INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

Buckling and Postbuckling Strength of CSCSTHIN Rectangular Plate

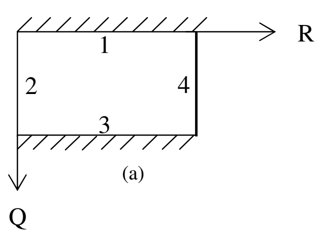
Dr. Ibearugbulem M. O.
Engineer & Senior Lecturer, Civil Engineering Department, Federal University of Technology, Owerri, Imo State, Nigeria
Dr. Ezeh J. C.
Engineer & Associate Professor, Civil Engineering Department, Federal University of Technology, Owerri, Imo State, Nigeria
Dr. Ettu L. O.
Engineer & Senior Lecturer, Civil Engineering Department, Federal University of Technology, Owerri, Imo State, Nigeria
Oguaghamba O. A.
Engineer & Lecturer, Civil Engineering Department, Imo State University, Owerri, Imo State, Nigeria
Ph.D. Student, Civil Engineering Department, Federal University of Technology, Owerri, Imo State, Nigeria

Abstract:
The objective of this study is to analyse the buckling and postbuckling strength of CSCSthin rectangular plate. Here, the exact displacement and stress profiles of the plate were obtained by applying the direct integration theory to the Kirchhoff's linear governing differential equation and von Karman's non–linear governing differential compatibility equation respectively. With these, the buckling and postbuckling load expression of the CSCS plate was obtained by applying work principle to the Von Karman's non–linear governing differential equilibrium equation. CSCS thin rectangular plate strength was obtained in terms of its yield/maximum stress. Other related parameters of the plate such as: displacement parameter, Wuv, stress coefficient, Wuv2 and load factor, Kcxwere determined. Results of this study show that for a CSCS plate material having yield stress of 250MPa, failure would occur only at 1.4h postbuckling out of plane deflection, contrary to the presumed critical buckling load. That is, CSCS plate would tolerate up to 56 MPa of additional load prior to its material and structural failure. Hence, CSCSpossesses additional strength beyond its critical buckling strength as had earlier predicted.

Keywords: *Buckling, Coupled Equations, Direct Integration, Postbuckling, Work Principle, Yield stress*

1. Introduction

Thin rectangular plate has four edges. The edges may be free, clamped, simply supported or mixed. Capital letters C, F, and S are commonly used to abbreviate or designate clamped edge, free edge, and simply supported edge respectively. The labelling and naming of CSCS plate in relation to the edge conditions, e along their strips are explained in Fig 1.



(a)

Edge	Label	Plate type
$e_1e_2e_3e_4{}^n$	$e_Ce_Se_Ce_S$	CSCS

(b)

Figure 1: Procedure for naming plate with different edge conditions

Postbuckling of plates may readily be understood through an analogy to a simple grillage model, as shown in Fig.2. In the grillage model, the continuous plate is replaced by vertical columns and horizontal ties. Under loading on the x – edges, the vertical columns will buckle. If they were not connected to the ties, they would buckle at the same load and no postbuckling reserve would exist. However, the ties are stretched as the columns buckle outward, thus restraining the motion and providing postbuckling reserve. The columns nearer to the supported edge are restrained more by the ties than those in the middle. This occurs too in a real plate, as more of the longitudinal in-plane compression is carried nearer the edges of the plate than in the center. Thus, the grillage model provides a working analogy for both the source of the postbuckling reserve and its most important result; i.e., re-distribution of longitudinal stresses.

