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The M/M/1/N Interdependent Queueing Model with Controllable Arrival Rates, Reverse Balking and Reverse Reneging

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Abstract:
In this Paper, an M/M/1/N interdependent queueing model with controllable arrival rates, reverse balking and reverse reneging is considered. This model is much useful in analysing the particular situations arising at the places like a data voice transmission, computer communication system etc. The steady state solution and the system characteristics are derived for this model. The analytical results are numerically illustrated and the effects of the nodal parameters on the system characteristics are studied.

Keywords: *Finite Capacity, Interdependent Controllable Arrival and Service rates, Reverse balking, Reverse reneging, Single Server, Bivariate Poisson process.*

1. Introduction

Queue is an unavoidable phenomenon of modern life that we encounter at every step in our daily life. In real practice, it is often likely that an arrival become discouraged when queue is long and may not wish to enter the queue. This type of arrival is called balking. The notion of customer balking appears in queuing theory in the works of Haight [1]. He has analysed M/M/1 queue with balking in which queue length is infinite. Jain and Rakesh Kumar [2] have studied M/M/1/N queueing system with reverse balking (a queueing system that indicates the probability of balking will be low when the queue size is more). On the other hand, a customer may enter the queue, but after a time lose patience and decide to leave the queue. This type of arrival is called reneging. Rakesh Kumar and Bhupender Kumar Som [3] have studied an M/M/1/N queuing system with reverse balking and reverse reneging (a queueing system that indicates the probability of reneging will be less when there are more number of customers in the system).

Along with several other assumptions, it is customary to consider that the arrival and service processes are independent. However, in many particular situations, it is necessary to consider that the arrival and services processes are inter dependent. A queueing model in which arrivals and services are correlated is known as interdependent queuing Model. Much work has been reported in the literature regarding interdependent standard queuing model with controllable arrival rates. K. Srinivasa Rao, Shobha and P. Srinivasa Rao [4] have discussed M/M/1/ ∞ interdependent queuing model with controllable arrival rates. A. Srinivasan and M. Thiagarajan [5,6], have analysed M/M/1/K interdependent queuing model with controllable arrival rates, M/M/C/K/N interdependent queuing Model with controllable arrival rates balking, reneging and spares. Recently S. Sasikala and M. Thiagarajan [7] have studied the M/M/c/N interdependent queuing Model with controllable arrival rates and reverse balking.

In this paper, an M/M/1/N interdependent queueing model with controllable arrival rates is considered with the assumption that the arrival and service processes of the system are correlated and follows a bivariate Poisson process. Here the arrival rate is considered as, λ_0-a faster rate of arrival and λ_1 –a slower rate of arrival. Whenever the queue size reaches a certain prescribed number R, the arrival rate reduces from λ_o to λ_1 and it continues with that rate as long as the content in the queue is greater than some other prescribed integer r ($r \geq 0$ & $r < R$). When the content reaches r, the arrival rate changes back to λ_o and the same process is repeated. In section 2, the description of the model is given stating the relevant postulates. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived in section5, the analytical results are numerically illustrated.

2. Description of the Model

a. Consider a single server finite capacity queuing system in which the customers arrive according to the Poisson flow of rates λ_0 and λ_1 and the service times are exponentially distributed with rate μ.

It is assumed that the arrival process [$X_1(t)$] and the service process [$X_2(t)$] of the system are correlated and follows a bivariate Poisson process having the joint probability mass function of the form

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_i + \mu - \in)t} \sum_{j=0}^{\min(x_1, x_2)} \frac{(\in t)^j [(\lambda_i - \in)t]^{x_1 - j} [(\mu - \in)t]^{(x_2 - j)}}{j!(x_1 - j)!(x_2 - j)!} \quad (2.1)$$

where $x_1, x_2 = 0,1,2,\ldots\ldots,0 < \lambda_1, \mu$; $0 \le \in < \min(\lambda_1, \mu), i = 0, 1$

with parameters $\lambda_0, \lambda_1, \mu$ and $\in$ as mean faster arrival rate, mean slower arrival rate, mean service rate and mean dependence rate (covariance between the arrival and service process) respectively.

b. The capacity of the system is finite (N)

c. The queue discipline is First –come, First –serve.

d. When the system is empty a customer may balk with probability q' and may enter with probability p' (=1 - q').

e. When there is at least one customer in the system, the customers balk with a probability $\left(1 - \frac{n}{N-1}\right)$ and join the system with probability $\frac{n}{N-1}$.

f. In reneging concept, the customers wait up-to certain time t and may leave the system before getting service due to impatience. The reneging time t are independently, identically and exponentially distributed with parameter β.

