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A Conceptual Mathematical Model of High Density Non-Metallic Reaction Wheels for Cube SAT Balancing Systems

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Abstract:
The reaction wheel setup is generally used in micro, mini, cube satellites to achieve balancing and to maintain its coordinates in a three-dimensional plane main, which is the space itself. These reaction wheels are generally designed in such a way that the density and the angular momentum of the total setup is more enough to balance the external forces, but since these wheels are made up of metals to possess high density they are equally attracted towards the earth's magnetic field which makes a misalignment in its orbit. Hence the major aim of this research work is to design a mathematical model of a reaction wheel setup, that withstands the external forces acting on the cube sat and nullifies the effect of angular displacement of the cube sat by applying counter torque in the opposite direction and also to stay in its orbit irrespective of the earth's magnetic field.

Keywords: *High density reaction wheels, Nonmetallic Reaction wheels, Satellite balancing systems*

1. Introduction
The Reaction wheel (a.k.a, momentum wheel) plays a major role on the balancing systems of cube satellites to maintain their coordinates and to avoid flipping in the space. Since there is no base point or any stable surface on the space, these satellites need to overcome the external forces and disorientations for maintaining its position and a stable orbit around the earth or any other planets. To overcome these forces, the reaction wheel system should be used in the miniature cube sat. These reaction wheels, work on the principle of Newtonian laws (2^{nd} and 3^{rd} specifically) to achieve the action of balancing. In other words, the reaction wheels in the satellite will produce a torque in the opposite direction of the angular displacement in the satellite so that it aligns with its original position irrespective of the external force.

2. Basic Mathematical Parameters to be Considered
The reaction wheel must own more inertia and density in order to get the required counter torque. Also some other dependent parameters such as driving torque, wheel mass, overall mass and the total area allocation, should be taken into account for an effective design of a reaction wheel. For an effective derivation of the relationship between the parameters involved in the design the proportionality constants should be found in between them. Then using them as a base, effective relation could be derived.

2.1. General Relations
The design of reaction wheels for cube sat balancing systems need the data such as dimensions of the cube sat and the total weight (w.r.t environment) that is to be balanced. Other such relations are as follows;

2.1.1. Torque
The torque produced is responsible for the rotation and generation of the counter torque in the system. In the derivation of the general torque relation the angular velocity and the force applied over plays a major role. It can be found that two formulae which can be derived for obtaining the torque relation.
EQ.1 with respect to the force applied and the geometry:
The first relation involves the product of radius of the wheel from its center distance and the force applied tangentially to the wheel.

$$\tau = F \times r$$

Where,

F = Force applied tangentially

r = radius of the wheel

EQ.2 with respect to the power P (in watts):

The second relation involves the ratio of Power and the product of numerical constants and the angular velocity.

$$\tau = {}^{P}/_{2\pi\omega}$$

Where,

P = Power in watts

ω = Angular velocity

Now equating both the equations, we get the power and force required are the dependent parameters and similar relations should be obtained for all other parameters.

2.1.2. Geometry and Momentum

The geometry of the reaction wheel usually resembles a flat plate, but there are some projections over the face of the plate, from its center distance. This geometry is carried out to reach good momentum and proper balancing of the masses through increasing the torque (τ),

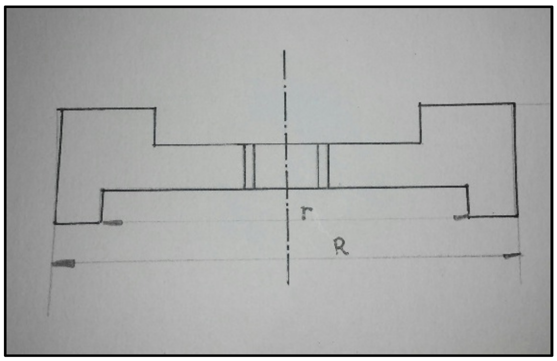

Figure 1: basic geometry of a reaction wheel

Similarly, the angular momentum (L) is the main factor which makes the wheel to balance the cube sat against angular distortion, it is represented by the relation;

$$L = I \times \omega$$

Where,

I = inertia of the wheel

(The inertia of the wheel greatly depends upon its geometry)

2.1.3. Inertia

The inertia (I) of the wheel mainly depends upon its shape and geometric model. But in general all the reaction wheel system will have a common shape of a flat disc, since here the major point is to eliminate the usage of metal wheels and replace it with the non-metallic materials but with the same density of the metals, special geometry and shape should be designed for attaining the required inertia of the wheel.

