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A Mathematical Model For Toxic Waste In A River And Its Remediation By Freshening

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Abstract:

We present a simple mathematical model for river pollution and examine the effect of freshening on the humiliation of toxin The model consists of a pair of coupled feedback diffusion advection equations for the toxin and dissolved oxygen concentrations, respectively. The coupling of these equations occurs because of reactions between oxygen and poison to produce harmless compounds. Here we consider the steady-state case in one spatial dimension. For simplified cases the model is solved logically We also present a numerical approach to the solution in the general case. The extension to the transient spatial model is relatively straight forward. The study is motivated by the crucial problem of water pollution in many countries and specifically within the The krishna River in India. For such real situations, simple models can provide decision support for planning limitations to be imposed on farming and city practices.

Key words: Toxin, Freshening, Foretel, Aeration

1. Introduction

Water pollution from human activities, either industrial or domestic, is a major problem in many countries [1]. Every year, approximately 25 million persons die as a result of water pollution. Developing models to enable us to understand how to control and foretell water quality is of crucial interest. When assessing the quality of water in a river, there are many factors to be considered: the level of dissolved oxygen; the presence of nitrates, chlorides, phosphates; the level of suspended solids; environmental hormones; chemical oxygen demand, such as heavy metals, and the presence of bacteria. Pollutants from agricultural operations can be a significant contributor to the impairment of surface and groundwater quality [2]. Mathematical water quality models date back to the 1920s: in 1925, the well-known model of Streeter and Phelps [3] described the balance of dissolved oxygen in rivers. Subsequently this model has been amended in various ways (cf. [4]). The primary objective of the present study was to investigate the alleviation of toxic waste by freshening within a flowing river contaminated by distributed sources and the associated depletion of dissolved oxygen. The particular river whose water quality was the motivation for the study is the The krishna River in India. It is assumed that the pollutants are largely biological wastes which undergo various biochemical and biodegradation processes using dissolved oxygen. For fish to survive we require dissolved oxygen concentrations everywhere to remain at least at 30% of the saturated value [5] and so this helps to set limits on how much toxic waste can be tolerated. Simplified models for this real situation will aid decisions concerning future restrictions to be imposed on farming and urban practices.

Parameters	SI Units
L is the polluted length of river (m)	325,000a
D_p is the dispersion coefficient of pollutant in the x direction (m^2 day^{-1})	3,456,000a
D_x is the dispersion coefficient of dissolved oxygen in the x direction (m^2 day^{-1}), taken to be the same as D_p	3,456,000
v is the water velocity in the x directi on (m day^{-1})	43,200a
A is the cross-section area of the river (m^2)	2100a
q is the rate of pollutant addition along the river (kg m^{-1} day^{-1})	0.06a
K_1 is the degradation rate coefficient at 20^0C for pollutant (day^{-1})	8.27c
K_2 is the de-aeration rate coefficient at 20^0C for dissolved oxygen (day^{-1})	44.10b

Parameters	SI Units
α is the mass transfer of oxygen from air to water (m^2 day^{-1})	16.50b
S is the saturated oxygen concentration (kg m^{-3})	0.01b

Table 1: Variables and parameter values

This is calculated as a product: re-Aaeration rate XA'; the re-aeration rate is 0:055 day^{-1b} and A' (the top surface area per unit length) isthe product of the river width of 300 m with the unit length (1 m).

[a][8]

[b] [7]

[c] Based on the molecular weights in the chemical reaction $K_1=(3/16)K_2$

[d] Estimated

2.Description Of The Mathematical Model

We model the flow in the river as being one-dimensional, using a single spatial parameter x(m) to describe the distance down the river from its source. Quantities, such as toxin or oxygen concentrations, are only allowed to vary along the length of the river and they are treated as homogeneous across the river cross-section. This assumption is justified by fulfilling Dobbins' criterion [6]. For the present we allow for variation with time t (days); however, in the latter part of the work we focus on seeking steady-state solutions and so we will drop the time dependence. We use a single quantity to measure water pollution, the concentration P(x, t)(kg m^{-3}). Dissolved oxygen within the river has concentration X(x, t)(kg m^{-3}). This latter quantity is crucial both for the survival of aerobic communities living in aquatic systems and also for the potential remediation of some of the unwanted pollutants by oxidation.

