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Effect of Thermal Radiation on Steady MHD Thermal Slip Boundary Layer Flow over a Flat Plate with Variable Fluid Properties

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Abstract:
The present study is an attempt to describe the physical properties of a two dimensional MHD steady fluid flow over a flat plate with thermal radiation and thermophoresis. In this paper, thermal conductivity and viscosity is considered as temperature dependent. Here the Prandtl number is treated as a variable rather than a constant. The non-dimensional equations are solved by fourth order Runge-kutta method along with Shooting technique. The results are described by the graphs and with its descriptions.

Keywords: *Radiation, Thermophoresis, Variable thermal conductivity, Slip*

1. Introduction

The boundary layer flows over a stretching sheet are very useful in the engineering applications such as flows occur in the extrusion process, glass fiber and paper production, hot rolling, wire drawing, electronic chips, crystal growing, plastic manufactures, and aerodynamic extrusion of plastic sheets. Convective phenomena are directly influenced by this surface and dynamic causing significant air mixing with horizontal as well as vertical turbulences. Very large number of research papers is available for boundary layer flows with heat transfer induced by a stretching sheet. Rajeswari et al. [1] have investigated chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a vertical porous surface in presence of suction. Mahdy [2] has studied the effect of chemical reaction and heat generation or absorption on double diffussive convection from vertical truncated cone in a porous media with variable viscosity. Pal and Talukdar [3] have studied perturbation analysis of unsteady magnetohydrodyanamic convective heat mass transfer in boundary layer slip flow past a vertical permeable plate with a thermal radiation and chemical reaction. Further the effect of thermal radiation, heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in presence of transverse magnetic field was investigated by Makinde and Ogulu [4]. Mahmoud [5] presented variable viscosity and chemical reaction effect on mixed convection heat and mass transfer along a semi-infinite vertical plate. Al-odat and Al-Azab [6] studied the influence of chemical reaction on transient MHD free convection over a moving vertical plate. Choudhury and Jha [7] have established the effect of chemical reaction on MHD micro-polar fluid flow past a vertical plate in slip-flow regime. The effect of thermal radiation on electrical conducting fluid and mass transfer in a rotating system with periodic suction along a vertical infinite plate studied by Parida et.al [8]. Rout et al. [9] studied the influence of chemical reaction and radiation on MHD heat and mass transfer fluid flow over a moving vertical plate in presence of heat source with convective boundary condition. Singh and Naveen Kumar [10] studied the free convection effects on flow past an exponentially accelerated vertical plate. Yao et al. [11] have studied heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. Recently, Rahman et. al [12,13], have studied many thermal boundary layer fluid flow problems with variable viscosity and thermal conductivity.

2. Mathematical Formulation

A two dimensional MHD boundary layer flow of a viscous incompressible, electrically conducting fluid moving over the surface of a semi-infinite impermeable flat plate is considered with a uniform velocity U_∞ in presence of heat source and radiation. The viscosity and thermal conductivity of the fluid are assumed to be functions of temperature. The external electric field is assumed to be zero and the magnetic Reynolds number is assumed to be small. Hence, the induced magnetic field is small compared with the externally applied magnetic field. The left surface of the plate is being heated by convection from a hot fluid at temperature T_f that gives a heat transfer coefficient h_f as a function of x, with its strength: $h_f(x) = h_{f_0} x^{-\frac{1}{2}}$, $h_{f_0} \neq 0$ and T_∞ is the temperature of the fluid away from the plate. Here the x-axis be taken along the direction of plate and y-axis is normal to it. A magnetic field is applied in the direction perpendicular to the plate with varying strength M as a function of x, which is given by $M(x) = M_0 x^{-\frac{1}{2}}$, $M_0 \neq 0$, where x is the co-ordinate along the plate. As here there is no applied electric field; Hall effect and Joule heating effect are neglected. Within the framework of the above-noted assumptions subject to the Boussinesq approximation can be given by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \left(\frac{\sigma_0 M^2(x)}{\rho}\right)(u - U_\infty) \quad .....(2)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y} + Q_0(T - T_\infty) \quad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(V_T C) \quad (4)$$