General inertia equation of a step turned plate can be written as;

$$I = \frac{\rho\pi}{2} \times h \times (R^4 + r^4)$$

Where,

ρ = density of the material

h = Height of the wall

R = radius of the outer ring

r = radius of the inner ring

By considering the total size, shape and mass of the cube sat the wheel is designed with proper inertia.

2.1.4. Stability against the Working Stresses

The reaction wheels are to be subjected to continuous rotation in some cases to withstand the angular distortion caused by the solar winds. In these cases, the wheel is to be operated under high angular velocity (ω), this action of working under high angular velocity against the inertial force subjects the wheel to stress and deformation or misalignment with the motor axis. To overcome these issues, the reaction wheel has to be designed to withstand the stresses.

The stress acting on the wheel while working under high angular velocity is given by;

$$\sigma(allowable) = \frac{3+\nu}{4}\rho\omega^2(R^2 + \frac{1-\nu}{3+\nu}r^2)$$

Where,
ν = poisson ratio
Where the allowable stress ('σ' can be found or related using the above equation)
So all these design parameters are to be considered before designing the appropriate reaction wheel for the specified cube sat.

2.2. Proportionality of the Parameters
The common relations between the design parameters are seen above, but before designing the whole wheel setup, the proportional parameter are to be found to ensure that any of the changes in one parameter should not affect the other. This consideration is done in order to eliminate the dependence while designing.

Property	Mass	Weight	Input power	Inertia	Density
τ			α		α
R	α			α	
r	α			α	
ω		i	α		
σ		α	i		α

Table 1: Proportionality parameters

Here 'α' refers to proportionality and 'i' refers to inverse-proportionality. Before designing these things are to be taken into account to avoid misconceptions while designing.

3. Desideratum of a Good Reaction Wheel
The reaction wheel system in any miniature satellite is an important part of it and it cannot be replaced while it is on a working satellite. Hence these satellite balancing systems should be given more importance and should be designed and manufactured with greater concentration and with minimal tolerances. Some of the major requirements of the reaction wheel setup are as follows;

3.1. Size Consideration and Positioning
General cube satellites have standard dimensions such as 6U (10x20x30cm) and 12U (20x20x30cm), so the reaction wheel setup should have one wheel for each axis, i.e.; in total for three axes (x, y, z axis), there should be three wheels in whole. These wheels are to be positioned at the three of six perpendicular faces of the cube sat setup.

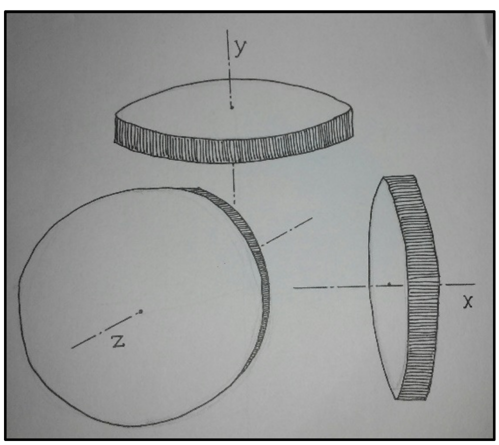

Figure 2: Reaction wheel arrangement for each axis

3.2. High Dense Wheel
The reaction wheel should have some mechanical properties to perform its required task. The main aim is to provide counter torque to the whole body when it is subjected to the angular distortion. To meet with its task, the wheel should store the necessary angular momentum to maintain a steady state position in the space where there is no base point for balance. Since the action of gravity is much more less or not present in the space, the rotation rate of 1 rpm is sufficient to maintain the position w.r.t the action of external force. Hence the wheel should be made of dense material.

