ISSN 2278 – 0211 (Online)

Nonlinear Solution Techniques and Load Stepping Schemes for Modified Cam Clay Model

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Abstract:
Critical state models are developed from the application of plasticity to soil mechanics and they are the most widely used soil constitutive model. The model is highly nonlinear in nature and in finite element analysis, with a nonlinear material model; an iterative and incremental scheme is required to solve the resulting system of nonlinear equations. Variety of incremental and iterative schemes are developed in the past to integrate the load-displacement curve for nonlinear material models and each of them has advantages and limitations over other while using with a specific constitutive model. As critical state models are nonlinear in nature in the elastic as well as plastic parts of the constitutive relation, the performance of different load stepping schemes for finite element analysis particularly with critical state models is to be done and can be an active area of study. In the present study the review of the literature on the use of various nonlinear solution techniques and load stepping schemes used for general plasticity models and critical state soil constitutive models are carried out. It is observed that very little literature is available which study the influence of the nonlinear solution techniques and load stepping schemes on the solution of nonlinear finite element problems with critical state models. From the available literature it can be concluded that critical state models are very sensitive to the load step size and iteration tolerance as well. A comparative study on the performance of some of the mostly used nonlinear solution techniques and load stepping schemes with modified cam clay model is also carried out. The solution schemes which are included in the study are: Newton-Raphson methos, Modified Newton-Raphson methos, Accelerated modified Newton-Raphson methods and Arclength method. The study reveals that the performance of Arclength method as compared to other methods is better while using them with modified cam clay model.

Keywords: *Critical state models, cam clay model, finite element analysis, nonlinear solution techniques, load stepping schemes*

1. Introduction

Soil is a highly nonlinear material and its behaviour can be well described within the elastoplastic framework. For realistic numerical analysis of a problem in soil mechanics an elastoplastic model is required. Critical state models are the most widely used soil constitutive models, which are developed from the application of plasticity to soil mechanics. The model is highly nonlinear in nature and a numerical tool like finite element is required for the application of this model to solve a problem in soil mechanics. In finite element analysis, with a nonlinear material model, an iterative and incremental scheme is required to solve the resulting system of nonlinear equations. There are series of solution schemes developed for solving nonlinear equation system. In context to the solution of nonlinear finite element problems, the most conventional method is the Newton-Raphson method in any of its different versions as it has the property of quadratic convergence. However, Newton-Raphson method or any of its versions may not be always adequate and competent in solving certain problems involving nonlinearity. Hence, there may be a need of relatively advanced techniques of solving nonlinear finite element problems.

To integrate the load-displacement curve for nonlinear material models, different incremental and iterative schemes are developed in the past and each of them has advantages and limitations over other while using with a particular constitutive model. The accuracy, efficiency, and stability of both the schemes are strongly influenced by the load increment size and material model used in the analysis. Critical state models are nonlinear in nature in the elastic as well as plastic parts of the constitutive relation. Therefore, the performance of different load stepping schemes for finite element analysis particularly with critical state models is to be done and can be an active area of study. In this study a brief review of the available literature on the principal methods for nonlinear solution techniques and load stepping schemes for solving nonlinear finite element equations in general and critical state model equations in particular is presented. A comparative study on the performance of some of the mostly used nonlinear solution techniques and load stepping schemes are done with modified cam clay model.

2. Review of Nonlinear Solution Techniques and Load Stepping Schemes

2.1. Solution Schemes for Solving General Nonlinear Finite Element Equations

Crisfield [9] developed an incremental/iterative solution technique in conjunction with the modified Newton-Raphson method in both standard and accelerated forms. The procedure was used to handle snap-through and snap-back buckling phenomena in nonlinear structural analysis. Watson and Holzer [26] studied the quadratic convergence of the method proposed by Crisfield [9] and reported the method to be robust by solving the nonlinear equations of equilibrium for space trusses. A method for introducing line searches into the arc-length solution procedure was described by Crisfield [10] and demonstrated its improved convergence characteristics in relation to the standard arc-length method. Simple single-parameter acceleration was also developed using line search concepts. The method was applied to analyze shallow shells, reinforced concrete beams, and slabs. An automatic incremental solution schemes for genera1 nonlinear analysis in static analysis of structure was presented by Bathe and Dvorkin [4]. The method was reported to perform well in the solution of problems like softening, stiffening, collapse and post-buckling analysis. Simo and Taylor [21] derived the consistent tangent operator for use with the Newton-Raphson scheme, which was able to give a full quadratic rate of convergence for simple yield criteria. Borst [6] developed an improved arc-length method and used it for analyzing strain softening solids. A critical study and improvement over the existing automatic incremental solution algorithms of nonlinear finite element equations for the static response of structures were presented by Bellini and Chulya [5]. It was reported that the algorithm performed well for elastic-perfectly plastic problem but further investigation was needed for highly nonlinear materials. A new simplified arc-length method was developed and its performance of analysis was compared with other available arc-length methods by Forde and Stiemer [11]. The Crisfield's method [10] and the new simplified method were reported to be the most robust arc-length procedures for the solution of highly nonlinear problems, with severe local alteration of stiffness. Lee [16] studied the numerical behaviour associated with the line search method in nonlinear finite element analysis and reported it to be erratic. The method was implemented and modified to optimize its usefulness for a general class of problems in conjunction with quasi-Newton updates using a general purpose finite element program. Wagner [25] discussed new strategies for path-following algorithms and derived a quadratic predictor within arc-length procedures to allow large steps leading to a reduction of the number of iterations in the nonlinear process.

