



ISSN 2278 – 0211 (Online)

## Fuzzy Soft Hyper K-ideal of a Hyper K-algebra

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### **Abstract:**

*Fuzzy soft hyper K-ideals of a hyper K-algebra are introduced and related properties are discussed. Relation between fuzzy soft hyper K-ideals and its level sets are investigated. Image and pre images of the fuzzy soft hyper k-ideal under a fuzzy soft homo morphism are studied.*

**Keywords:** hyper k-algebra, hyper k-ideal, level sets, fuzzy soft homomorphism

### **1. Introduction**

The concept of hyper structure was introduced in 1934 by French mathematician, Marty [3], R.A. Borzooei et.al introduced the notion of a hyper K-algebra and hyper K-ideals. Based on the notion of fuzzy soft K-algebra and K-ideals by [5] Muhammad Abram, N.O. Alsheri and Rania Saeed Alghamdi, fuzzy soft hyper K-algebra and fuzzy soft hyper k-ideals of hyper k-algebras are introduced by present author. In this paper fuzzy soft hyper k-ideals and related properties are discussed. Examples are provided and characterization of fuzzy soft hyper k-ideals in terms of its level sets. is also dealt.

### **2. Preliminaries**

In this section we first give fundamental definition and results in hyper K-algebra.

- Definition 2.1

By a hyper K-algebra we mean a nonempty set H endowed with a hyper operation “ $\circ$ ” and a Constant 0 satisfying the following axioms

$$(HK1) (x \circ z) \circ (y \circ z) < x \circ y$$

$$(HK2) (x \circ y) \circ z = (x \circ z) \circ y$$

$$(HK3) x < x$$

$$(HK4) x < y \ \& \ y < x \implies x = y \text{ for all } x, y, z \in H \text{ where } x < y \text{ is defined by } 0 \in x \circ y \ \& \ \text{for every } A, B \subseteq H \ A < B \text{ is defined by for } a \in A \text{ and there exists } b \in B \text{ such that } a < b.$$

$$(HK5) 0 < x \text{ for all } x \in H.$$

Then  $(H, \circ, 0)$  is called a hyper K-algebra.

Let  $(H, \circ, 0)$  be a hyper k-algebra and  $S$  be a subset of H containing 0. If S is a hyper K-algebra with respect to the hyper operation ‘ $\circ$ ’ on H, we say that S is a hyper K-subalgebra of H.

- Definition 2.2

A fuzzy set  $\mu$  in a set X is a function  $\mu : X \rightarrow [0, 1]$ .

- Definition 2.3

For a fuzzy set  $\mu$  in X and  $\alpha \in [0, 1]$

$$\text{Define } U(\mu; \alpha) = \{ x \in H / \mu(x) \geq \alpha \}$$

Which is called a level set of  $\mu$ .

Molodtsov [9] defined the notion of soft sts in the following way. Let U be an initial universe and E be a set of parameters.

Let P(U) denote the power set of U. Let A be a nonempty subset of E. A pair  $(f, A)$  is called a fuzzy soft set in the following way.

A pair  $(f, A)$  is called a fuzzy soft set over U where f is a mapping given by  $f: A \rightarrow I^U$  where  $I = [0, 1]$ . In general, for every  $\varepsilon \in A$   $f(\varepsilon) = f_\varepsilon$  is a fuzzy set of U and is called a fuzzy value set of parameters.

To solve decision making problems based on fuzzy soft sets Feng et al introduced the notion called t-level soft sets of fuzzy soft sets.

- Definition 2.4

Let  $(f, A)$  be a fuzzy soft set over U. For each  $t \in [0, 1]$  the set  $(f, A)^t = (f^t, A)$  is called an t – level soft set of  $(f, A)$  where  $f_\varepsilon^t = \{x \in U / f_\varepsilon^t(x) \geq t\}$  for all  $\varepsilon \in A$  Clearly  $(f, A)^t$  is a soft set over U.

• Definition 2.5

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over  $U$ . we say that  $(f, A)$  is a fuzzy soft subset of  $(g, B)$  and write  $(f, A) \subseteq (g, B)$  if

- i)  $A \subseteq B$
- ii)  $f(\mathcal{E}) \subseteq g(\mathcal{E})$  for all  $\mathcal{E} \in A$ .