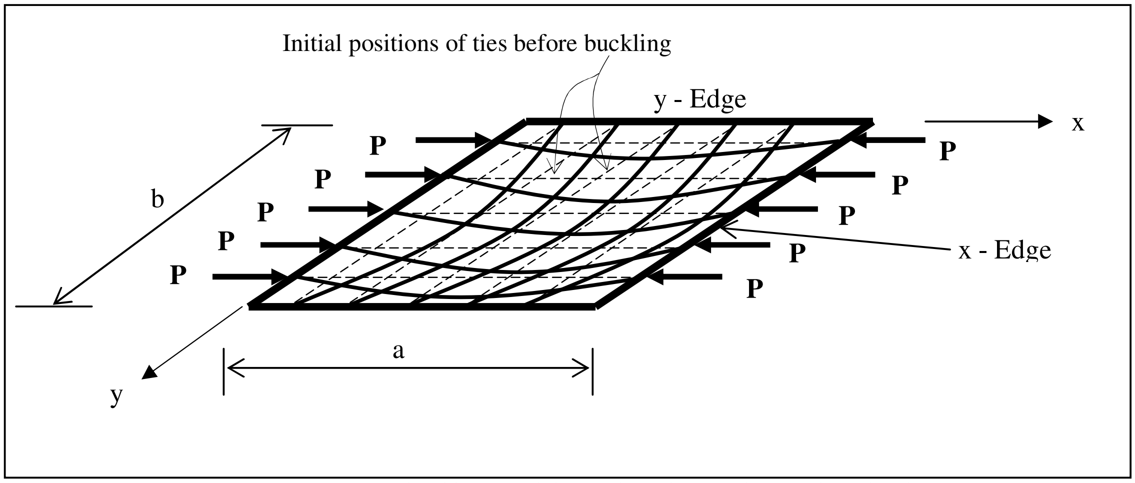


Figure 2: Post-buckling model of a thin plate under in-plane loads

In his context, Chaje (1974) defined postbuckling load as the increase in stiffness with increase in deflection characteristic of the plate. This represents possible resistance of axial load by plate at excess of the critical load subsequent to buckling. Hence, the postbuckling response of thin elastic plates is very important in engineering analysis. Therefore, concerted effort to thoroughly studying thin plates postbuckling behaviour becomes imminent.

Postbuckling load analysis of thin plates accounts for the membrane stretching and their corresponding strains and stresses, while buckling analysis accounts also for the membrane stretching but do not consider the corresponding strains and stresses developed by the stretching. Postbuckling load analysis of plate involves nonlinear large-deflection plate bending theory, contrary to buckling load study which is based on classical or Kirchhoff's linear theory of plates. Researchers have not done much on postbuckling behaviour of thin plates as its analysis involves nonlinear large-deflection plate theory, which usually reduces to two indeterminate nonlinear governing differential equations originally derived by Von Karman in 1910 (Rhodes, 2002, 2003). These equations are written as follows:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2\,\partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{h}{D}\left[\frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 \phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right] \tag{2}$$

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{dxdy} + N_y\frac{\partial^2 w}{\partial y^2} \tag{3}$$

where, ϕ is the stress function, w is deflection function, h is the plate's thickness and D is flexural rigidity. Equation 1 is the "Compatibility Equation". It ensures that in an elastic plate the in-plane and out-of-plane displacements are compatible. Equations 2 and 3 are based on equilibrium principles of stress and in-plane loads respectively. They are termed "Equilibrium equations" (Rhodes, 2002, 2003). Equations 1 and 2 are usually called Von Karman's coupled equations.

The exact solutions of these equations have been a rigor from the conceptual time to the recent time, in which the coupled solutions would give the buckling/postbuckling load of plates from which the true failure load is determined. This exact solutions of these equation is imminent, as the critical load predicted by buckling analysis is adjudged unsatisfactory (Chaje, 1974; Szilard, 2004).

Despite these revelations, very few researchers have made effort to solving these coupled equations to obtain the expressions for the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression. Researchers such as: Von Karman *et. al.* (1932), Marguerre (1937), Levy (1942), Timoshenko and Woinowsky – Krieger (1959), Volmir(1967), Iyengar (1988), Ventsel and Krauthammer (2001), Chai (2001); and Yoo and Lee (2011) have tried to solve these equations to obtain the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under uniaxial compression. They tried to solve the problem by assuming double trigonometric solutions for deflection, w and stress,ϕ functions to solve the governing differential equations of thin rectangular plates. In which case, the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression they obtained would also be said to be assumed, as the solutions of the governing differential equations of the plate (deflection and stress functions) were assumed abinitio. No researcher has bothered to solve for these parameters by the direct solution of these coupled governing differential equations.

In addition, these researchers restricted themselves to the use of either direct variational or indirect variational energy methods to finally evaluate the buckling/postbuckling load of this simply supported edges thin rectangular plate. None of the researchers considered applying direct work principle to finally evaluate the buckling/postbuckling loads of the CSCS plate or any other plate.

Von Karman evaluated the final buckling/postbuckling loads characteristics of SSSS plate by solving the equilibrium equation 3, after assuming trigonometric functions for deflection and stressVon Karman *et. al.* (1932). Marguerre (1937), Timoshenko and Woinowsky – Krieger (1959) and Volmir (1967) also assumed doubled trigonometric functions of deflection and stress; and employed the principle of minimum potential energy, rather than the equilibrium equation to furnish the final solution for the same SSSS plate. Iyengar (1988), Ventsel and Krauthammer (2001), Chai (2001) and Yoo and Lee(2011)also assumed doubled trigonometric functions of deflection and stress used Galerkin's energy methods to obtain the final buckling/postbuckling load of SSSS plate.