The postulates of the model are

i. The probability that there is no arrival and no service with Reverse balking and reverse reneging during a small interval of time *h*, when the system is in faster rate of arrivals is $1 - [(\lambda_0 - \in)p' + [(\mu - \in) + N\beta]]h + o(h)$

ii. The probability that there is one arrival and no service with Reverse balking and reverse reneging during a small interval of time *h*, when the system is in faster rate of arrivals is $(\lambda_0 - \in)p'h + o(h)$

iii. The Probability that there is no arrival and no service with Reverse balking and reverse reneging during a small interval of time *h* at state *n*, when the system is in faster rate of arrival is
$1 - \left[\left(\frac{n}{N-1}\right)(\lambda_0 - \in)p' + [(\mu - \in) + (N - (n-1))\beta]\right]h + o(h)$

iv. The Probability that there is no arrival and no service with Reverse balking and reverse reneging during a small interval of time *h* at state *n*, when the system is in slower rate of arrival is
$1 - \left[\left(\frac{n}{N-1}\right)(\lambda_1 - \in)p' + [(\mu - \in) + (N - (n-1))\beta]\right]h + o(h)$

v. The probability that there is no arrival and one service with reverse balking and reverse reneging during a small interval of time *h* state *n*, when the system is either in faster or slower rate of arrivals is $[(\mu - \in) + (N - n)\beta]h + o(h)$

vi. The Probability that there is one arrival and one service with Reverse balking and reverse reneging during a small interval of time *h*, when the system is either in faster or slower rate of arrivals is $\in h + o(h)$

3. The Steady State Equations

Let $P_n(0)$ be the steady state probability that there are *n* customers in the system when the arrival rate is λ_0 and $P_n(1)$ be the steady state probability that there are *n* customers in the system when the arrival rate is λ_1. We observe that $P_n(0)$ exists when *n* = 0,1,2,.....*r* -*1*,*r* both $P_n(0)$ & $P_n(1)$ exist when *n = r+1, r+2, ...R-1* and $P_n(1)$ exists when *n=R, R+1,....N*. Further $P_n(0) = P_n(1) = 0$ if n>N. With this dependence structure, the steady state equations are

$$0 = -(\lambda_0 - \in)p'P_0(0) + [(\mu - \in) + N\beta]P_1(0) \quad (3.1)$$

$$0 = -\left[\left(\frac{1}{N-1}\right)(\lambda_0 - \in) + [(\mu - \in) + N\beta]\right]P_1(0) + [(\mu - \in) + (N-1)\beta]P_2(0) + (\lambda_0 - \in)p'p_0(0) \quad (3.2)$$

$$0 = -\left[\left(\frac{n}{N-1}\right)(\lambda_0 - \in) + [(\mu - \in) + (N - (n-1))\beta]\right]P_n(0) + [(\mu - \in) + (N - n)\beta]P_{n+1}(0) + \left(\frac{n-1}{N-1}\right)(\lambda_0 - \in)P_{n-1}(0) \quad , n = 2,3,\ldots\ldots, \quad r - 1 \quad (3.3)$$

$$0=-\left[\left(\frac{r}{N-1}\right)(\lambda_0-\in)+[(\mu-\in)+(N-(r-1))\beta]\right]P_r(0) \tag{3.4}$$
$$+[(\mu-\in)+(N-r)\beta]\mathrm{P}_{r+1}(0)+[(\mu-\in)+(N-r)\beta]\mathrm{P}_{r+1}(1)+\left(\frac{r-1}{N-1}\right)(\lambda_0-\in)P_{r-1}(0)$$

$$0=-\left[\left(\frac{n}{N-1}\right)(\lambda_0-\in)+[(\mu-\in)+(N-(n-1))\beta]\right]P_n(0)$$
$$+[(\mu-\in)+(N-n)\beta]\mathrm{P}_{n+1}(0)+\left(\frac{n-1}{N-1}\right)(\lambda_0-\in)P_{n-1}(0) \tag{3.5}$$
$$n=r+1,r+2,......,\quad R-2$$