• Definition 2.6

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over  $U$ . Then their extended intersection is a fuzzy soft set denoted by  $(h, C)$  where  $C = A \cup B$

$$\text{and } h(\mathcal{E}) = \begin{cases} f_{\mathcal{E}} & \text{if } \mathcal{E} \in A - B \\ g_{\mathcal{E}} & \text{if } \mathcal{E} \in B - A \\ f_{\mathcal{E}} \cap g_{\mathcal{E}} & \text{if } \mathcal{E} \in A \cap B \end{cases} \text{ for all } \mathcal{E} \in C.$$

This is denoted by  $(h, C) = (f, A) \tilde{\cap} (g, B)$

• Definition 2.7

If  $(f, A)$  and  $(g, B)$  are two fuzzy soft sets over the same inverse  $U$  then “ $(f, A)$  AND  $(g, B)$  is a fuzzy soft set denoted by  $(f, A) \wedge (g, B)$  and is defined by  $(f, A) \wedge (g, B) = (h, A \times B)$  where  $h(a, b) = f(a) \cap g(b)$  for all  $(a, b) \in A \times B$ . Here  $\cap$  is the operation of fuzzy intersection.

• Definition 2.8

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over  $U$ . Then their extended union denoted by  $(h, C)$  where  $C = A \cup B$  is defined by

$$h(\mathcal{E}) = \begin{cases} f_{\mathcal{E}} & \text{if } \mathcal{E} \in A - B \\ g_{\mathcal{E}} & \text{if } \mathcal{E} \in B - A \\ f_{\mathcal{E}} \cup g_{\mathcal{E}} & \text{if } \mathcal{E} \in A \cap B \end{cases} \text{ for all } \mathcal{E} \in C.$$

This is denoted by  $(h, C) = (f, A) \tilde{\cup} (g, B)$

• Definition 2.9

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over a common universe  $U$  with  $A \cap B \neq \emptyset$ . Then their restricted intersection is a fuzzy soft set  $(h, A \cap B)$  denoted by  $(f, A) \cap (g, B) = (h, A \cap B)$  where  $h(\mathcal{E}) = f(\mathcal{E}) \cap g(\mathcal{E})$  for all  $\mathcal{E} \in A \cap B$ .

• Definition 2.10

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over a common universe  $U$  with  $A \cap B \neq \emptyset$ . Then their restricted union is denoted by  $(f, A) \cup (g, B)$  and is defined by  $(f, A) \cup (g, B) = (h, C)$  where  $C = A \cap B$  and for all  $\mathcal{E} \in C$ ,  $h(\mathcal{E}) = f(\mathcal{E}) \cup g(\mathcal{E})$ .

• Definition 2.11

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft sets over  $H$ . Then  $(f, A)$  OR  $(g, B)$  is a fuzzy soft set denoted by  $(f, A) \cup (g, B)$  and is defined by  $(f, A) \vee (g, B) = (O, A \times B)$  where  $O(\alpha, \beta) = f(\alpha) \vee g(\beta)$  for all  $\alpha, \beta \in A \times B$  Here  $\vee$  is the operation of union of two fuzzy soft sets.

• Definition 2.12

A non-empty subset  $I$  of a hyper  $K$ -algebra  $H$  is called a hyper  $K$ -ideal of  $H$  if it satisfies

- i)  $0 \in I$
- ii) (for all  $x, y \in H$ )  $x \circ y < I, y \in I \rightarrow x \in I$ .

• Definition 2.13

A fuzzy set  $\mu$  in  $H$  is called a fuzzy hyper  $K$ -ideal if it satisfies the following conditions

(F1) ( for all  $x, y \in H$  )  $x < y \Rightarrow \mu(x) \geq \mu(y)$

(F2) (for all  $x, y \in H$ )  $\mu(x) \geq \min \left\{ \inf_{a \in x \circ y} \mu(a), \mu(y) \right\}$

**3. Fuzzy Soft Hyper K-ideals**

Let  $(f, A)$  be a fuzzy soft set over  $H$  where  $H$  is a hyper  $K$ -algebra.

Let  $A$  be a subset of  $E$ . If there exists  $\mathcal{E} \in A$  such that  $f(\mathcal{E})$  is a fuzzy hyper  $K$ -ideal of  $H$  then we say that  $(f, A)$  is a fuzzy soft hyper  $K$ -ideal based on the parameter  $\mathcal{E}$ .