Our model is composed of two coupled advection dispersion equations. These equations account for the evolution of the toxin and the dissolved oxygen concentrations, respectively. The rates of change of the concentration with position x and time t are expressed as

$$\frac{\partial(AP)}{\partial t} = D_P \frac{\partial^2(AP)}{\partial x^2} - \frac{\partial(vAP)}{\partial x} - K_1 \frac{X}{X+k} AP + qH(x), \ (x < L \le \infty, t > 0)$$

$$\frac{\partial(AX)}{\partial t} = D_X \frac{\partial^2(AX)}{\partial x^2} - \frac{\partial(vAX)}{\partial x} - K_1 \frac{X}{X+k} AP + \alpha(s-x), \ (x < L \le \infty, t > 0)$$

Where H(x) is the Heaviside function $H(x) = \begin{cases} 1, & 0 < x \\ 0, & otherwise \end{cases}$

The parameters associated with these equations and suitable modelling values are given in Table 1. These equations are standard and are developed in chennai[7]. The first equation includes both addition of pollutant at a rate qH.x/, and its removal by oxidation. The river has been divided into two sections: upstream x < 0 near the source, where it is assumed that there is no added pollution and downstream 0 < x < L where pollution is added at a rate q. For simplicity, the addition of toxin which is strictly a function of time and position, will be taken to be constant along the downstream portion of the river. The second equation is a mass balance for dissolved oxygen, with addition through the surface at a rate proportional to the degree of saturation of dissolved oxygen (S-X), and consumption during the oxidation of the toxin The rate of depletion of toxin concentration P, due to the biochemical reaction with dissolved oxygen, has been described using a ``Michaelis-Menten'' term $-K_1 \frac{X}{X+k} AP$. This term enables pollution to be removed at a rate proportional solely to the pollution concentration when oxygen levels are high. However, at low levels of oxygen the reaction must also be proportional to the oxygen concentration, as also allowed for by this term. In the second equation, the coefficients of the corresponding dissolved oxygen concentration depletion term differ because of the different weights of oxygen and toxin involved in the reaction.

To simplify the equations, we set the values of the cross-sectional area of the river A, the downstream velocity of the river v, the rate of addition of toxin q, the rate of transfer of oxygen through the surface of the river α, the saturated oxygen concentration, and the dispersion rates of toxin and dissolved oxygen, D_P and D_X , respectively, to be constant. Hence forth we will consider only the steady-state solutions, for which the left-hand sides of Eqs. (2.1) and (2.2) vanish. For these the only variation is with the distance downstream on the river and so we write P(x, t) = $P_s(x)$ and X(x, t)= $X_s(x)$.

We first consider various special cases for which the equations simplify and can be solved analytically and then describe apreliminary numerical approach to solving the more general problem. For the present we will ignore the restriction x < L,due to the finite length of the river.

3. Analytic steady-state solutions for special cases

3.1. Zero Dispersion

We begin by considering the case when the dispersion can be taken to be negligible, $D_P = 0$, $D_X = 0$. For this case theequations reduce to

$$\frac{d(vAP_s(x))}{dx} = -K_1 \frac{X_s(x)}{X_s(x) + k} AP_s(x) + q, \qquad (x > 0)$$

$$\frac{d(vAX_s(x))}{dx} = -K_2 \frac{X_s(x)}{X_s(x) + k} AP_s(x) + \alpha(S - X_s(x)), \qquad (x > 0)$$

with boundary conditions $P_s(0) = 0$ and $X_s(0) = S$. For this case there is no pollution upstream because of the absence of dispersion (i.e. $P_s(x) = 0$ and $X_s(x) = S$ for $x < 0$). In the case where, additionally, the half-saturated oxygen demandconcentration for toxin decay kis negligible $(k \approx 0)$, we can calculate the downstream toxin concentration to be