$$0=-\left[\left(\frac{R-1}{N-1}\right)(\lambda_0-\in)+[(\mu-\in)+(N-(R-2))\beta]\right]P_{R-1}(0) \tag{3.6}$$
$$+\left(\frac{R-2}{N-1}\right)(\lambda_0-\in)\ \mathrm{P}_{R-2}(0)$$

$$0=-\left[\left(\frac{r+1}{N-1}\right)(\lambda_1-\in)+[(\mu-\in)+(N-r)\beta]\right]P_{r+1}(1)+[(\mu-\in)+(N-(r+1)\beta]\mathrm{P}_{r+2}(1) \tag{3.7}$$

$$0=-\left[\left(\frac{n}{N-1}\right)(\lambda_1-\in)+[(\mu-\in)+(N-(n-1))\beta]\right]P_n(1)+[(\mu-\in)+(N-n)\beta]P_{n+1}(1) \tag{3.8}$$
$$+\left(\frac{n-1}{N-1}\right)(\lambda_1-\in)P_{n-1}(1)\qquad n=r+2,r+3,......,\quad R-1$$

$$0=-\left[\left(\frac{R}{N-1}\right)(\lambda_1-\in)+[(\mu-\in)+(N-(R-1))\beta]\right]P_R(1)+[(\mu-\in)+(n-R)\beta]P_{R+1}(1) \tag{3.9}$$
$$+\left(\frac{R-1}{N-1}\right)(\lambda_0-\in)P_{R-1}(0)+\left(\frac{R-1}{N-1}\right)(\lambda_1-\in)\ \mathrm{P}_{R-1}(1)$$

$$0=-\left[\left(\frac{n}{N-1}\right)(\lambda_1-\in)+[(\mu-\in)+(N-(n-1))\beta]\right]P_n(1)+[(\mu-\in)+(N-n)\beta]\mathrm{P}_{n+1}(1)$$
$$+\left(\frac{n-1}{N-1}\right)(\lambda_1-\in)P_{n-1}(1)\qquad \mathrm{n=R+1,\ R+2,.......,\quad N-1} \tag{3.10}$$

$$0=(\lambda_1-\in)P_{N-1}(1)-[(\mu-\in)+\beta]P_N(1) \tag{3.11}$$

From (3.1) to (3.6) we get

$$P_n(0)=\begin{cases} p'\dfrac{(n-1)!}{(N-1)^{n-1}}\displaystyle\prod_{l=1}^{n}\frac{\lambda_0-\in}{[\mu-\in+(N-(l-1))\beta]}P_0(0) & \text{n = 1,2,...... , r - 1, r} \\[2ex] p'\dfrac{(n-1)!}{(N-1)^{n-1}}\displaystyle\prod_{l=r+1}^{n}\frac{\lambda_0-\in}{[\mu-\in+(N-(l-1))\beta]}P_0(0)-1 \\[2ex] -\dfrac{1}{\displaystyle\prod_{t=r+1}^{n-1}[\mu-\in+(N-t)\beta]}\left[\begin{matrix}\left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{n-r-1}(n-1)\,p_{n-r-1}+\\ \left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{\text{n-r-2}}(\text{n - 1})\ \text{p}_{n-r-2}[\mu-\in+(N-r)\beta]\\ +\left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{\text{n-r-3}}(\text{n - 1})\ \text{p}_{n-r-3}[\mu-\in+(N-r)\beta]\\ [\mu-\in+(N-(r+1)\beta]\\ +...+\left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{\text{n-(n-1)}}(\text{n - 1})\ \text{p}_{n-(n-1)}[\mu-\in+(N-r)\beta]..\\ ..[\mu-\in+(N-(n-3))\beta]+\displaystyle\prod_{t=r+1}^{n-1}[\mu-\in+(N-(t-1))\beta]\end{matrix}\right]P_{r+1}(1)P_0(0) \\[2ex] & \text{n = r + 2, r + 2,........ , R - 1}\end{cases} \tag{3.12}$$