If for all  $\mathcal{E} \in A$ ,  $f_{\mathcal{E}}$  is a fuzzy hyper  $K$ -ideal of  $H$ , then we say that  $(f, A)$  is a fuzzy soft hyper  $K$ -ideal on  $H$ .

• Example 3.1

Let  $H = \{0, a, b, c\}$  be a hyper  $K$ -algebra with the following cayley Table.

$\circ$	0	a	b	c
0	{0}	{0}	{0}	{0}
a	{a}	{0}	{a}	{a}
b	{b}	{0}	{0}	{0}
c	{c}	{0,a}	{c}	{0, a, c}

Let  $(f, A)$  be a fuzzy soft set over  $H$  where  $A = \{e_1, e_2\}$

Define  $f_{e_1}$  on  $H$  by  $f_{e_1}(0) = 0.9, f_{e_1}(a) = 0.2, f_{e_1}(b) = f_{e_1}(c) = 0.6$

Define  $f_{e_2}$  on  $H$  by  $f_{e_2}(0) = 0.7, f_{e_2}(a) = 0.2, f_{e_2}(b) = 0.5, f_{e_2}(c) = 0.4$

We see that  $f_{e_1}, f_{e_2}$  are fuzzy hyper  $K$ -ideals on  $H$ . Thus  $(f, A)$  is a fuzzy soft hyper  $k$ -ideal on  $H$ .

- Proposition 3.2

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft hyper K-ideals over  $H$ . then their extended intersection  $(f, A) \tilde{\cap} (g, B)$  is also a fuzzy soft hyper K-ideal over  $H$ .

Proof:

Let  $(f, A) \tilde{\cap} (g, B) = (h, C)$  where  $C = A \cap B$

$$\text{Then } h(e) = \begin{cases} f(e) & \text{if } e \in A - B \\ g(e) & \text{if } e \in B - A \\ f(e) \cap g(e) & \text{if } e \in A \cap B \end{cases}$$

Since  $(f, A)$  and  $(g, B)$  are fuzzy soft hyper K-ideals of  $H$ ,  $h(e)$  is a fuzzy hyper K-ideal of  $H$  if  $e \in A - B$  or  $B - A$ .  
Let  $e \in A \cap B$ .

Then  $h(e) = f(e) \cap g(e)$ .

Let  $x, y \in H$  and  $x < y$ .

$$\text{and } h_e(x) = \min \{ f_e(x), g_e(x) \}$$

$$\geq \min \{ f_e(y), g_e(y) \} = h_e(y)$$

Therefore  $h_e(x) \geq h_e(y)$

We prove the second condition of fuzzy hyper k-ideal

$$h_e(x) = \min \{ f_e(x), g_e(x) \}$$

$$\geq \min \{ \min \{ a \in x \circ y \} f_e(a), f_e(y) \}, \min \{ a \in x \circ y \} g_e(a), g_e(y) \}$$

$$= \min \{ \min \{ a \in x \circ y \} [ \min \{ f_e(a), g_e(a) \} ], \min \{ f_e(y), g_e(y) \} \}$$

$$= \min \{ a \in x \circ y \} [ \min \{ f_e(a), g_e(a) \} ], \min \{ f_e(y), g_e(y) \}$$

$$= \min \{ a \in x \circ y \} (f_e \cap g_e)(a), (f_e \cap g_e)(y)$$

$$= \min \{ a \in x \circ y \} h_e(a), h_e(y)$$

$$\text{Thus } h_e(x) \geq \min \{ a \in x \circ y \} h_e(a), h_e(y)$$

$\Rightarrow h(e)$  is a fuzzy hyper K-ideal on  $H$ .

Hence  $(f, A) \tilde{\cap} (g, B)$  is a fuzzy soft hyper K-ideal on  $H$ .

- Proposition 3.3

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft hyper K-ideals on  $H$  then  $(f, A) \text{ AND } (g, B)$  is a fuzzy soft hyper K-ideal on  $H$ .

Proof:

$(f, A) \text{ AND } (g, B)$  is defined by  $(f, A) \wedge (g, B) = (h, A \times B)$

Where  $h(a, b) = f(a) \wedge g(b)$  where  $(a, b) \in A \times B$ .

