auction revenue if $V = 0.4$

Fig. 4, shows the relationship between the reserve price R and the auction revenue. The revenue is increasing in the reserve price up to R above the firm's maximum willing to pay.

For consumer's utility, let the consumer's utility denoted by K where K depends on the number of winning bidders but it does not depend on the winning bid value since an increase in the number of winning bidders leads to more competition as well as cheaper service with better quality (in this case, the number of winning bidders is one if $\text{R} < \text{V}$ and zero if $\text{R} > \text{V}$).

Since the winning bid value is a sunk cost, not a marginal cost, the microeconomic theory shows that the sunk cost does not affect the price consumers pay. Therefore, consumer's utility does not depend on the winning bid value. The consumer utility can be written as follows.

$$\text{Benefit to consumers} = \text{K if V} > \text{R}$$
$$\text{Benefit to consumers} = 0 \text{ if V} < \text{R}$$

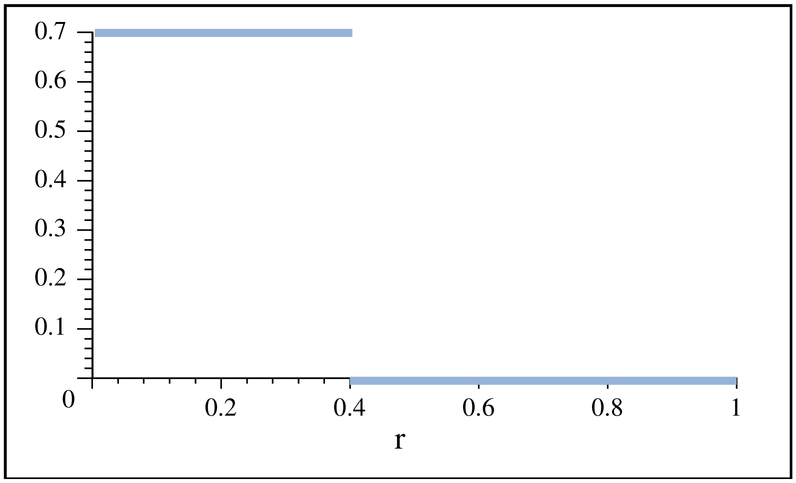

Figure 5: Reserve price R and benefit to consumers

Fig. 5, shows that the consumer's utility is a constant for $R < V$ and equal to 0 if $R > V$.

4.2. Optimal Reserve Price in Complete Information Setting

According to the relationship between firm's payoff, government's revenue and benefit to consumers studied in the previous section, we make the following conclusions.

- If the reserve price is too high ($R > V$), there is no auction winner and it thereby hurts the government's revenue, consumers and the firms.
- The reserve price has no effect on consumers if there is a winner ($0 < R < V$).

The firm's payoff is lower if the reserve price rises.

Objective	Optimal reserve price
Government's revenue	V
Benefit to consumers	$0 - V$
Benefit to firm	0

Table 1: Optimal reserve price

Table 1 shows the optimal reserve prices in various cases. For maximum revenue, the optimal reserve price is V. If $R = V$, the government's revenue is maximized at V. The firm's payoff is the lowest at 0. For maximum benefit to consumers, optimal R is between 0 and V. Finally, to maximize the firm's payoff, the optimal R is zero.

We can conclude that the optimal reserve price depends on auction objective and the firm's payoff from winning the auction V.

5. Optimal Reserve Price in Imperfect Information Setting

In the auction design literature, the optimal reserve price will maximize the seller's revenue in the imperfect information setting [vii]. Moreover, factors that affect the optimal reserve price are buyer and seller's payoffs, the number of auctioned items, the number of buyers and sellers, risk attitudes of buyers and seller and the distribution of buyer's maximum willingness to pay. Normally, the auctioned items are spectrum licenses. The seller is the government. Buyers are participating firms. Bidder's maximum willingness to pay is the firm's additional payoff or profit from the spectrum license [vi].

In reality, the government does not know the firm's payoff from winning the auction (V) but the government may be able to estimate the distribution of V. We consider a simple model to show an impact of reserve price to auction result. To simplicity of this model, we make the following assumptions.

- The government has one license for sale by an auction with the reserve price of R.
- In this auction, the bidder is a single firm. The buyer's maximum affordable value is V where V is distributed uniformly from 0.5 to 1.5.