Boundary conditions:

$$u(x,0) = L\frac{\partial u}{\partial y}(x,0),\ v(x,0) = 0,\ -K(T)\frac{\partial T}{\partial y}(x,0) = h_f[T_w - T(x,0)],\ C_w(x,0) = Ax^\lambda + C_\infty$$

$$u = U_\infty,\ T = T_\infty,\ C = C_\infty \quad \text{when } y \to \infty \quad (5)$$

The subscripts *w* and ∞ refer to the wall and boundary layer edge, respectively. L is the slip length, C_w is the species concentration at the plate surface, A>0 is a constant, V_T is the thermophoretic velocity and λ is a real number and $K(T)$ is the thermal conductivity coefficient which is function of T.

For a viscous fluid the viscosity dependence on temperature T is of the form

$$\mu(T) = \mu_\infty\left(\frac{1}{1 + \gamma(T - T_\infty)}\right) = \frac{\mu_\infty \theta_r}{\theta_r - \theta} \quad (6)$$

γ is the variable thermal conductivity parameter, μ_∞ is the thermal conductivity of the fluid far away from the plate, Where $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$, $A = \frac{\gamma}{\mu_\infty}$, and $T_r = T_\infty - \frac{1}{\gamma}$

The thermal conductivity varies linearly with temperature for liquid metals. So the thermal conductivity is given by

$$K(T) = K_\infty\left(1 + \varepsilon\left(\frac{T - T_\infty}{T_w - T_\infty}\right)\right) \quad (7)$$

Where T_w is the reference temperature at the plate, ε is the variable thermal conductivity parameter and K_∞ is the thermal conductivity of the fluid far away from the plate.

The thermophoretic velocity V_T is given as

$$V_T = -\frac{t_p \mu(T)}{\rho T_r}\frac{\partial T}{\partial y} \quad (8)$$

Where T_r is some reference temperature and t_p is the thermophoretic coefficient.

For a similarity solution of equation (1) - (5), defining an independent variable η and a dependent variable f as

$$\eta = y\sqrt{\frac{U_\infty}{\nu_\infty x}}, \quad \psi = \sqrt{\nu_\infty x U_\infty}\, f(\eta) \quad (9)$$

By using the Rosseland approximation the value q_r is given by

$$q_r = -\frac{4\sigma^*}{3k_s}\frac{\partial T^4}{\partial y} \quad (10)$$

Where σ^* is the Stefan-Boltzman constant and k_s is the Rosseland mean absorption coefficient. Assuming the temperature difference within the flow are sufficiently small, by Taylor series expansion neglecting the higher order terms, T^4 can be expressed as a linear function of temperature of the form

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \qquad (11)$$

By using (10) and (11)

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k_s}\frac{\partial^2 T}{\partial y^2} \qquad (12)$$

Introducing the following non-dimensional parameters

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\quad Pr_\infty = \frac{\mu_\infty C_p}{K_\infty},\ \tau = \frac{-t_p(T_w - T_\infty)}{T_r},\quad R = \frac{16\sigma^* T_\infty^3}{3k_s K_\infty},$$

$$Sc = \frac{\nu_\infty}{D},\ \nu_\infty = \frac{\mu_\infty}{\rho}\ Gr_x = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2},\ Gc_x = \frac{g\beta^*(C_w - C_\infty)x}{U_\infty^2},\ Nc = \frac{C_\infty}{C_w - C_\infty},$$

$$S_x = \frac{Q_0 x}{U_\infty \rho c_p},\ Ha = M_0\sqrt{\frac{\sigma_0}{\rho U_\infty}},\ a = \frac{h_{f_0}}{K_\infty}\sqrt{\frac{\nu_\infty x}{U_\infty}},\ \delta = L\sqrt{\frac{U_\infty}{x\nu_\infty}} = Kn_{x,L}\, Re_x^{\frac{1}{2}}\ \text{(liquid)}$$

$$\delta = \frac{2-\sigma}{\sigma} Kn_{x,\delta}\, Re_x^{\frac{1}{2}}\ \text{(gas)}$$