Researchers in later years very often assumed doubled trigonometric functions of deflection and stress and used a similar type of approach, i.e., combining an exact solution of the compatibility equation with either evaluation and minimization of the potential energy, or an approximate solution (for example, using Galerkin's method, Ritz method or Rayleigh-Ritz method) of the equilibrium equation.
In all these, none of these researchers obtained the displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx} associated with CSCS plate buckling and postbuckling characteristics, or any other plate. This situation has been the bane of comprehensive solution of the buckling/postbuckling characteristics of plates, as the actual yield/maximum stress of the plate could not be obtained, which this paper addressed.

2. The Direct Integration Approach for Exact General Deflection and Stress Profile for Buckling and Postbuckling of CSCS Plate

Oguaghamba (2015) used direct integral calculus approach and evaluated equation 3 to obtain the exact general displacement function of a buckled plate. The deflection function, W in its non – dimensional coordinates: R and Q is given as:

$$W(R,Q) = \Lambda \sum_{m=0}^{4}\sum_{n=0}^{4} U_m R^m V_n Q^n \tag{4}$$

where non – dimensional coordinates:R and Q in equation 4 relates to the usual independent coordinates x and y by the relation:

$$x = aR:\ \ 0 \le R \le 1 \ and\ y = bQ:\ \ 0 \le Q \le 1 \tag{5}$$

$\mathbf{U_m}$ **and** $\mathbf{V_n}$ are coefficients to be determined.
Solving equation 1 by direct integral calculus approach, the stress distribution of the plate at buckling and postbuckling load regimes is obtained(Oguaghamba, 2015). This expression in non-dimensional coordinates, R and Q is given as:

$$\phi(R,Q) = \frac{Ep^2\Lambda^2}{(1+2p^2+p^4)}\Bigg[\Bigg[\left(\frac{{U_1}^2}{24}R^4 + \frac{U_1U_2}{30}R^5 + \frac{1}{180}\left[2{U_2}^2 + 3U_1U_3\right]R^6 + \frac{1}{210}\left[2U_1U_4 + 3U_2U_3\right]R^7 + \frac{1}{1680}\left[16U_2U_4 + 9{U_3}^2\right]R^8 + \frac{U_3U_4}{126}R^9 + \frac{{U_4}^2}{315}R^{10}\right) \times \left(\frac{{V_1}^2}{24}Q^4 + \frac{V_1V_2}{30}Q^5 + \frac{1}{180}\left[2{V_2}^2 + 3V_1V_3\right]Q^6 + \frac{1}{210}\left[2V_1V_4 + 3V_2V_3Q^3\right]Q^7 + \frac{1}{1680}\left[16V_2V_4 + 9{V_3}^2\right]Q^8 + \frac{V_3V_4}{126}Q^9 + \frac{{V_4}^2}{315}Q^{10}\right) - \left(\frac{U_0U_2}{12}R^4 + \frac{1}{60}\left[3U_0U_3 + U_1U_2\right]R^5 + \frac{1}{180}\left[6U_0U_4 + 3U_1U_3 + {U_2}^2\right]R^6 + \frac{1}{210}\left[3U_1U_4 + 2U_2U_3\right]R^7 + \frac{1}{840}\left[3{U_3}^2 + 7U_2U_4\right]R^8 + \frac{U_3U_4}{168}R^9 + \frac{{U_4}^2}{420}R^{10}\right)\left(\frac{V_0V_2}{12}Q^4 + \frac{1}{60}\left[3V_0V_3 + V_1V_2\right]Q^5 + \frac{1}{180}\left[6V_0V_4 + 3V_1V_3 + {V_2}^2\right]Q^6 + \frac{1}{210}\left[3V_1V_4 + 2V_2V_3\right]Q^7 + \frac{1}{840}\left[3{V_3}^2 + 7V_2V_4\right]Q^8 + \frac{V_3V_4}{168}Q^9 + \frac{{V_4}^2}{420}Q^{10}\right)\Bigg]\Bigg] - \frac{N_{cx}b^2}{2h}Q^2 \tag{7}$$