Foster [12] proposed a solution procedure involving the use of a constant arc-length solution scheme with line searches and accelerations. The method was compared with the modified Newton–Raphson solution scheme and reported to increase the numerical stability with substantial reduction of the CPU time required for convergence in the finite element modeling of reinforced concrete. An incremental scheme with automatic load step size control for solving the global equations associated with nonlinear finite element analysis was presented by Abbo and Sloan [1]. In the scheme the displacement increments for each load increment were estimated twice by using Euler scheme and modified Euler scheme and the difference of these two were used to control the step size. The algorithm was reported to be fast, robust, simple to implement and it was possible to control the load path error independently of the number of coarse load increments supplied by the user.

In the part I of his paper Geers [14] proposed solution control techniques, including fully automatic load estimation, adaptation and correction, to handle unstable equilibrium path arising from nonlinear response of structures. A comparative analysis of several automated solution control methods was presented and compared to classical global arc-length procedures by Geers [15] in the part II of his paper.

Alfano and Crisfield [2] developed efficient, fully implicit quasi-static algorithms for the analysis of delamination, which were based on a local-control arc-length method. The numerical performances of the developed algorithms had been compared with those of the cylindrical arc-length method. A 'double-line-search' (DLS) procedure was developed with which a significant improvement of convergence and stability were achieved.

A good literature review of new developments and modifications of the arc-length method in the field of nonlinear finite element analysis of reinforced concrete structures was given by Bashir-Ahmed and Ziao-zu [3]. From the review it was reported that the basic arc-length method or any of its modified form had not been validated in the literature for three-dimensional solid modeling of reinforced concrete structures in general or for strain softening in compression in particular.

Cardoso and Fonseca [8] briefly described the generalized displacement control (GDC) method for solving nonlinear equilibrium equations with both limit and snap-back points. It was shown that the GDC method could be viewed as an orthogonal arc-length method with additional features. The method was reported to be reliable and proposed as a consistent alternative to most existing iterative techniques, such as the arc-length method and any of its modified forms.

2.2. Solution Schemes for Solving Nonlinear Finite Element Equations of Soil Constitutive Models

The performance of any of the methods described above was done only with the analysis of structures and no studied was done with any soil constitutive models including critical states models. As critical state models are the most widely used soil constitutive models for analyzing geomechanic problems with finite element method [7], the study of application of various solution schemes with these models are essential.

Gen and Potts [13] performed a study to compare a number of incremental and iterative schemes with critical state models, which includes the tangent stiffness method (forward Euler method), the initial stress method, the Newton-Raphson method and modified Newton-Raphson method. It was concluded that different problems may require different solution strategies while using these models.

A detailed comparative study of the tangent stiffness, modified Newton–Raphson and visco-plastic schemes was given by Potts and Ganendra [18] by considering several boundary value problems.

Sloan et al. [22] described briefly some single parameter, initial stiffness acceleration schemes for elastoplasticity. Their performance was compared by conducting numerical analysis of some benchmark problems in Soil Mechanics. Two most common yield criteria in Soil Mechanics, namely Tresca and Mohr-coulomb yield criteria, were used in the analysis. A brief review of the various accelerated iterative schemes in nonlinear finite element analysis in elastoplasticity was also presented. On the basis of their study, the performance of the initial stiffness scheme with the modified Thomas accelerator was reported to be the method of choice.

Sloan and Abbo [23] presented an algorithm for performing coupled Biot consolidation analysis with automatic time stepping and error control. The local solution error was estimated from the difference between a pair of first and second order integration schemes. Automatic subincrementation of the user defined trial time steps were done to keep this time stepping error in the displacements close to the specific tolerance. Hence the algorithm was able to automatically choose small time steps whenever the behaviour is strongly nonlinear, and therefore a few iterations per time step for elastoplastic consolidation problems was needed. It was also reported that the new solution algorithm computes not only the displacements and pore pressures, but also their rates. A number of practical examples were presented which illustrated the utility of the scheme for linear and nonlinear problems.