From (3.7) to (3.11) we get

$$P_n(1) = \left\{ \begin{array}{l} 1 + \dfrac{1}{\prod_{t=r+2}^{n-1}[(\mu-\in)+(N-t)\beta]}\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{n-r-1}(n-1)P_{n-r-1} \\ +\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-2}}(n-1)P_{n-r-2}[(\mu-\in)+(N-r)\beta] \\ +\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-3}}(n-1)P_{n-r-3}[(\mu-\in)+(N-r)\beta][(\mu-\in)+(N-(r+1))\beta] \\ +\ldots+\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-R+1}}(n-1)P_{n-R+1}[(\mu-\in)+(N-r)\beta] \\ \ldots[(\mu-\in)+(N-(n-3))\beta]+\prod_{t=r+1}^{n-1}[\mu-\in+(N-(t-1))\beta] \\ \qquad\qquad\qquad \text{n} = \text{r}+1, \text{r}+2, \ldots, \text{R} \\ \dfrac{1}{\prod_{t=r+2}^{n-1}[(\mu-\in)+(N-t)\beta]}\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{n-r-1}(n-1)P_{n-r-1} \\ +\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-2}}(n-1)P_{n-r-2}[(\mu-\in)+(N-r)\beta]+\ldots \\ +\left(\dfrac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-R}}(n-1)P_{n-R}[(\mu-\in)+(N-r)\beta][(\mu-\in)+(N-(r+1))\beta] \\ \ldots[(\mu-\in)+(N-(R-2))\beta] \end{array} \right\} P_{r+1}(1)P_0(0)$$

n=R+1,R+2,....,N

(3.13)

where
$$P_{r+1}(1) = \frac{P'\dfrac{(R-1)!}{(N-1)^{R-1}}(\lambda_0-\in)^R\prod_{l=1}^{r+1}\dfrac{1}{[(\mu-\in)+(N-(l-1))\beta]}}{\begin{array}{l}\dfrac{(R-1)!}{r!}\left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{R-r-1}+\dfrac{(R-1)!}{(r+1)!}\left(\dfrac{\lambda_0-\in}{(N-1)}\right)^{R-r-2}[(\mu-\in)+(N-r)\beta]+\ldots \\ +(R-1)\left(\dfrac{\lambda_0-\in}{(N-1)}\right)[(\mu-\in)+(N-r)\beta]\ldots[(\mu-\in)+(N-(R-3))\beta]+ \\ +\prod_{t=r}^{R+2}[(\mu-\in)+(N-t)\beta].\end{array}}$$

The probability $P_0(0)$ that the system is empty can be calculated from the normalizing condition $P(0)+P(1)=1$

$$
P_0(0)=\frac{1}{1+\sum_{n=1}^{R-1} p^{\prime}\,\frac{(n-1)^1}{(N-1)^{n-1}}\prod_{l=1}^{n}\frac{\lambda_0-\in}{[\mu-\in+(N-(l-1))\beta]}P_0(0)}
$$

$$
-\left[1+\sum_{n=r+2}^{R-1}\frac{1}{\prod_{t=r+1}^{n-1}[\mu-\in+(N-t)\beta]}\left[\begin{array}{l}
\left(\frac{\lambda_0-\in}{(N-1)}\right)^{n-r-1}(n-1)\,p_{n-r-1}\\
+\left(\frac{\lambda_0-\in}{(N-1)}\right)^{\text{n-r-2}}(\text{n - 1})\ \text{p}_{n-r-2}[\mu-\in+(N-r)\beta]\\
+\left(\frac{\lambda_0-\in}{(N-1)}\right)^{\text{n-r-3}}(\text{n - 1})\ \text{p}_{n-r-3}[\mu-\in+(N-r)\beta]\\
[\mu-\in+(N-(r+1)\beta]\\
+\ldots+\left(\frac{\lambda_0-\in}{(N-1)}\right)^{\text{n-(n-1)}}(\text{n - 1})\ \text{p}_{n-(n-1)}[\mu-\in+(N-r)\beta]\ldots\\
..[\mu-\in+(N-(n-3))\beta]+\prod_{t=r+1}^{n-1}[\mu-\in+(N-(t-1))\beta]
\end{array}\right]\right.
$$