$\mathbf{U_m}$ **and** $\mathbf{V_n}$coefficients in equation 4 and were determined by Oguaghamba (2015) using the Benthem's boundary conditions ofCSCS plate as follows:
$U_o = 0;\ U_1 = U_4;\ U_2 = 0; U_3 = -2U_4;\ U_4 = U_4;\ V_o = 0;\ V_1 = 0;\ V_2 = V_4; V_3 = -2V_4;\ V_4 = V_4$
Hence, the CSCS plate displacement and stress profiles in buckling and postbuckling regimes are obtained by substituting these coefficients into equations 4 and 7 as:

$$W(R,Q) = W_{uv}(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$W(R,Q) = W_{uv}h_1(R,Q) \tag{8}$$

where, $W_{uv} = \Lambda U_4 V_4$

$$h_1(R,Q) = (R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \tag{9}$$

$$\phi(R,Q) = \frac{\varphi W_{uv}^2}{3175200}\left[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(14Q^6 - 36Q^7 + 39Q^8 - 20Q^9 + 4Q^{10}) - 3(-14R^6 + 6R^7 + 6R^8 - 5R^9 + R^{10})(14Q^6 - 48Q^7 + 57Q^8 - 30Q^9 + 6Q^{10})\right] - \frac{N_{cx}b^2}{2h}Q^2 \tag{10}$$

$$\phi(R,Q) = \varphi W_{uv}^2 h_2(R,Q) - \frac{N_{cx}b^2}{2h}Q^2 \tag{11}$$

where,

$$\varphi = \frac{Ep^2}{(1+2p^2+p^4)} \tag{12}$$

W_{uv}^2 = Stress function coefficient for a plate in postbuckling regime
Λ^2 = Consolidated coefficient factor of stress in postbuckling regime
$h_2(R,Q)$ = Non-dimensional stress shape (profile) function of the slightly bent plate, given as:

$$h_2(R,Q) = \frac{1}{3175200}\left[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(14Q^6 - 36Q^7 + 39Q^8 - 20Q^9 + 4Q^{10}) - 3(-14R^6 + 6R^7 + 6R^8 - 5R^9 + R^{10})(14Q^6 - 48Q^7 + 57Q^8 - 30Q^9 + 6Q^{10})\right] \tag{13}$$

Expressions for the deflection and stress functions factors, W_{uv} and W_{uv}^2 of the plate behaviour under pre – buckling, buckling and post buckling regimes deduced by Oguaghamba (2015) is given as:

$$W_{uv} = 51.2\alpha h; W_{uv}^2 = W_{uv}^2 = 2621.44\alpha^2 h^2 \tag{14}$$

3. Work Principle Application for Buckling and Postbuckling Load and Stress of CSCS Plate

Oguaghamba (2015) applied the work principle according to Ibearugbulem *et al.*(2013, 2014) to equation 2 in non – dimensional coefficient and obtained the exact general buckling and postbuckling load, N_{cx} (R, Q) of thin rectangular plates in non – dimensional coordinates as in equation 15.

$$N_{cx} = -\frac{\frac{49}{484}\int\int_{0,0}^{1,1}\left(\frac{\partial^4 h_1}{p^2 \partial R^4}\cdot h_1 + 2\frac{\partial^4 h_1}{\partial R^2 \partial Q^2}\cdot h_1 + p^2\frac{\partial^4 h_1}{\partial Q^4}\cdot h_1\right)\mathrm{dRdQ}}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)dRdQ}\frac{\pi^2 D}{b^2} +$$
$$\frac{\frac{294}{121}(1-\mu^2)p^2 W_{uv}^2}{(1+2p^2+p^4)h^2}\frac{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial Q^2}\frac{\partial^2 h_2}{\partial R^2} + \frac{\partial^2 h_1}{\partial R^2}\cdot\frac{\partial^2 h_2}{\partial Q^2} - 2\frac{\partial^2 h_1}{\partial R\,\partial Q}\frac{\partial^2 h_2}{\partial R\,\partial Q}\right)\cdot h_1 \mathrm{dRdQ}}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)\mathrm{dRdQ}}\frac{\pi^2 D}{b^2} \quad (15)$$

where the first and the second terms account for critical buckling load of the plate and the gain in load of the plate at postbuckling regime respectively.