In the next section, we will use this model to study the effect of reserve price to firm's payoff, government's revenue and consumer's benefit, and social welfare.

5.1. Effect of Reserve Price to Firm's Payoff, Government's Revenue and Consumer's Benefit

In this model, since there is only one bidder, the bidding strategy is to bid R if $V > R$ and not to participate if $V < R$. The firm's payoff is specified as follows.

$$\text{Firm's payoff} = V - R \text{ if } V > R$$
$$\text{Firm's payoff} = 0 \text{ if } V < R \textit{ and does not participate}$$

Therefore, the firm's expected payoff is

$$\text{Expected payoff} = (\mathrm{E}[\mathrm{V}|\mathrm{V} > R] - \mathrm{R}) \cdot \mathrm{prob}(\mathrm{V} > R) + 0 \cdot \mathrm{prob}(\mathrm{V} < R)$$

where $\mathrm{E}[\cdot]$ is the expectation function and prob is the probability function. When V is distributed between 0.5 to 1.5 billion baht,

For $\mathrm{R} < 0.5$, $\mathrm{E}[\mathrm{V}|\mathrm{V} > R] = 1$ and $\mathrm{prob}(\mathrm{V} > R) = 1$.

For $0.5 > R > 1.5$, $\mathrm{E}[\mathrm{V}|\mathrm{V} > R] = (0.5 + \mathrm{R})/2$

and $\mathrm{prob}(\mathrm{V} > R) = 1.5 - \mathrm{R}$

For $\mathrm{R} > 1.5$, $\mathrm{prob}(\mathrm{V} > R) = 0$

Therefore,

$\text{Firm's expected payoff} = 1 - \mathrm{R}$ for $\mathrm{R} < 0.5$

$\text{Firm's expected payoff} = ((1.5 + \mathrm{R})/2 - \mathrm{R})(1.5 - \mathrm{R})$ for $0.5 > R > 1.5$

$\text{Firm's expected payoff} = 0$ for $\mathrm{R} > 1.5$

Fig.6, shows the relationship between the firm's payoff and R

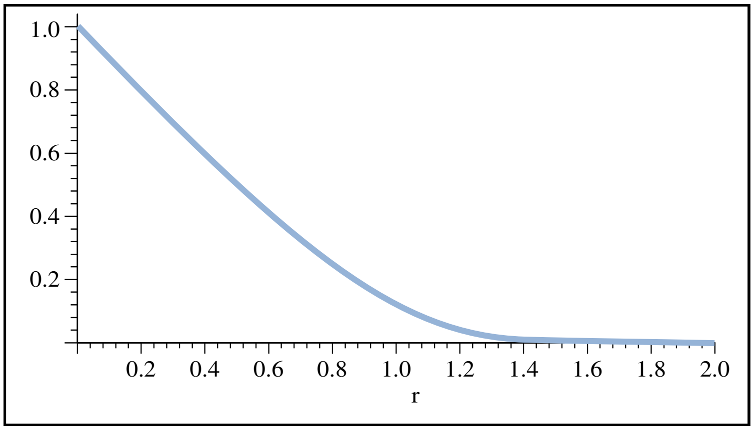

Figure 6: Firm's payoff and R

For the government's revenue, the government receives the auction proceed if there is an auction winner. The auction has a winner when $\mathrm{V} > R$ where $\mathrm{prob}(\mathrm{V} > R)$ is equal to one if $\mathrm{R} < 1$, equal to $2 - \mathrm{R}$ if R is between 1 and 2 and equal to zero otherwise. Therefore,