Where $Kn_{x,L} = \frac{L}{x}$, $Kn_{x,\delta} = \frac{\lambda}{x}$, $Re_x^{\frac{1}{2}} = \frac{U_\infty x}{\nu_\infty}$

Using (6)- (12) in the system of Eq. (1) - (5), the reduced non-dimensional Eqs. are given by

$$\frac{\theta_r}{\theta_r - \theta} f''' + \frac{\theta_r}{(\theta_r - \theta)^2} f''\theta' + \frac{1}{2} f'' f - Ha^2(f' - 1) + Gr_x\,\theta + Gc_x\,\varphi = 0 \qquad (13)$$

$$(1 + R + \varepsilon\theta)\theta'' + \varepsilon(\theta')^2 + \frac{1}{2}\, Pr_\infty\, f\,\theta' + Pr_\infty\, S_x\,\theta = 0 \qquad (14)$$

$$\phi'' + \frac{1}{2} Sc\, f\,\phi' - \tau Sc\,\frac{\theta_r}{\theta_r - \theta}\left(\theta''\phi + \theta'\phi' + \frac{\phi(\theta')^2}{\theta_r - \theta} + Nc\left(\theta'' + \frac{(\theta')^2}{\theta_r - \theta}\right)\right) = 0$$

…… (15)

Corresponding boundary conditions are

$$f(0) = 0,\ f'(0) = \delta f''(0),\ \theta'(0) = a\left[\frac{\theta(0) - 1}{1 + \varepsilon\theta(0)}\right],\ \phi(0) = 1$$

$$f'(\infty) = 1,\ \theta(\infty) = 0,\ \phi(\infty) = 0 \qquad (16)$$

Again $Pr = \frac{\mu(T)C_p}{K(T)} = \frac{\theta_r}{(\theta_r - \theta)(1 + \varepsilon\theta)} Pr_\infty$ (17)

With the use of equation (17) the non-dimensional temperature equation (14) can be written as

$$(1 + R + \varepsilon\theta)\theta'' + \varepsilon(\theta')^2 + \frac{1}{2}\, Pr\left(1 - \frac{\theta}{\theta_r}\right)(1 + \varepsilon\theta)(f\,\theta' + S_x\,\theta) = 0 \qquad (18)$$

Where the value of θ_r cannot be equal to zero or $\theta(0)$ as for $\theta_r \in (0, \theta(0))$ no solutions could be found. The value $\theta_r \to \infty$ was managed by setting $1/\theta_r = 0$ in the governing equation.

3. Result Discussion

The solution of the above non-dimension equations are solved by using fourth order Runge-kutta method along with shooting technique. The numerical results of non-dimensional velocity, temperature and concentration computed for various values of physical parameters Ha = 1, Gr = Gc = 5,R = 0.2, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, ε = 0.5, τ = 0.1 and δ = 0.5 are shown.

VELOCITY PROFILE - In Figure 1 it is observed that there is no change of velocity due to thermal conductivity parameter ε but the velocity is lower for the case of no-slip than the presence of slip. Effects of the thermal radiation parameter R on the velocity is displayed in Figure 2. Increasing the radiation enhances the heat transfer rate as well as the velocity boundary layer thickness which leads to more velocity. In Figure 3, for θ_r > 0, velocity increase with the increase of θ_r. But from the Figure 4 it is observed that for θ_r <0, velocity profiles decrease on the boundary with the increase of magnitude of θ_r .