Substituting the expressions of $h_1(R,Q)$ and $h_2(R,Q)$ into equation 15; solving out the resulting integrand expressions gave the buckling and postbuckling load expression for a CSCS thin rectangular plate as:

$$N_{cx} = \left[\left(\frac{1.00048614}{P^2} + 2.42975207 + 5.16917842P^2\right) + 5.43794706\times10^{-5}\frac{(1-\mu^2)p^2 W_{uv}^2}{(1+2p^2+p^4)h^2}\right]\frac{D\pi^2}{b^2} \quad (16)$$

Introducing the expression of W_{uv}^2 given in equation 14 into equation 16; the buckling and postbuckling load expression for a CSCS thin rectangular plate reduced to:

$$N_{cx} = \left[\left(\frac{1.00048614}{P^2} + 2.42975207 + 5.16917842P^2\right) + 1.29722793\times10^{-1}\frac{p^2\alpha^2}{(1+2p^2+p^4)}\right]\frac{D\pi^2}{b^2} \quad (17)$$

$$N_{cx} = K_{cx}\frac{D\pi^2}{b^2} \quad (18)$$

$$K_{cx} = \left(\frac{1.00048614}{P^2} + 2.42975207 + 5.16917842P^2\right) + 1.29722793\times10^{-1}\frac{p^2\alpha^2}{(1+2p^2+p^4)} \quad (19)$$

where, K_{cx} is the buckling and postbuckling load coefficient.

Oguaghamba [14] also evaluated the inplane and bending buckling and postbuckling yield stress developed by the CSCS as:

$$\sigma_{cri} = \left[\left(\frac{1.00048614}{P^2} + 2.42975207 + 5.16917842P^2\right) + 1.29722793\times10^{-1}\frac{p^2\alpha^2}{(1+2p^2+p^4)}\right]\frac{D}{hb^2}\pi^2 +$$
$$51.20\left[\frac{1.125}{P^2} + 0.5623\right]\frac{D}{hb^2}\alpha \quad (20)$$

4. Results and Discussions

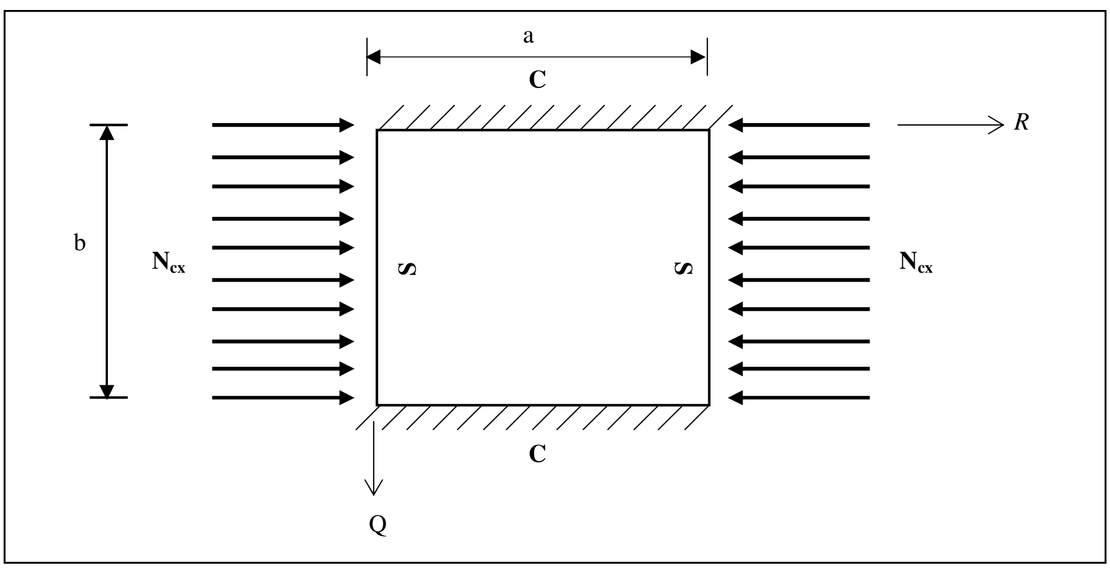


Figure 2: CSCS – Thin Rectangular Plate under Uniaxial Load

Fig. 2 shows a CSCS thin rectangular plate subjected to uniaxial compression loads on the R - edges. The interest is to evaluate the buckling and postbuckling load of the plate.

Iyengar [10]; Ventsel and Krauthammer [11]; Szilard [4]; and Yoo and Lee [13] in their separate works obtained the buckling and postbuckling load of SSSS – thin rectangular plate only as:

$$\mathbf{N_{cx}} = \left[\left(\frac{\mathbf{1}}{\mathbf{P^2}} + \mathbf{2} + \mathbf{P^2}\right) + \mathbf{3}\left(\frac{\mathbf{1}}{\mathbf{p^2}} + \mathbf{p^2}\right)\frac{\mathbf{W_{11}^2}}{\mathbf{h^2}}\frac{(\mathbf{1}-\boldsymbol{\mu}^2)}{\mathbf{4}}\right]\frac{\mathbf{D\pi^2}}{\mathbf{b^2}} \quad (\mathbf{21})$$