Let  $x < y$

Then  $f_a(x) \geq f_a(y)$  and  $g_b(x) \geq g_b(y)$

$$h_{(a,b)}(x) = (f_a \cap g_b)(x)$$

$$= \min \{ f_a(x), g_b(x) \}$$

$$\geq \min \{ f_a(y), g_b(y) \}$$

$$= (f_a \cap g_b)(y)$$

$$= h_{(a,b)}(y)$$

$$h_{(a,b)}(x) \geq h_{(a,b)}(y)$$

$$h_{(a,b)}(x) = \min \{ f_a(x), g_b(x) \}$$

$$\geq \min \{ \min \{ w \in x \circ y \} f_a(w), f_a(y) \}, \min \{ w \in x \circ y \} g_b(w), g_b(y) \}$$

$$= \min \{ \min \{ w \in x \circ y \} [ \min \{ f_a(w), g_b(w) \} ], \min \{ f_a(y), g_b(y) \} \}$$

$$= \min \{ w \in x \circ y \} [ \min \{ f_a(w), g_b(w) \} ], \min \{ f_a(y), g_b(y) \}$$

$$= \min \{ w \in x \circ y \} (f_a \cap g_b)(w), (f_a \cap g_b)(y)$$

$$= \min \{ w \in x \circ y \} h_{(a,b)}(w), h_{(a,b)}(y)$$

$$\text{Thus } h_{(a,b)}(x) \geq \min \{ w \in x \circ y \} h_{(a,b)}(w), h_{(a,b)}(y)$$

Thus  $(f, A) \text{ AND } (g, B)$  is a fuzzy soft hyper K-ideal on  $H$ .

- Proposition 3.4

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft hyper K-ideal over  $H$ . Then  $(f, A) \text{ OR } (g, B)$  is a fuzzy soft set hyper K-ideal over  $H$ .

Proof:

$(f, A) \text{ OR } (g, B) = (O, A \times B)$  Where  $O_{(\alpha, \beta)}(x) = (f_\alpha \cup g_\beta)(x)$

Let  $x < y$

Then  $f_\alpha(x) \geq f_\alpha(y)$

and  $g_\beta(x) \geq g_\beta(y)$

$$O_{(\alpha, \beta)}(x) = (f_\alpha \cup g_\beta)(x)$$

$$= \max \{ f_\alpha(x), g_\beta(x) \}$$

$$\begin{aligned} &\geq \max \{ f_\alpha(y), g_\beta(y) \} f_\alpha \cup g_\beta(y) \\ &= O_{(\alpha,\beta)}(y) \\ O_{(\alpha,\beta)}(x) &= \max \{ f_\alpha(x), g_\beta(x) \} \\ &\geq \min \{ f_\alpha(x), g_\beta(x) \} \end{aligned}$$

The rest of the proof follows from the previous proposition.

We state the following propositions without proof.

- Proposition 3.5

Let (f, A) and (g, B) be two fuzzy soft hyper K-ideals on H and  $A \cap B \neq \emptyset$ . Then their restricted intersection  $(f, A) \cap (g, B)$  is a fuzzy soft hyper K-ideal on H.

- Proposition 3.6

Let (f, A) and (g, B) be two fuzzy soft hyper K-ideals on H. Then their restricted union  $(f, A) \cup (g, B)$  is a fuzzy soft hyper K-ideal on H.

- Definition 3.7

The relative complement of a fuzzy soft set (f, A) denoted by  $(f, A)'$  is defined by  $(f, A)' = (f', A)$  where  $f' : A \rightarrow F(P(U))$  by  $f'(\alpha) = 1 - f(\alpha)$  for all  $\alpha \in A$ .

- Note 3.8

The relative complement of a fuzzy soft hyper K-ideal is not a fuzzy soft hyper K-ideal.

- Proposition 3.9

Let (f, A) be a fuzzy soft hyper K-ideal on H and B is a subset of A Then  $(f|_B, B)$  is a fuzzy soft hyper k-ideal on H

Proof:

Obvious.

The following example shows that there exists a fuzzy soft set over H such that

- (f, A) is not a fuzzy soft hyper K-ideal on H.
- There exists a subset B of A such that  $(f|_B, B)$  is a fuzzy soft hyper K-ideal on H.

Consider the example as given in 3.1 and H denotes the set of hotels

Consider the set of parameters  $A = \{ \text{cheap, costly, moderate} \}$ ,  $B = \{ \text{cheap, moderate} \}$

Let (f, A) be the fuzzy soft set over H

Then  $f(\text{cheap})$ ,  $f(\text{costly})$ ,  $f(\text{moderate})$  are fuzzy sets defined as follows.