TEMPERATURE PROFILE -The dimensionless temperature for different values of ε is displayed in Figure 5 and it decreases as the variable thermal conductivity parameter ε increases. The Figure 6 shows the dimensionless temperature for different values of *R,* and it indicates that the temperature distribution increases as the radiation parameter *R* increases. The Figure7 displays the plate surface temperature increases with an increase in the local surface convection parameter due to convective heat exchange between the hot fluids on the lower side of the plate to the cold fluid on its upper surface; consequently, the thickness of the thermal boundary layer is enhanced. The temperature is higher for the case of no-slip than the presence of slip. The effect of Schmidt number Sc on temperature is displayed in Figure 8 and it is observed that temperature increases as increase in Sc.

CONCENTRATION PROFILE - From Figure 9, it is observed that the dimensionless concentration slightly decreases as the radiation parameter *R* increases. The thermophoretic parameter τ is expected to alter the concentration boundary layer significantly. The dimensionless concentration decreases by increasing τ value which is displayed in Figure 10.

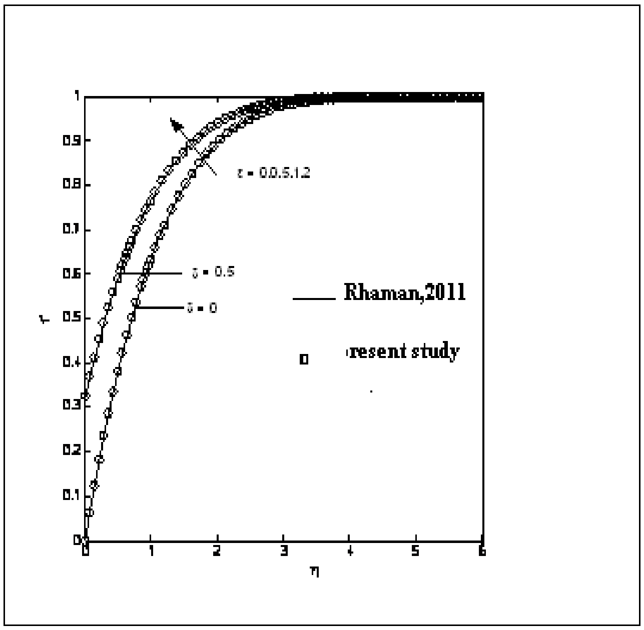

Figure 1: Comparison of velocity distribution with Rahman for various values of ε and δ. Ha = 1, Gr = Gc = 5,R = 0.2, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1.

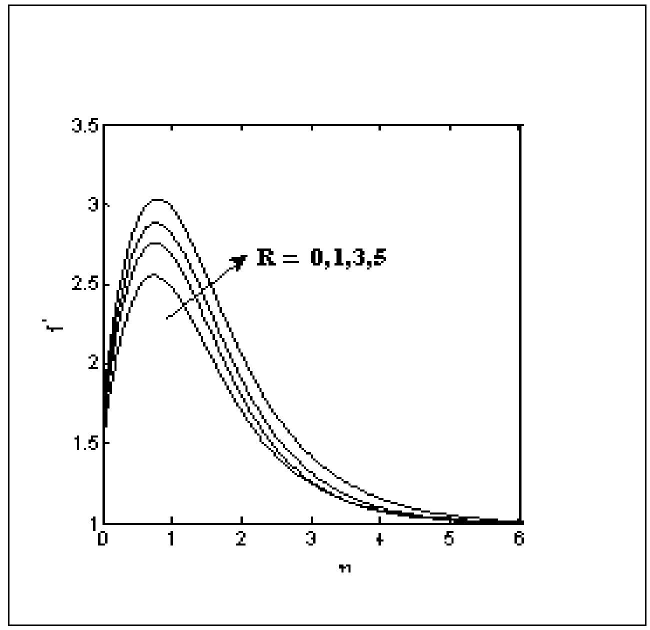

Figure 2: velocity for various values of R. Ha = 1, Gr = Gc = 5, ε =0.5 , θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