The stress function coefficient, W_{uv}^2 in the present study is well defined in equation 14 for CSCS plate. This is not the case for the stress function coefficient, W_{11}^2 in the literature formulation for SSSS plate, as given in equation 21; while that for CSCS plate has not been studied. Hence, the stress function coefficient, W_{11}^2 in the literature formulation has no empirical interpretation. This leaves the literature formulation as a mere theoretical exercise rather than real life adventure. The present study clearly defined these parameters: the displacement parameter, W_{uv}, stress coefficient,${W_{uv}}^2$ and load factor, K_{cx}. With this parameters, the present study obtained the critical yield stress of the CSCS plate under buckling and postbuckling loads as given in equation 20. Therefore, equations 19 and 20 can be used to obtain the actual value of the buckling and postbuckling load and critical yield stress of an SSSS plate, knowing other parameters: deflection coefficient, α; Poisson ration, μ; breadth, b; aspect ratio, p and thickness, h of the plate.

For instance, an ASTM grade A36 thin rectangular steel plate possessing CSCS edge conditions; subjected to uniformly distributed in-plane load on its R – edge, b and having the following physical and geometric properties as: breadth, b = 4000mm; thickness of plate, h = 20mm; yield load, σ_{ys} = 250 MPa; Ultimate Stress, σ_u = 400 – 550 MPa; Poisson's ratio, μ = 0.30; Modulus of elasticity, E = 200 GPa; density of plate, ρ = 7,800 kg/m^3. The buckling and postbuckling load coefficient and critical yield stress of the plate through unit aspect ratio and deflection coefficients range: $0 \leq \alpha \leq 5.0$ are shown in Fig. 3 and Fig. 4 respectively.

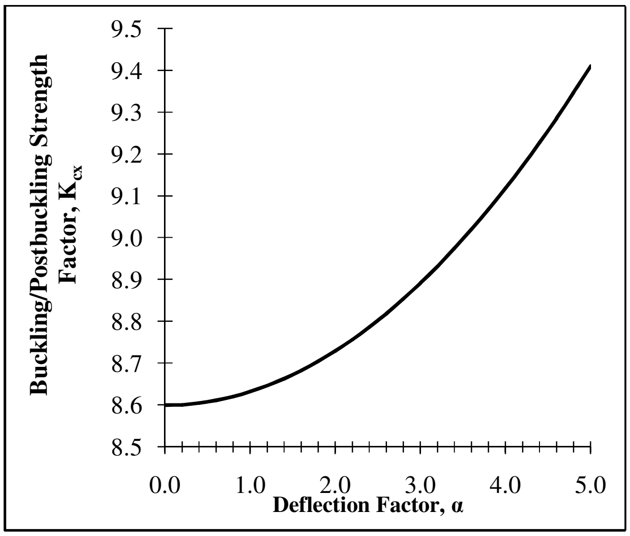


Figure 3: Buckling and Postbuckling Load Coefficient, K_{cx} and Deflection Factor, α at aspect ratio of unity for CSCS – Plate

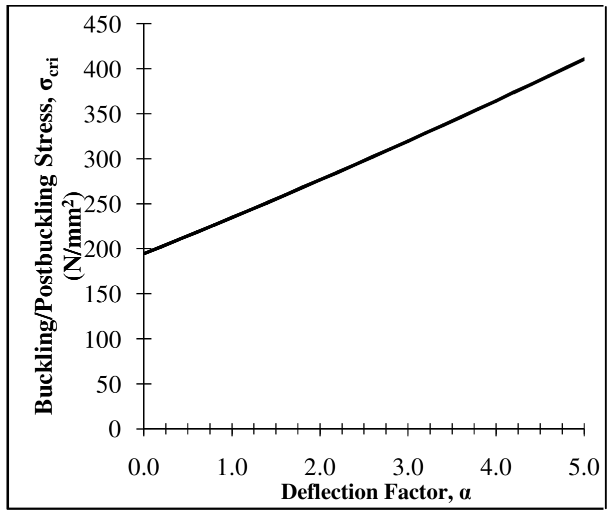


Figure 4: Buckling and postbuckling load critical yield/maximum stress, $\sigma_{cri}$and deflection Factor, α at aspect ratio of unity for CSCS – Plate

In Fig. 3, the graph shows that the buckling and postbuckling load parameter, K_{cx} increases quadratically as the out of plane deflection factor, α increases. The buckling and postbuckling load parameters, K_{cx} are higher at other aspect ratios lower than 1.0. Thus, the behaviour of buckling and postbuckling load parameter which is a function of the buckling and postbuckling load means that the buckling and postbuckling load would continue to increase as the out of plane deflection increases. This is contrary to the literature's hypothesis that the axial stiffness reduces, as the plate as a whole sustains increase in load after buckling or deflection (Oguaghamba, 2015).