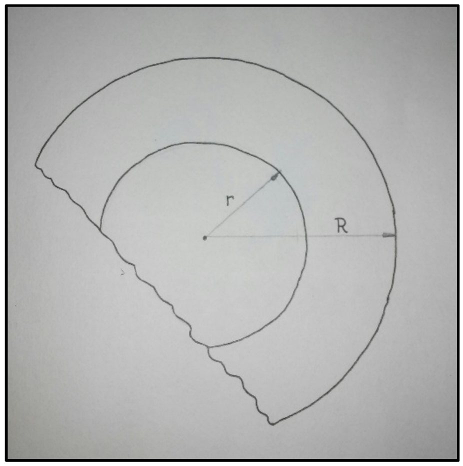

Figure 3: Basic wheel geometry

And the main point is that these reaction wheels are to be made of non-metals and hence they are not affected by the magnetic field of the earth. If at all the wheel is made of electrically conductive materials or metals then the electromagnetic field of the earth will induce eddy current in the satellite balancing system, which will surely affect the performance of the system.

3.3. Motor and Motor Control

Generally, these motor systems are used to rotate the wheels and to control the rpm. To maintain the precision and accuracy stepper motor is to be used but the motors act differently in space than in earth, since these motors uses windings of super conducting wire to produce the magnetic field, the field intersects with the magnetic field of the earth, leading to the inefficiency of the system.

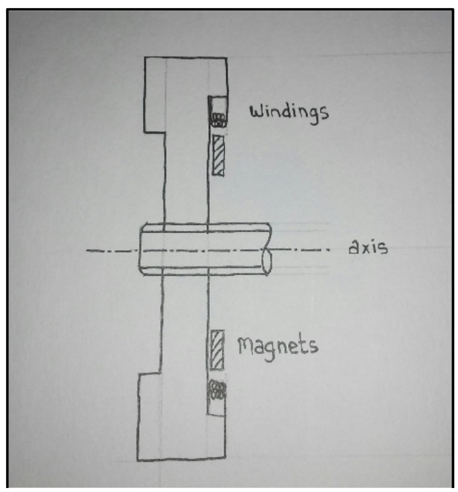

Figure 4: Wheel integrated hybrid motor

For these reasons, instead of using conventional motors hybrid wheel integrated motors are to be used for better actions. This wheel integrated motors are kept apart from the direct interaction hence efficient and since it has no driving shaft and conventional motor the space could be saved in here and there will be no loss in torque transmission.

4. Preferred Material

The reaction wheels are not to be made of electrically conductive material (to be more specific, metals), at the same time the wheel should store proper angular momentum and should have the working temperature range of -40º to 80ºC. For these reasons the conventional cube sat systems uses Highly Dense Urethane wheels, but hey suffer from a major disadvantage of storing momentum w.r.t its density.

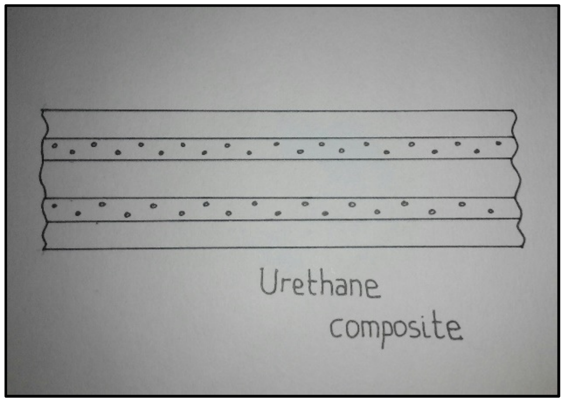

Figure 5: Composite reaction wheel setup

To overcome these drawbacks, the urethane wheels should be replaced with the modern Urethane composite materials. This composites involves the sandwich of urethane with ceramic materials to increase its density.

4.1. Role of Composite Materials

The high density urethane composite material can store very high angular momentum equivalent to the metals and the major advantage of this material is that they are easy to machine and non-corrosive when exposed to humidity. Since it is a non-metal and relatively has high density it is not affected by the earth's magnetic field and easy to operate. Also it is relatively cheap and w.r.t the density of the wheel, the system is stronger and heavier.

5. Test for Sustainability and Design of External Hull

The reaction wheel should be designed in such a way that it should sustain itself in harsh conditions in space such as solar wind, interstellar dust and against the electromagnetic field of the earth. To achieve this sustainability parameter proper design procedures has to be carried on. The design procedures not only involve the design of the reaction wheels, but also the design of external hull plays a major role in cube sat balancing systems.