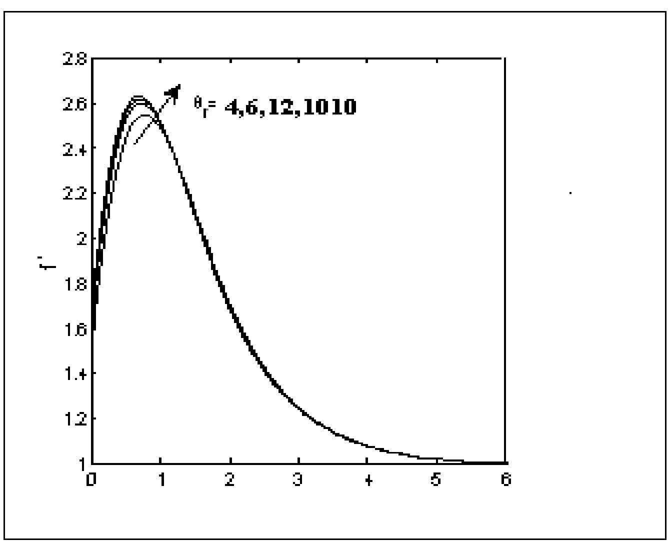

Figure 3: velocity for various values of θ_r.>0. Ha = 1, Gr = Gc = 5, ε =0.5, R =0.2,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

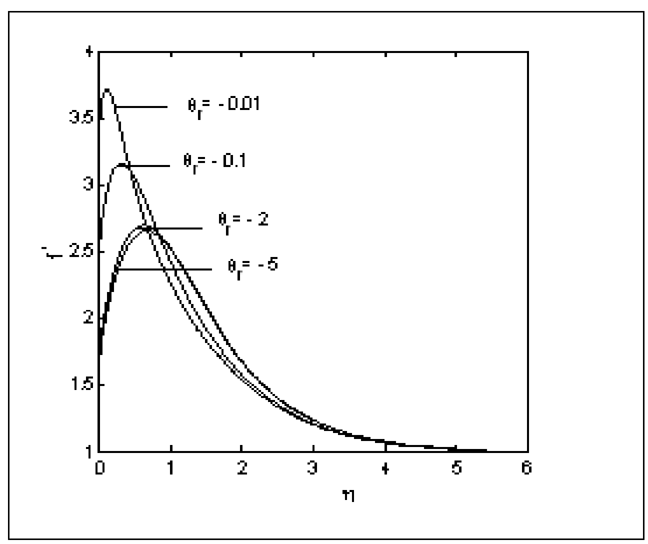

Figure 4: velocity for various values of θ_r<0. Ha = 1, Gr = Gc = 5, ε =0.5 ,R =0.2,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

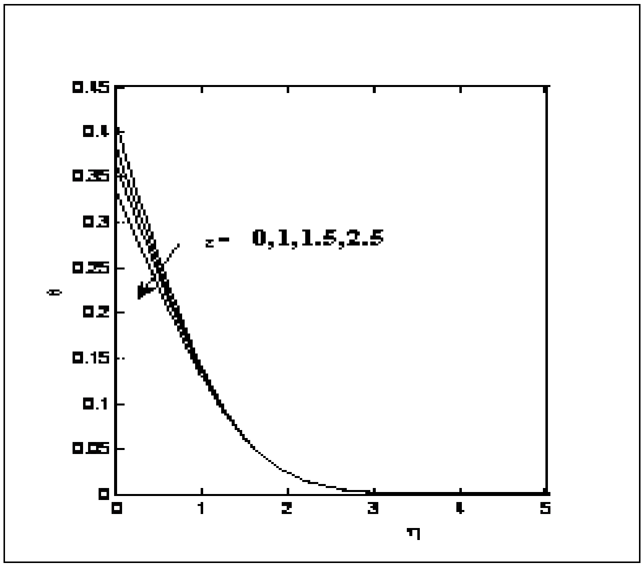

Figure 5: Temperature for various values of ε. Ha = 1, Gr = Gc = 5, R = 0.2, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