$$
+1+\sum_{n=r+2}^{R}\left[\begin{array}{l}
\frac{1}{\prod_{t=r+2}^{n-1}[(\mu-\in)+(N-t)\beta]}\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{n-r-1}(n-1)P_{n-r-1}\\
+\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-2}}(n-1)P_{n-r-2}[(\mu-\in)+(N-r)\beta]\\
+\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-3}}(n-1)P_{n-r-3}[(\mu-\in)+(N-r)\beta][(\mu-\in)+(N-(r+1))\beta]\\
+\ldots+\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-R+1}}(n-1)P_{n-R+1}[(\mu-\in)+(N-r)\beta]\\
\ldots.[(\mu-\in)+(N-(n-3))\beta]+\prod_{t=r+1}^{n-1}[\mu-\in+(N-(t-1))\beta]
\end{array}\right.
$$

$$
\left.+\sum_{n=R+1}^{N}\left[\begin{array}{l}
\frac{1}{\prod_{t=r+2}^{n-1}[(\mu-\in)+(N-t)\beta]}\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{n-r-1}(n-1)P_{n-r-1}\\
+\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-r-2}}(n-1)P_{n-r-2}[(\mu-\in)+(N-r)\beta]+\ldots\\
+\left(\frac{\lambda_1-\in}{(\mu-\in)(N-1)}\right)^{\text{n-R}}(n-1)P_{n-R}[(\mu-\in)+(N-r)\beta][(\mu-\in)+(N-(r+1))\beta]\\
\ldots.[(\mu-\in)+(N-(R-2))\beta]
\end{array}\right]\right]P_{r+1}(1)
$$

(3.14)

4. Characteristics of the Model

The probability $P(0)$ that the system is in faster rate of arrival is

$$P(0) = \sum_{n=0}^{N} P_n(0)$$

Since $P_n(0)$ exists only when n=0,1,2,...,.r-1,r, r+1, r+2,...,R-2, R-1, we get

$$P(0) = P_0(0) + \sum_{n=1}^{r} \mathrm{P}_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \quad (4.1)$$

The Probability that the system is in slower rate of arrival is

$$P(1) = \sum_{n=0}^{N} P_n(1) = \sum_{n=0}^{r} P_n(1) + \sum_{n=r+1}^{R-1} P_n(1) + \sum_{n=R}^{N} P_n(1) \quad (4.2)$$

Since $P_n(1)$ exists only when n=r+1, r+2, ...,R-2, R-1,...,N we get

$$P(1) = \sum_{n=r+1}^{N} P_n(1) \quad (4.3)$$

The expected number of customers in the system is give by

$$L_s = L_{so} + L_{s1} \quad (4.4)$$

where

$$L_{so} = \sum_{n=0}^{r} nP_n(0) + \sum_{n=r+1}^{R-1} nP_n(0) \quad (4.5)$$

and

$$L_{s1} = \sum_{n=r+1}^{R-1} \mathrm{n}P_n(1) + \sum_{n=R}^{N} \mathrm{n}P_n(1) \quad (4.6)$$

Therefore

$$L_s = \sum_{n=0}^{r} \mathrm{n}P_n(0) + \sum_{n=r+1}^{R-1} \mathrm{n}P_n(0) + \sum_{n=r+1}^{N} \mathrm{n}P_n(1) \quad (4.7)$$

From (3.12) and (3.13), we get

$$L_S = \sum_{n=1}^{R-1} \mathrm{n}p' \frac{(n-1)^1}{(N-1)^{n-1}} \prod_{l=1}^{n} \frac{\lambda_0 - \in}{[\mu - \in + (N-(l-1))\beta]} P_0(0)$$

$$- \left[(r+1) + \sum_{n=r+2}^{R-1} n \frac{1}{\prod_{t=r+1}^{n-1} [\mu - \in + (N-t)\beta]} \left[\begin{array}{l} \left(\frac{\lambda_0 - \in}{(N-1)}\right)^{n-r-1} (n-1) p_{n-r-1} \\ + \left(\frac{\lambda_0 - \in}{(N-1)}\right)^{\text{n-r-2}} (\text{n - 1})\ \text{p}_{n-r-2} [\mu - \in + (N-r)\beta] \\ + \left(\frac{\lambda_0 - \in}{(N-1)}\right)^{\text{n-r-3}} (\text{n - 1})\ \text{p}_{n-r-3} [\mu - \in + (N-r)\beta] \\ [\mu - \in + (N-(r+1)\beta] \\ + \ldots + \left(\frac{\lambda_0 - \in}{(N-1)}\right)^{\text{n-(n-1)}} (\text{n - 1})\ \text{p}_{n-(n-1)} [\mu - \in + (N-r)\beta] \ldots \\ ..[\mu - \in + (N-(n-3))\beta] + \prod_{t=r+1}^{n-1} [\mu - \in + (N-(t-1))\beta] \end{array} \right] \right.$$