$\circ$	0	a	b	c
Cheap	0.9	0.2	0.6	0.5
Moderate	0.7	0.1	0.5	0.5
Costly	0.9	0.8	0.6	0.7

Table 1

$(f, A)$  is not a fuzzy soft hyper K-ideal on H.

Since  $f(\text{costly})$  is not a fuzzy hyper K-ideal on H

$c \circ a = \{ \circ, a \}$  hence  $c < a$  but  $f_{\text{costly}}(c) \not\geq f_{\text{costly}}(a)$

whereas  $f(\text{cheap})$  and  $f(\text{moderate})$  are fuzzy hyper K-ideals on H.

We provide a characterization for fuzzy soft hyper k-ideals

- Theorem 3.10

$(f, A)$  is a fuzzy soft hyper K-ideal on H if and only if its level subset  $U(f_e, \alpha)$  is a soft hyper K-ideal on H for all  $\alpha \in [0, 1]$ .

Proof:

Let (f, A) be a fuzzy soft hyper K-ideal on H.

Let  $e \in A$

Then  $f_e$  is a fuzzy hyper K-ideal on H

We claim that non-empty level set  $U(f_e, \alpha)$  is a hyper K-ideal for all  $\alpha \in [0, 1]$

$U(f_e, \alpha) \neq \emptyset \Rightarrow$  there exists  $a \in U(f_e, \alpha)$

$\Rightarrow f_e(a) \geq \alpha$

Since  $0 \leq x$  for all  $x \in H$

$0 \leq a$  in particular

$f_e(0) \geq f_e(a) \geq \alpha$

$\Rightarrow 0 \in U(f_e, \alpha)$

Let  $x, y \in H$  be such that  $x \circ y \in U(f_e, \alpha)$  and  $y \in U(f_e, \alpha)$  then there exists  $a' \in x \circ y$  and  $b \in U(f_e, \alpha)$  such that  $a' < b$

$\Rightarrow f_e(a') \geq f_e(b) \geq \alpha$

$\Rightarrow f_e(a') \geq \alpha$  for all  $a' \in x \circ y$

Therefore  $\inf_{a' \in x \circ y} f_e(a') \geq \alpha$  also  $f_e(y) \geq \alpha$

Since  $f_e$  is a fuzzy hyper K-ideal

$$f_e(x) \geq \min \{ \inf_{a' \in x \circ y} f_e(a'), f_e(y) \} \geq \alpha$$

Implies  $x \in U(f_e, \alpha)$

Thus  $x \circ y \in U(f_e, \alpha)$  and  $y \in U(f_e, \alpha)$

$$\Rightarrow x \in U(f_e, \alpha)$$

Hence  $U(f_e, \alpha)$  is a hyper K-ideal.

Conversely,

Let  $(f, A)$  be a fuzzy soft set on  $H$ . Let  $e \in A$ . If the level set  $U(f_e, t)$  for all  $t \in [0, 1]$  is a hyper K-ideal then  $(f, A)$  is a fuzzy soft hyper K-ideal based on  $e$ .

$f_e$  is a fuzzy set on  $H$ .

Let  $x, y \in H$  be such that  $x < y$

We will show that  $f_e(x) \geq f_e(y)$

Consider  $U(f_e, f_e(y))$

Let  $x \circ y \in U(f_e, f_e(y))$

and  $y \in U(f_e, f_e(y))$

$U(f_e, f_e(y))$  is a hyper K-ideal,  $\Rightarrow x \in U(f_e, f_e(y))$

$$\Rightarrow f_e(x) \geq f_e(y)$$

$$\text{Let } \alpha = \min \{ \inf_{a \in x \circ y} f_e(a), f_e(y) \}$$

Then  $f_e(y) \geq \alpha$

$$y \in U(f_e, \alpha)$$

$$\text{for all } b \in x \circ y, f_e(b) \geq \inf_{a \in x \circ y} f_e(a)$$

$$\geq \min \{ \inf_{a \in x \circ y} f_e(a), f_e(y) \}$$

$$= \alpha$$

Therefore  $f_e(b) \geq \alpha$

$$\Rightarrow b \in U(f_e, \alpha)$$

$$\Rightarrow x \circ y \subseteq U(f_e, \alpha)$$

$$\Rightarrow x \circ y < U(f_e, \alpha)$$

since  $A \subseteq B \Rightarrow A < B$  in a hyper K-algebra and  $y \in U(f_e, \alpha)$

$$\Rightarrow x \in U(f_e, \alpha)$$

$$\Rightarrow f_e(x) \geq \alpha$$

$$f_e(x) \geq \alpha = \min \{ \inf_{a \in x \circ y} f_e(a), f_e(y) \}$$

Thus  $f_e$  is a fuzzy hyper K-ideal.