A brief review of the use of different load stepping schemes for elastoplastic constitutive models and implementation of critical state models in finite element method was presented by Sheng and Sloan [20]. It was reported that in spite of strong influence of the iterative/incremental schemes on the efficiency and accuracy of the nonlinear finite element analysis, very few comparative studies of these schemes for critical state models had been done. They investigated the performance of various incremental, iterative and automatic solution schemes by analyzing a number of problems using critical state model. The various schemes they studied include: (1) Incremental schemes: Forward Euler scheme without drift correction and Forward Euler scheme with drift correction, (2) Iterative schemes: Newton–Raphson scheme, Modified Newton–Raphson scheme, and Accelerated Modified Newton–Raphson scheme, (3) Automatic schemes: Automatic incremental scheme, and Automatic iterative scheme.

Some of the important conclusion made from the studies of Sheng and Sloan are: (i) The Newton-Raphson scheme was reported to be more robust than modified Newton-Raphson scheme, but less efficient in terms of CPU time. (ii) The accelerated modified Newton-Raphson schemes were reported to be efficient in terms of CPU time over the Newton-Raphson and Modified Newton-Raphson schemes for runs with large load steps, but were not very robust. (iii) The performance of all iterative schemes was reported to be sensitive to the load step size and the iteration tolerance. (iv) The automatic schemes were reported to be robust, efficient and accurate when applied to critical state models.

Minkoff and Kridler [17] compared two adaptive time stepping methods (the local error method and the pore pressure method) to a constant step size scheme run with small time steps. The methods were tested on the Terzaghi consolidation problem, which is a classic one-dimensional test problem with single phase flow. It was reported that the pore pressure method was the cheapest of the three methods and the local error method was able to achieve excellent solution accuracy.

Seifert and Schmidt [19] investigated the convergence properties of three different line-search methods within an automatic time increment in finite element calculations. An elastic–plastic and an elastic–viscoplastic version of Gurson's porous plasticity model were used for the assessment of the algorithms. It was reported that the inclusion of line-search could reduce the number of increment.

Verhoosel et al. [24] proposed a general framework for implementation of arc-length constraints on the basis of the energy release rate. Three different energy release rate constraints were derived and their applicability was shown by means of five numerical examples. The study revealed the possibility of use of the proposed constraints for robust tracing of the equilibrium path for problems in which material instabilities are involved. This was reported to be an important improvement with respect to conventional arc-length methods. The study concluded with the ability of the proposed framework for the derivation of arc-length constraints for various types of constitutive behaviour and kinematics relations.

2.3. Critical Review on Literature Study

From the literature study the following points may be drawn:

1. Very little literature is available which study the influence of the nonlinear solution techniques and load stepping schemes for the solution of nonlinear FE problems in Soil Mechanics.
2. Only Gen and Potts [13], and Sheng and Sloan [20] studied the effect of various solution schemes while applied to critical state soil constitutive models.
3. These studies were done using explicit integration algorithms for the model.

3. Numerical Example

3.1. Introduction

To compare the performance of various solution schemes a practical Soil Mechanics problem is solved. The implicit integration algorithm and FEA template developed by Devi (2013) is used to integrate the MCCM equations. The numerical example carried out is an uncoupled elastoplastic analysis to simulate the laboratory drained triaxial compression tests. For this purpose, a quarter of the cylindrical specimen of 0.5 unit in diameter and 1.0 unit in length is discretized into 8 number of eight nodded, is oparametric, quadrilateral elements. The soil is considered to be lightly over-consolidated with an OCR of 1.2 (NCC).

3.2. Soil Properties and Initial Condition

The soil is first consolidated to P_c=60 kN/m^2, where P_c is the pre consolidation pressure. This is then followed by swelling to P_0=50 kN/m^2, where P_0 is the present consolidation pressure, to give an *OCR* of 1.2. The other soil parameters are as follows:

Initial axial pressure (σ_{ro}) = initial radial pressure (σ_{ao}) = 50 kN/m^2,
Initial void ratio (e_o) =1.50 kN/m^2
Slope of the sweeling curve (κ) = 0.02,
Slope of the vergin consolidation curve (λ) = 0.2,
Slope of the critical state line (M) = 1.2,
Poisson's ratio (μ) = 0.3

The load is applied in 10, 20 and 50 equal steps. A converged solution was not achieved with any schemes with 10 number of load steps. The converged solution with 20 number of load increments was obtained only with arclength method. The results are obtained using different solution schemes and compared with established data [20].

3.3. Analysis of Results

Comparison of the results are done in terms of
(i) The errors in strain calculation
(ii) The time required in the analysis and
(iii) The number of iteration required for a converged solution.