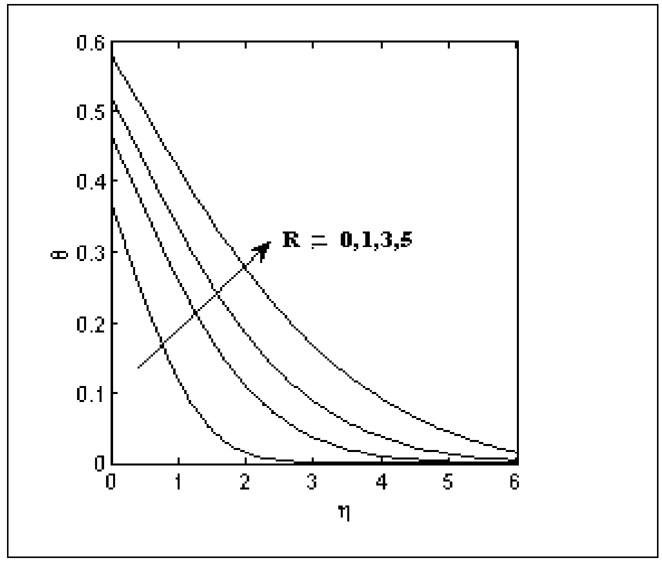

Figure 6: Temperature for various values of R. Ha = 1, Gr = Gc =5 ε = 0.5, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

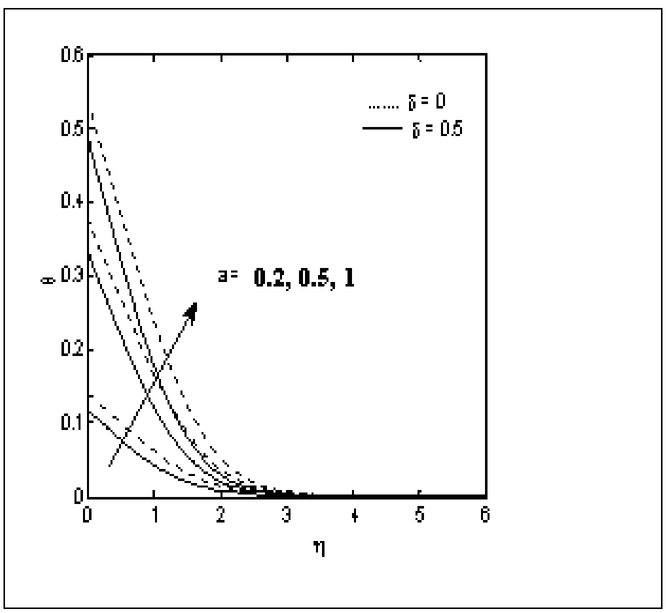

Figure 7: Temperature for various values of a. Ha = 1, Gr = Gc = 5, ε = 0.5, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2, R = 0.5,τ = 0.1 and δ = 0.5