We see that fuzzy soft hyper K-ideal is not a fuzzy soft hyper k-subalgebra. Consider the example given in 3.1 and the Table1  $f(\text{cheap})$  is a fuzzy hyper k-ideal but it is not a fuzzy hyper k sub algebra based on the same parameter cheap. Since the level set of the fuzzy set  $f(\text{cheap})$  for  $\alpha = 0.5$  is  $\{0, c, b\}$  and  $c \circ c = \{0, a, c\} \not\subseteq \{0, c, b\}$  Hence the level set is not a hyper k-subalgebra

A fuzzy soft hyper k-subalgebra need not be a fuzzy soft hyper k-ideal Let  $(f, A)$  be a fuzzy soft set and  $A = \{\text{costly}\}$  We see that  $f(\text{costly})$  is a hyper k-subalgebra but it is not a hyper k-ideal

- Definition 3.11

Let  $\varphi: X \rightarrow Y$  and  $\vartheta: A \rightarrow B$  be two functions  $A$  and  $B$  are parametric sets from the crisp sets  $X$  and  $Y$  respectively Then the pair  $(\varphi, \vartheta)$  is called a fuzzy soft function from  $X$  to  $Y$

- Definition 3.12

Let  $(f, A)$  and  $(g, B)$  be two fuzzy soft hyper K-ideals over  $H_1$  and over  $H_2$  respectively and let  $(\varphi, \vartheta)$  be a fuzzy soft function from  $H_1$  to  $H_2$

1) The image of  $(f, A)$  under the fuzzy soft function  $(\varphi, \vartheta)$  denoted by  $(\varphi, \vartheta)(f, A) = (\varphi(f), \vartheta(A))$  where for all  $k \in \vartheta(A)$ ,  $y \in H_2$   
 $(\varphi(f))_k(y) = \bigvee \varphi(x) = y \quad \bigvee \vartheta(a) = k \quad f_a(x)$  if  $x \in \vartheta^{-1}(y)$ ,  $0$  otherwise

2) The preimage of  $(g, B)$  under the fuzzy soft function  $(\varphi, \vartheta)$  denoted by  $(\varphi, \vartheta)^{-1}(g, B)$  is the fuzzy soft set over  $H_1$  defined by  $(\varphi, \vartheta)^{-1}(g, B) = (\varphi^{-1}(g), \vartheta^{-1}(B))$  where  $\varphi^{-1}(g)_a(x) = g(\varphi(x))$  for all  $a \in \vartheta^{-1}(B)$ , for all  $x \in H_1$

- Definition 3.13

Let  $(\varphi, \vartheta)$  be a fuzzy soft function from  $H_1$  to  $H_2$ . If  $\varphi$  is a homomorphism from  $H_1$  to  $H_2$  then  $(\varphi, \vartheta)$  is said to be a fuzzy soft homomorphism If  $\varphi$  is an isomorphism from  $H_1$  to  $H_2$  and  $\vartheta$  is a 1-1 mapping from  $A$  to  $B$  then  $(\varphi, \vartheta)$  is said to be a fuzzy soft isomorphism

- Theorem 3.14

Let  $(\varphi, \vartheta)$  be a fuzzy soft homomorphism from  $(H_1, 0_1)$  to  $(H_2, 0_2)$  If  $(f, A)$  is a fuzzy soft hyper K ideal over  $H_1$  then  $(\varphi, \vartheta)(f, A)$  is a fuzzy soft hyper k-ideal over  $H_2$

Proof:

Let  $k \in \vartheta(A)$  and  $y_1, y_2 \in H_2$  If  $\varphi^{-1}(y_1) = \varphi^{-1}(y_2)$  is a empty set then the proof is obvious