$\text{Government revenue} = \mathrm{R}$ for $\mathrm{R} < 0.5$

$\text{Government revenue} = \mathrm{R}(1.5 - \mathrm{R})$ for $0.5 < R < 1.5$

$\text{Government revenue} = 0$ for $\mathrm{R} > 1.5$

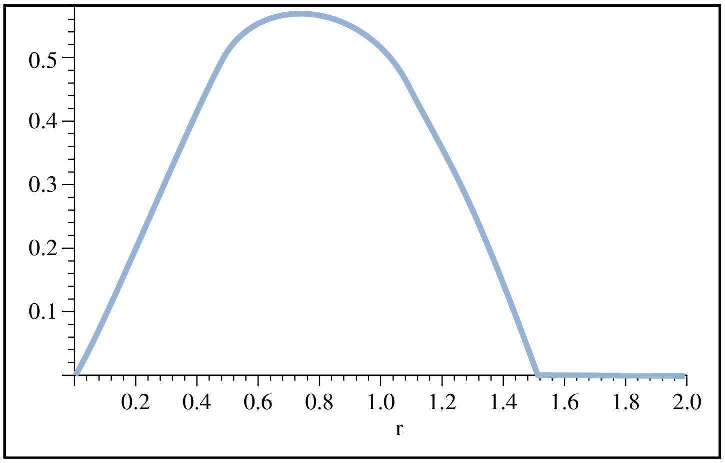

Figure 7: Government's revenue and R

According to Fig. 7, the relationship between the government's revenue and R has a bell-shape and the maximum point is at $\mathrm{R} = 0.75$. To calculate consumer's utility, let consumer's utility is equal to K where K depends on the number of winning bidders and does not depend on the winning bid value since an increase in the number of winning bidders leads to more competition and better quality of service. In this scenario, the number of winning bidders is one if $\mathrm{R} < V$ and zero if $\mathrm{R} > V$.

Since the winning bid value is a sunk cost, not a marginal cost, the microeconomic theory shows that the sunk cost does not affect the price consumers pay. Therefore, consumer's utility does not depend on the winning bid value. The consumer utility can be written as follows.

Benefit to consumers $= \mathrm{K}$ if $\mathrm{R} < 0.5$

Benefit to consumers $= \mathrm{K} \cdot \mathrm{prob}(\mathrm{V} > R) = \mathrm{K}(1.5 - \mathrm{R})$ if $0.5 < R < 1.5$

Benefit to consumers $= 0$ if $\mathrm{R} > 1.5$

Fig.8, shows the relationship between the consumer's benefit and reserve price.

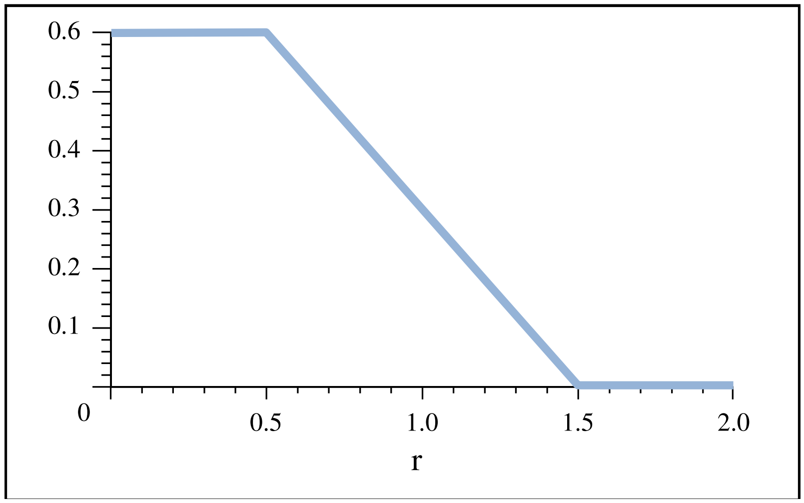

Figure 8: Consumer's benefit and R

5.2. Effect of Reserve Price to Total Welfare

This section studies total welfare of the state which is the sum of firm's payoff, government's revenue and consumer's benefit. The total welfare can be written as follows.

$$\mathrm{W} = \mathrm{a} \cdot \text{government revenue} + (1 - \mathrm{a})(\text{firm's payoff} + \text{consumer's benefit})$$

where a is the weight with value between 0 and 1. Value of a reflects how much weight the government put on the government's revenue relative to firm's and consumer's payoffs. a = 1 implies that the government concerns only about the government's revenue. a = 0 imples that the government does not consider the government's revenue at all.

Fig.9, shows the total welfare function and the reserve price R for a = 0, 0.3, 0.7 and 1, respectively. This figure demonstrates that when the government put less weight on the government's revenue and a is small, the optimal R is equal to zero, and in the case the government concerns more about the revenue, the optimal R is positive. Moreover, the optimal R is increasing in a.