However, this hypothesis is clarified in Fig. 4. The linear relationship in the yield stress behaviour against out of plane deflection explained that the plate would resist extra in-plane load after buckling, while reduces in material stiffness. That is, the plate resists further in-plane load due to postbuckling reserve but loses stiffness due to in-plane bending stress developed. Where the in-plane load bending stress is not considered, the plate would behave as if it had higher yield stress, which it does not. Fig. 4 also show that for a CSCS plate material having yield stress of 250MPa, failure of such plate under in-plane loading would not occur until the out of plane deflection of the plate is about (1.4h). It is at this point that the induced stress in the plate would reach the failure stress for the plate material, which may cause failure of the plate. The buckling and postbuckling stresses, σ_{cri} are even higher at other aspect ratios lower than 1.0, as shown in Tabe 1. CSCS plate possesses such increased load resistance because the edges which are simply supports and clamped allow stretching of the longitudinal fibers of the plate on deformation of the transverse fibers. In this way, the longitudinal fibers of the plate would undergo stress redistribution under load, as well as develop transverse tensile stresses after buckling. These tensile stresses provide the postbuckling reserve load. Thus, additional load may often be applied to some geometric deformation without reaching material yield stress or imposing structural damage to the plate.

However, as the structural requirements for plates are that the structure should not be so flexible that the behaviour causes alarm or discomfort to the users; other structural criteria may be applied to select the applicable load below this yield stress, which is also far above the buckling load.

For instance, at zero deformation (critical buckling load), the yield stress of the plate for aspect ratio of unity is 194.462 MPa. This is below the design yield stress of the plate. Extra 56 MPa on 1.4h deformation can be tolerated by the plate prior to material and structural. This happens at postbuckling regime.

p	$0.00 \leq \alpha \leq 1.80$									
	0	**0.20**	**0.40**	**0.60**	**0.80**	**1.00**	**1.20**	**1.40**	**1.60**	**1.80**
	$\sigma_{cri} - Values\ (MPa)$									
0.5	174.666	198.421	222.214	246.044	269.911	293.817	317.759	341.740	365.758	389.813
0.6	159.872	177.184	194.542	211.946	229.395	246.890	264.431	282.017	299.649	317.326
0.7	158.395	171.823	185.303	198.834	212.418	226.053	239.740	253.479	267.269	281.112
0.8	165.107	176.014	186.977	197.996	209.070	220.201	231.387	242.629	253.927	265.281
0.9	177.560	186.738	195.974	205.269	214.621	224.032	233.500	243.027	252.611	262.254
1.0	194.462	202.404	210.404	218.463	226.580	234.756	242.991	251.284	259.637	268.047
p	$2.00 \leq \alpha \leq 3.80$									
	2.00	**2.20**	**2.40**	**2.60**	**2.80**	**3.00**	**3.20**	**3.40**	**3.60**	**3.80**
	$\sigma_{cri} - Values\ (MPa)$									
0.5	413.906	438.037	462.205	486.410	510.653	534.934	559.252	583.608	608.002	632.433
0.6	335.050	352.819	370.633	388.494	406.400	424.351	442.349	460.392	478.481	496.615
0.7	295.006	308.952	322.950	336.999	351.100	365.254	379.459	393.715	408.024	422.384
0.8	276.691	288.156	299.677	311.254	322.887	334.576	346.321	358.121	369.978	381.890
0.9	271.955	281.713	291.530	301.404	311.337	321.328	331.377	341.483	351.648	361.871
1.0	276.517	285.045	293.632	302.277	310.981	319.744	328.566	337.446	346.384	355.382
p	$5.00 \leq \alpha \leq 5.00$									
	4.00	**4.20**	**4.40**	**4.60**	**4.80**	**5.00**				
	$\sigma_{cri} - Values\ (MPa)$									
0.5	656.901	681.407	705.951	730.532	755.150	779.807				
0.6	514.795	533.021	551.292	569.609	587.972	606.381				
0.7	436.796	451.260	465.776	480.343	494.963	509.634				
0.8	393.858	405.882	417.962	430.097	442.289	454.536				
0.9	372.152	382.491	392.887	403.342	413.855	424.426				
1.0	364.438	373.553	382.726	391.959	401.249	410.599				