$$+ (r+1) + \sum_{n=r+2}^{R} n \left[\begin{array}{l} \frac{1}{\prod_{t=r+2}^{n-1} [(\mu - \in) + (N-t)\beta]} \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{n-r-1} (n-1) P_{n-r-1} \\ + \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{\text{n-r-2}} (n-1) P_{n-r-2} [(\mu - \in) + (N-r)\beta] \\ + \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{\text{n-r-3}} (n-1) P_{n-r-3} [(\mu - \in) + (N-r)\beta][(\mu - \in) + (N-(r+1))\beta] \\ + \ldots + \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{\text{n-R+1}} (n-1) P_{n-R+1} [(\mu - \in) + (N-r)\beta] \\ \ldots.[(\mu - \in) + (N-(n-3))\beta] + \prod_{t=r+1}^{n-1} [\mu - \in + (N-(t-1))\beta] \end{array} \right.$$

$$\left. + \sum_{n=R+1}^{N} n \left[\begin{array}{l} \frac{1}{\prod_{t=r+2}^{n-1} [(\mu - \in) + (N-t)\beta]} \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{n-r-1} (n-1) P_{n-r-1} \\ + \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{\text{n-r-2}} (n-1) P_{n-r-2} [(\mu - \in) + (N-r)\beta] + \ldots \\ + \left(\frac{\lambda_1 - \in}{(\mu - \in)(N-1)}\right)^{\text{n-R}} (n-1) P_{n-R} [(\mu - \in) + (N-r)\beta] \\ [(\mu - \in) + (N-(r+1))\beta] \ldots [(\mu - \in) + (N-(R-2))\beta] \end{array} \right] \right] P_{r+1}(1) P_0(0)$$

Using Little's formula, the expected waiting time of the customers in the system is given by

$$W_s = \frac{L_s}{\overline{\lambda}} \quad \text{Where } \overline{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$$

5. Numerical Illustrations

For various values of $\lambda_0, \lambda_1, \mu, r, R, \in, q', p', N$ the values of $P_0(0)$, $P(0)$, $P(1)$ L_S and W_S are computed and tabulated in the following Table.

S. No	r	R	N	λ_0	λ_1	μ	$\in$	β	q'	p'	$P_0(0)$	P(0)	P(1)	L_S	W_S
1	2	5	8	3	3	4	0.5	0.1	0	1	0.608921	0.998233	0.001767	0.437727	0.145162
2	2	5	8	4	3	4	0.5	0.1	0	1	0.513809	0.993088	0.06912	0.585450	0.146616
3	2	5	8	4	4	4	0.5	0.1	0	1	0.512485	0.990528	0.009472	0.601150	0.150288
4	2	5	8	4	4	5	0.5	0.1	0	1	0.573636	0.996873	0.003127	0.492570	0.123143
5	2	5	8	4	4	5	0.5	0.1	0.2	0.8	0.627110	0.997267	0.002733	0.430791	0.107698
6	2	5	8	4	4	5	0.5	0.1	1	0	1.000000	1.000000	0.000000	0.000000	0.000000
7	2	5	8	4	4	5	1	0.1	0.2	0.8	0.589081	0.998045	0.001955	0.470818	0.117705
8	2	5	8	3	3	3	0.5	0.2	0	1	0.652130	0.999064	0.000936	0.383231	0.127744

Table 1

6. Conclusions

The observations made from the table 1 are

i. When the mean dependence rate decreases and the other parameters are kept constant, L_S and W_S decrease
ii. When the arrival rate increases and the other parameters are kept constant, Ls and Ws increase.
iii. When the service rate increases and the other parameters are kept constant, Ls and Ws decrease. It is also observed that the expected system size is zero when q' is 1 .
iv. When the balking rate decreases and other parameters are kept constant, Ls increases regularly and attains maximum when q' is zero.
v. When β increases and other parameters are kept constant, Ls and Ws decrease.

7. References

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