Let  $\varphi^{-1}(y_1)$  and  $\varphi^{-1}(y_2)$  be not empty sets and let  $\varphi^{-1}(y_1) < \varphi^{-1}(y_2)$  then there exists  $x_1$  and  $x_2 \in H_1$  such that  $\varphi(x_1) = y_1$  and  $\varphi(x_2) = y_2$  For  $x_1^1 \in \varphi^{-1}(y_1)$  there exists  $x_2^1 \in \varphi^{-1}(y_2)$  such that  $x_1^1 < x_2^1$  Since  $(f, A)$  is a fuzzy soft hyper k-ideal of  $H_1$   $f_a(x_1^1) \geq f_a(x_2^1)$  therefore  $\forall \varphi(x_1^1) = y_1 \quad f_a(x_1^1) \geq \forall \varphi(x_2^1) = y_2 \quad f_a(x_2^1)$   
 $\varphi(f)_k(y_1) = \forall \vartheta(a) = k \forall \varphi(x_1^1) = y_1 \quad f_a(x_1^1) \geq \forall \vartheta(a) = k (\forall \varphi(x_2^1) = y_2 \quad f_a(x_2^1))$   
 $= \varphi(f)_k(y_2)$

Let  $u$  and  $v \in H_1$

$$\varphi_{\varphi}(u) = y_1 \quad \varphi_{\varphi}(v) = y_2$$

Then  $f_a(u) \geq \min \{ \inf_{w \in (u \circ_1 v)} f_a(w), f_a(v) \}$

$$\text{Let } z \in y_1 \circ_2 y_2 = \varphi(u \circ_1 v)$$

Then  $z = \varphi_{\varphi}(w)$  where  $w \in u \circ_1 v$

$$\forall \varphi_{\varphi}(u) = y_1 \geq \forall \varphi_{\varphi}(v) = y_2, \varphi(w) = z \quad \min \{ \inf_{w \in u \circ_1 v} f_a(w), f_a(v) \}$$

$$\varphi(f)_k(u) = \forall \vartheta(a) = k \forall \varphi(u) = y_1 \quad f_a(u) \geq \forall \vartheta(a) = k \min \{ \inf_{w \in u \circ_1 v} f_a(w), f_a(v) \}$$

• Theroem 3.15

Let  $(\varphi, \vartheta)$  be a fuzzy soft homomorphism from  $(H_1, \circ_1)$  to  $(H_2, \circ_2)$  If  $(g, B)$  is a fuzzy soft hyper K ideal over  $H_2$  then  $(\varphi, \vartheta)^{-1}(g, B)$  is a fuzzy soft hyper k-ideal over  $H_1$

Proof:

Let  $(g, B)$  be a fuzzy soft hyper K ideal over  $H_2$  Let  $a \in \vartheta^{-1}(B)$  and  $u, v \in H_1$  be such that  $u < v$  which implies  $\varphi(u) < \varphi(v)$  and  $\varphi(u)$

$$= y, \varphi(v) = z, \varphi^{-1}(g)_a(u) = g_{\vartheta(a)}(\varphi(u))$$

$$= g_{\vartheta(a)}(y)$$

$$\geq g_{\vartheta(a)}(z)$$

$$= g_{\vartheta(a)}(\varphi(v))$$

$$= \phi - 1(g)_a(v)$$

$$\varphi^{-1}(g)_a(u) = g_{\vartheta(a)}(\varphi(u)) = g_{\vartheta(a)}(y)$$

$$\geq \min \{ \inf_{w \in y \circ_2 z} g_{\vartheta(a)}(w), g_{\vartheta(a)}(z) \}$$

$$= \min \{ \inf_{\varphi(t) \in \varphi(u) \circ_2 \varphi(v)} g_{\vartheta(a)}(\varphi(t)), g_{\vartheta(a)}(\varphi(v)) \} \text{ where } w = \varphi(t) \text{ for some } t \in u \circ_1 v$$

$$= \min \{ \inf_{\varphi(t) \in \varphi(u \circ_1 v)} \phi - 1(g)_a(t), \phi - 1(g)_a(v) \}$$

This completes the proof

#### 4. Conclusion

In this paper some properties of fuzzy soft hyper K-ideals of a hyper K-algebra are studied and the characterization of a fuzzy soft hyper-ideal in terms of its level sets is provided and any relation existing between the fuzzy soft hyperk-subalgebras and the fuzzy soft hyper k-ideals is analyzed. Image and pre image of a fuzzy soft hyper k-ideal under a fuzzy soft homomorphism are also studied. In the author's opinion these results maybe extended to intuitionistic hyper K-algebra.

#### 5. Acknowledgement

This research is financially supported by the University grants commission (SEROIU. G. C) Hyderabad, India under project No F MRP 5331/14.

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