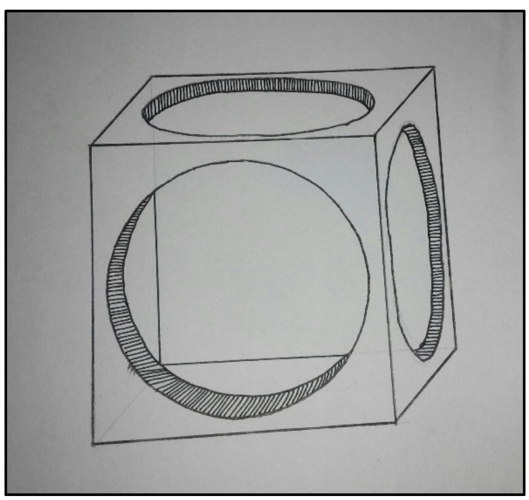

Figure 6: Hollow cube sat hull for reaction wheel setup

5.1. Design of External Hull

The external hull of the cube sac contains six faces in which three of them should be fixed with the reaction wheel setup in three axes respectively (x, y, z axes). The hull can be made through sheet metal operations such as bending, punching, etc. Since sheet metal is used in the construction of the hull it can be ensured that the total weight of the system can be minimized.

The three axes in which the reaction wheels are attached should be punched to maintain the balancing of unbalanced forces, this is done in order to maintain and distribute the weight of the system all over the body.

5.2. Sustainability of External Hull

The hull must maintain its temperature which is influenced by the interaction of the solar wind/incoming photons. Any increase in temperature will affect the rigid geometry of the hull and in turn damage the components present in it which is undesirable.

S.No	Property of Hull	Value
1	Mass	1.3 to 2 kg
2	Temperature sustainability	3500 K
3	Geometry	Cube / Cuboid
4	Dimensions	6U, 12U

Table 2: Suggested design data for designing an external hull for cube sat

The above mentioned parameters are the basic data that are to be taken into account before designing the hull.

6. Conventional Models Already in Use

At present three types of reaction wheels are in use for satellite balancing systems. These three systems are preferred than any other conventional systems because of their simplicity in design, manufacturing and their availability in market. Those three systems are as follows:

i. High density Urethane reaction wheels,
ii. MAI-101 miniature reaction wheel,
iii. Inline reaction wheel.

This new design of reaction wheel claims all the advantages of above specified wheels. Also implanting the concept of composite Urethane reaction wheel in satellite balancing system will hugely improve the performance of the system.

Based on the above three mentioned reaction wheels, the High density urethane reaction wheel and the inline reaction wheels are used in most common satellite sizes. The MAI-101 miniature reaction wheel is used in smaller satellites. The advantage of the composite urethane reaction wheel over other reaction wheel is that it can be easily designed, fabricated and installed into most satellites.

7. Advantages over the Metal Wheel

The new design of the composite urethane reaction wheel has comparatively more advantages over the conventional metal reaction wheel. Some of them are as follows:

- It has considerably good density when compared to normal urethane wheels and is equivalent to metallic wheels.
- Unlike the conductive reaction wheels, these urethane wheels are unaffected by the electromagnetic field of earth and hence, it has good precision and accuracy.
- It can store angular momentum equivalent to reaction spheres.
- It is cheap for both machining and fabrication.
- Since this system has inbuilt motors positioned inside inline reaction wheels they are kept away from the interactions of the earth’s magnetic field.

8. Future Works

The conventional system as depicted here can be revamped into several different models with a wide variety of applications. The mass of the system can be further reduced to accommodate several such components and make the system more compact in nature. The input power of the current conventional system is, though minimized greatly, still a moderate value comparatively, which can be further reduced to make the system more power efficient. The greater the efficiency of the system, the greater it’s lifetime. Thus, we can expect the machine to sustain for more periods.

Also, with further future developments we can expect the system to be capable of storing higher angular momentum, due to which good positioning can be achieved. These factors coupled with the upcoming future technologies can enable this system to be highly productive and effective.

9. Conclusion

The major goal of this research work is to develop a mathematical model/design that is compatible for most satellites. To reach this goal various conventional reaction wheel models were analyzed in detail through which the final conclusion of finding the proportionality parameters were decided. Through these steps, the design procedure was simplified and a system was designed by following the above mentioned parameters leading to the development of the composite Urethane reaction wheel system. Although this system has good tendency to store larger angular momentum with reduced weight (compared to the conventional model) certain parameters as mentioned above can be researched and developed for meeting the requirements of futuristic models.

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