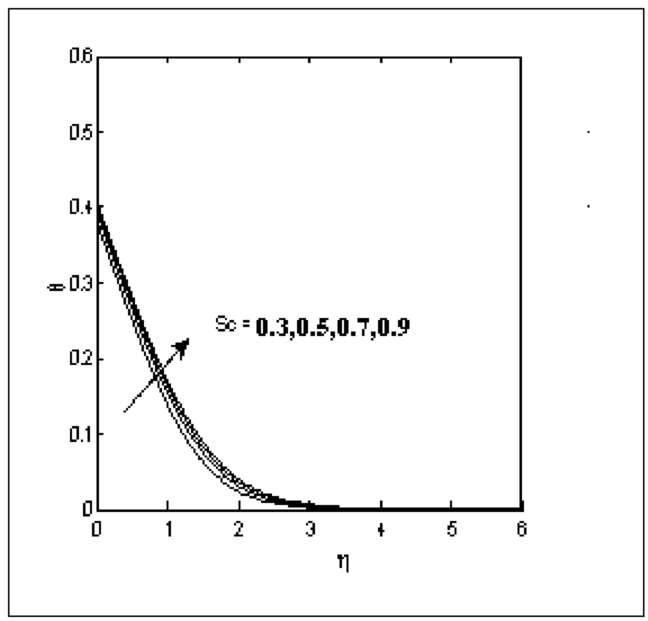

Figure 8: Temperature for various values of Sc. Ha = 1, Gr = Gc = 5, ε = 0.5, θ_r = 3, Pr = 1, S_x = 0.1,R= 0.5,Nc = 0.2,a = 0.5, τ = 0.1 and δ = 0.5

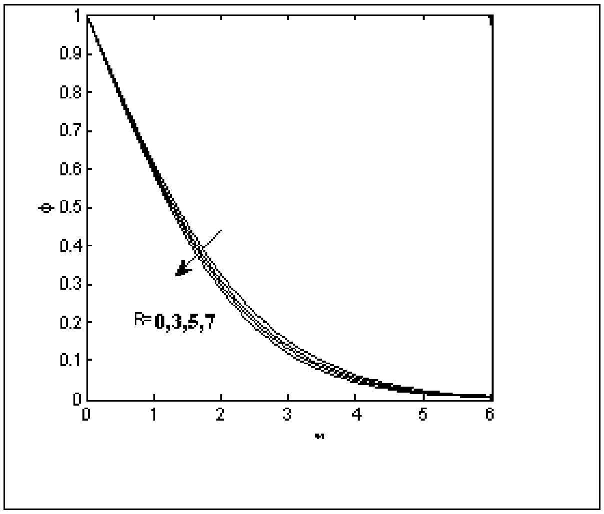

Figure 9: Concentration for various values of R with Ha = 1, Gr = Gc = 5,R = 0.2, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, ε = 0.5, τ = 0.1 and δ = 0.5

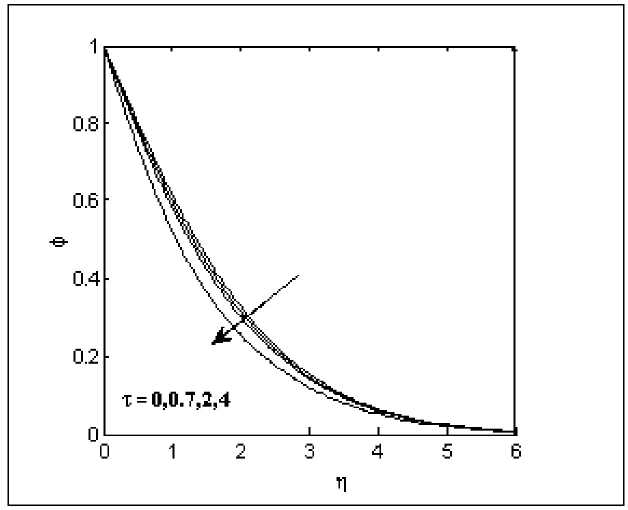

Figure10: Concentration for various values of τ with Ha = 1, Gr = Gc = 5,R = 0.2, θ_r = 3,Pr = 1,S_x = 0.1,Sc = 0.22,Nc = 0.2,a = 0.5, ε = 0.5, and δ = 0.5.

4. Conclusion

The effects of variable viscosity and variable thermal diffusivity on steady heat and mass transfer process in a two- dimensional MHD convective flow over a flat plate with partial slip at the surface subjected to the convective boundary condition with thermophoresis are studied. Finally, the following conclusions are drawn:

- The velocity and temperature distribution increases with higher value of the radiation parameter.
- The temperature is higher for the case of no- slip than the presence of slip
- The concentration decreases by increasing the thermophoretic parameter.

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