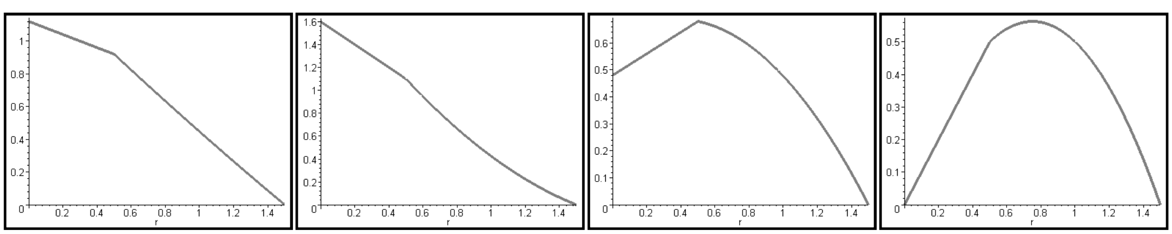

Figure 9: Total welfare and reserve price for $a = 0$, $a = 0.3$, $a = 0.7$ and $a = 1$, respectively

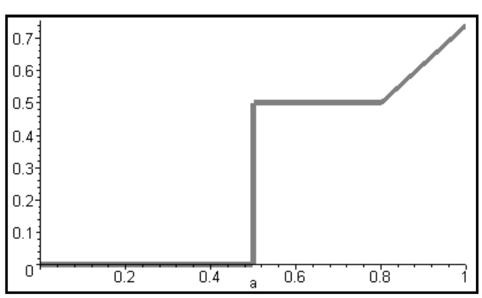

Figure 10: Optimal reserve price for a between 0 and 1

Fig.10, shows the optimal reserve price that maximizes the state's total welfare for a between 0 and 1. This figure demonstrates that if the government put less weight to the revenue (a < 0.5), the optimal reserve price is equal to zero. If a is between 0.5 and 0.8, the optimal reserve price is a constant equal to 0.5. If a is greater than 0.8, the optimal reserve price is greater than 0.5 and is increasing in a.

Next topic is a study of reserve price that maximizes revenue in a model with multiple bidders.

6. Optimal Reserve Price for Expected Revenue Maximization in Scenario with Multiple Bidders

In the previous section, we study the optimal reserve price when the government's objective includes maximizing the auction revenue, consumer's benefit and firm's payoff with a model with a single bidder. In this section, we study optimal reserve price to maximize revenue in case of multiple bidders.

We will study the reserve price using a mathematical model of the second-price auction since all four formats yields the same revenue and the second-price auction is the theoretically simplest auction.

Auction rules in this analysis are the following. Each bidder puts his bid in an envelope. The value in the envelope must be greater than or equal to the reserve price R. The bidder with the highest value is the winner. The winner needs to make monetary payment equal to the second-highest price.

There are N bidders in this auction indicated by $1,2,3,\ldots,N$ where bidder i values the auctioned item at V_i which is each bidder's private information. This value is distributed with cumulative density function $F(\cdot)$ and probability density function $f(\cdot)$. Economists have proved that in this auction, bidders will follow the strategy below.

Bidder i bids V_i if $V_i \geq R$

Bidder i will not participate if $V_i < R$

We can use the study of [Pendakur, K. (2002). Taking prices seriously in the measurement of inequality.Journal of Public Economics, 86(1), 47-69.] to prove that the expected revenue from this auction is

$$E[RV] = N\left(R\big(1-F(R)\big)G(R) + \int_r^{\infty} y\big(1-F(y)\big)g(y)dy\right)$$

where $G(\cdot) = F(\cdot)^{N-1}$ and $g(\cdot) = G'(\cdot)$.