Table 1: CSCS Plate Buckling and Postbuckling Strength, σ_{cri}

5. Conclusion

Whereas the previous study did not analysed the buckling and postbuckling load characteristics of CSCS plate, this paper analysed the buckling and postbuckling load characteristics of CSCS plate. Whereas the double trigonometric functions have been adjudged inadequate for the analysis of thin plates' postbuckling load characteristics, this study obtained exact displacement and stress profiles of buckling and postbuckling load characteristics of CSCS plate by direct integration of the governing differential equations of the plate and implored the work principle technique to finally evaluating the buckling and postbuckling load of CSCS plate. In addition to the buckling and postbuckling load and yield stress obtained for CSCS plate, the study obtained other parameters of the CSCS plate under buckling and postbuckling regimes such as: displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx}. With all these, the study explained stiffness loss behaviour of plate in postbuckling regime. Thus, the study found out that CSCS plate would accommodate more loads beyond the critical buckling load, prior to actual material failure in its postbuckling regime. For CSCS plate's of higher yield stress, failure would be due to geometric or permissible deflection criteria. The study also revealed that plate deforms along the transverse direction, leading to the stretching of the longitudinal fibers of the plate, when uniaxially loaded. In this way, the longitudinal fibers of the plate would undergo stress redistribution, as well as develop transverse tensile stresses. These tensile stresses provide the postbuckling reserve load.

6. References

i. Chai,H. (2001). Contact Buckling and Postbuckling of Thin Rectangular Plates, Journal of the Mechanics and Physics of Solids, vol. 49, pp. 209 – 30.

ii. Chaje,A. (1974). Principles of Structural Stability Theory, London: Prentice - Hall Inc.

iii. Ibearugbulem,O. M.,Ettu, L. O. andEzeh, J. C. (2003). Direct Integration and Work Principle as New Approach in Bending Analyses of Isotropic Rectangular Plates, The International Journal of Engineering and Science (IJES), vol. 2 (3), pp. 28 – 36.

iv. Ibearugbulem,O. M., Oguaghamba,O. A.,Njoku, K. O. andNwaokeorie, M. (2014). Using Line Continuum to Explain Work Principle Method for Structural Continuum Analysis, International Journal of Innovative Research and Development, vol. 3 (9), pp. 365 – 70.

v. Iyengar,N. G. (1988). Structural Stability of Columns and Plates, New York: Ellis Horwood Limited.

vi. Levy,S. (1942). Bending of Rectangular Plates with Large Deflections, National Advisory Council for Aeronautics,NACA Technical Report, Washington D.C., No. 737.

vii. Marguerre,K. (1937). Effective Width of Plates in Compression,National Advisory Council for Aeronautics, NACA Technical Note, Washington D. C., No. 833.

viii. Oguaghamba,O. A. (2015). Analysis of Buckling and Postbuckling Loads of Isotropic Thin Rectangular Plates,Ph.D. Thesis Submitted to Postgraduate School, Federal University of Technology, Owerri, Nigeria.

ix. Rhodes,J. (2002). Buckling of Thin Plates and Members — and Early Work on Rectangular Tubes, Thin-Walled Structures.vol. 40, pp. 87–108.

x. Rhodes,J. (2003). Some observations on the Postbuckling Behaviour of Thin-Walled Members,Thin-Walled Structures. vol. 41, pp. 207 – 26.

xi. Szilard,R. (2004). Theories and Application of Plates Analysis: Classical, Numerical and Engineering Methods, New Jersey: John Wiley and Sons Inc.

xii. Timoshenko, S. P. and Woinowsky - Krieger,S. (1959). Theory of Plates and Shells, 2nd ed., Auckland: McGraw-Hill Inc.

xiii. Ventsel,E. and Krauthammer,T. (2001). Thin Plates and Shells: Theory, Analysis and Applications, New York: Maxwell Publishers Inc.

xiv. Volmir,A. S. (1967).A Translation of Flexible Plates and Shells, Air Force Flight Dynamics Laboratory, Ohio: Wright-Patterson Air Force Base, Technical Report No. 66-216.

xv. Von Karman,T.,Sechler, E. E. and Donnel,L. H. (1932). Strength of Thin Plates in Compression, Trans ASME, vol. 54, pp. 53 – 4.

xvi. Yoo, C. H. and Lee, S. C. (2011). Stability of Structures: Principles and Applications, Amsterdam: Butterworth-Heinemann.