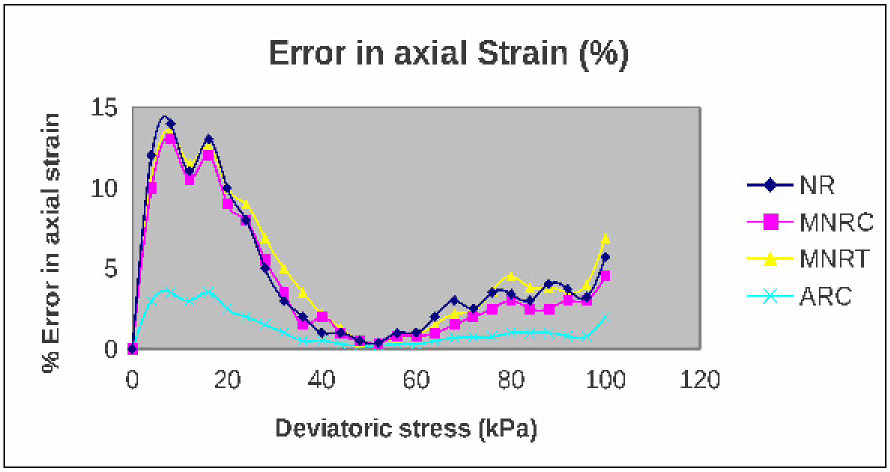

Figure 1: % error in axial strain vs Deviatoric stress

Figure 1 shows the percentage of error in axial strain from different solution schemes. The solution schemes considered include: (i) Newton-Raphson method(NR), (ii) Modified Newton-Raphson method with Chen accelerator(MNRC), (iii) Modified Newton-Raphson method with Thomas accelerator(MNRT), and (iv) Arclength method(ARC). The percentage error was calculated with respect to the one obtained by Sheng and Sloan [20]. Figure 2 shows the total iteration number and time required by different schemes for convergence. It can be noted that in each case the total load was applied in 50 number of equal increment till the sample fails. The failure was indicated by the non-convergence of the program.

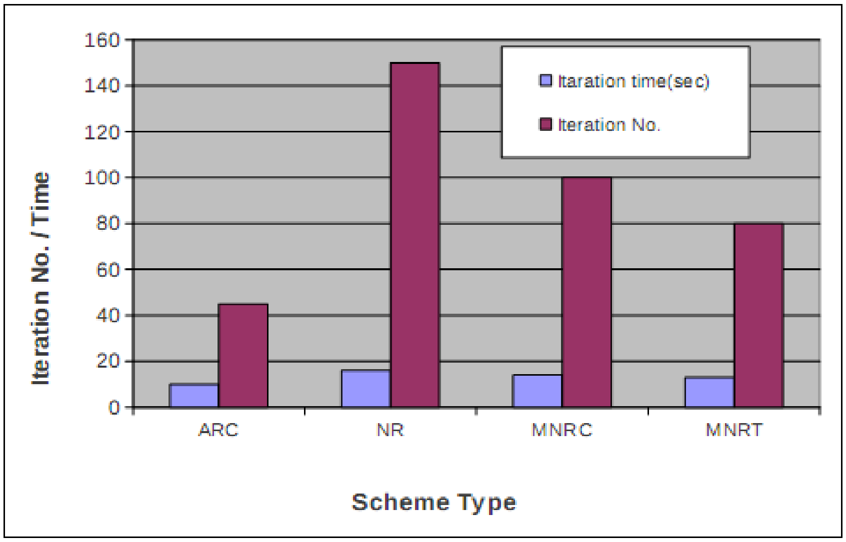

Figure 2: Iteration number and time vs scheme type

4. Conclusion

The principal methods for nonlinear solution techniques and load stepping schemes for solving nonlinear constitutive models are surveyed and brief review of the available literature is presented. The study of the effects of these solution schemes in the analysis of structures with finite element method started as early as 1980s. It is observed that arc-length method, in its standard as well as modified form, with or without line-search, dominates the whole domain of the iterative schemes. A very few literature is available which study the influence of the nonlinear solution techniques and load stepping schemes on the solution of nonlinear finite element problems while applied to soil constitutive models including critical state models. From the available literature it can be concluded that performance of load stepping schemes and nonlinear solution techniques while used with critical state models is very sensitive to the load step size and iteration tolerance, and can be an active area of study.

A comparative study on the performance of some of the mostly used nonlinear solution techniques and load stepping schemes with modified cam clay model is also carried out. The solution schemes which are included in the study are: Newton Raphson methos, Modified Newton Raphson methos, Accelerated modified Newton Raphson methods and Arclength method. The study reveals that the performance of Arclength method is better as compared to other methods while using them with modified cam clay model.

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