After differentiate E[RV] and equate it to zero, the reserve price R that maximizes revenue is identified by the following equation.

$$R - \frac{\big(1-F(R)\big)}{f(R)} = 0$$

If V_i is uniformly distributed on the interval [0,1], $f(R) = 1$ and $F(R) = R$. Thus, the revenue-maximizing reserve price is $R = 1/2$ as follows.

$$R - \frac{\big(1-F(R)\big)}{f(R)} = 0 \Rightarrow R - (1-R) = 0 \Longrightarrow R = \frac{1}{2}$$

From the equation,

$$E[RV] = N\left(R\big(1-F(R)\big)G(R) + \int_r^{\infty} y\big(1-F(y)\big)g(y)dy\right)$$

We can prove that

$$E[RV|RV > 0] = \frac{N\big(R\big(1-F(R)\big)G(R) + \int_r^{\infty} y\big(1-F(y)\big)g(y)dy\big)}{1-F(R)^N}$$

To simplify the calculation, assume that V_i is uniformly distributed on the interval [0,1]. Hence Hence, $F(R) = R$ce, iformly distributed on the interval rve price pected revenue from this auction is ual n., $G(R) = R^{N-1}$, $G(y) = N$ and $g(y) = y^{N-2}(N-1)$ and the optimal reserve price is $R = 1/2$. So,

$$E[RV|RV > 0] = \frac{N\left(\frac{1}{2^{N-3}} + \frac{N\cdot 2^{N+1} - 2^{N+1} - N^2 - N + 2}{N(N+1)2^{N+1}}\right)}{1 - \frac{1}{2^N}}$$

Therefore, the ratio between the reserve price and revenue is

$$\frac{\text{Reserve price}}{\text{Revenue}} = \frac{\frac{1}{2}}{E[R|R > 0]} = \frac{1}{2}\cdot\frac{1-\frac{1}{2^N}}{N\left(\frac{1}{2^{N-3}} + \frac{N\cdot 2^{N+1} - 2^{N+1} - N^2 - N + 2}{N(N+1)2^{N+1}}\right)}$$

This ratio depends on the number of bidders N. Table 8.2 shows the ratio of reserve price to winning bid. If there is one bidder, this ratio is equal to one. It is equal to 0.90 if there are two bidders and is decreasing as there are more bidders. As shown in Table 2 and Fig.11, the ratio of the optimal reserve price and the winning bid have negative relationships with the number of bidders which represents the level of competition. This figure demonstrates that the ratio of reserve price for winning bid is decreasing if the number of bidders increases since the auction with many bidders already has high competition and so the auctioneer does not need to set a high reserve price. In contrast, in an auction with a few bidders, weak competition among bidders makes the revenue small. The auctioneer should set a high reserve price to increase an auction revenue.

Number of bidders	Reserve price/winning bid
1	1
2	0.90
3	0.82
4	0.77
5	0.72
6	0.69
7	0.66
8	0.64
9	0.62

*Table 2: Relationship between the optimal reserve price and auction revenue**
**Note: figures in the table are calculated under assumptions of uniform distribution of V_i as in the model above. Thus, the optimal ratio for Thailand may be different from the figures in the table.*

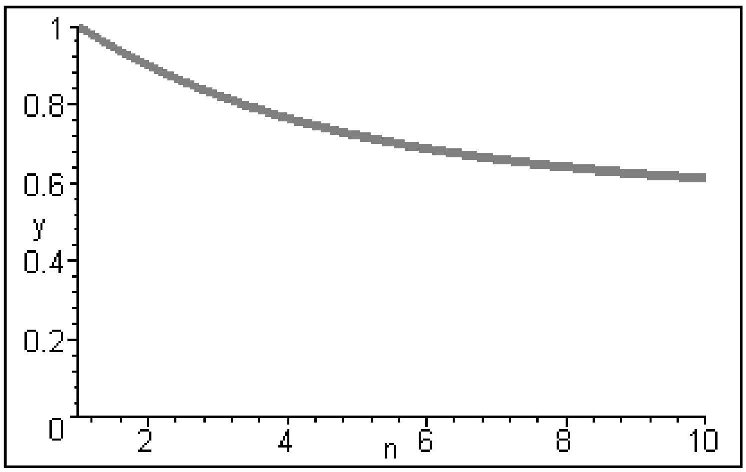

Figure 11: Ratio of reserve price to winning bid and the number of bidders

7. Future Research
For the future research, the ratios of reserve price to winning bid in many countries and effects of the radio will be studied. The distribution of the radio of reserve price to winning bid for the countries will be illustrated and analyzed.

8. Conclusion
Spectrum valuation is an important input for determining a proper reserve price of spectrum licenses. This paper aims to present a spectrum valuation method and describe how to determine a proper reserve price that makes the assignment of scare spectrum efficient and beneficial to consumers, licenses, and telecommunication industry as a whole.

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