$P_s(x) = (q / K_1 A)(1 - \exp(-K_1 x / v))$, which tends to the limit $q/K_1 A$. The corresponding dissolved oxygen concentration is

$$X_s(x) = S - \frac{K_2 q}{K_1}\left(\frac{1}{\alpha} - \left(\frac{1}{\alpha - K_1 A}\right)e^{\frac{-K_2 x}{v}}\right) - \left(\frac{K_2 q A}{\alpha - K_1 A}\right)e^{-\frac{\alpha x}{vA}}$$

Taking the downstream limit we have

$$\lim_{x \to \infty} X_s(x) = S - \frac{K_2 q}{\alpha K_1}.$$

Therefore downstream, in this simplified model, the dissolved oxygen requirement for fish survival, which is that X is greater than 30% of the saturated value S [5], is achieved for levels q which satisfy q < 0.7αK$_1$S/K$_2$. In this case, with our parameter values, the fish survival constraint is q < 0.015 kg m^{-1} day^{-1}

(The actual rate of toxin insertion in the The krishna River corresponds to q = 0.06 kg m^{-1} day^{-1}).

The dispersion-free equations can also be solved for small values of k. In this case

$$\left(P_s(x), X(x)\right)|_{\text{target x}} = \left(\frac{q}{K_1 A}\right) + \frac{\alpha k q}{\alpha K_1 q - q K_2}, S - \frac{q K_2}{\alpha K_1}.$$

This solution is not valid if $q \geq \alpha K_1 S / K_2$ and, in that case, the value of the dissolved oxygen concentration reaches zero at apoint $x = \bar{x}$ and, thereafter, for $x > \bar{x}, X_s(x) = 0$. Later work will provide ways of estimating $\bar{x}$.

3.2. Models Including Dispersion With Linear Kinetic

We now consider the case where dispersion terms are included, causing the second-order derivative terms to survive, but for which k is assumed to be negligible $(k \approx 0)$. Then the equations become

$$D_p \frac{d(AP_s(x))}{dx} - \frac{d(vAP_s(x))}{dx} - K_1 AP_s(x) + qH(x) = 0$$

$$D_x \frac{d^2(AX)}{dx^2} - \frac{d(vAX_s(x))}{dx} - K_2 AP_s(x) + \alpha(S - X_s(x)) = 0$$

We find the pollution concentration

$$P_s(x) = \begin{cases} \dfrac{q}{K_1 A}\left(1 - \left(\dfrac{\delta + \beta}{2\beta}\right)e^{(\delta - \beta)x}\right), & x \geq 0 \\[2ex] \dfrac{q}{K_1 A}\left(\dfrac{\beta - \delta}{2\beta}\right)e^{(\delta + \beta)x}, & x < 0 \end{cases}$$

Where $\delta = v/2D_p$ and $\beta = \left(\sqrt{v^2 + 4D_p K_1}\right)/2D_p$ and the dissolved oxygen concentration is

$$X_s(x) = \begin{cases} S - \dfrac{K_2 q}{K_1}\left[\left(\dfrac{\gamma + \eta}{2\eta\alpha} - \dfrac{\delta + \beta}{4\beta\eta A^*} + \dfrac{\delta - \beta}{4\beta\eta B^*}\right)e^{(\gamma - \eta)x} - \dfrac{\delta + \beta}{2\beta A^*}xe^{(\delta - \beta)x}\right], & x \geq 0 \\[2ex] S + \dfrac{K_2 q}{K_1}\left[\left(\dfrac{\gamma - \eta}{2\eta\alpha} - \dfrac{\delta + \beta}{4\beta\eta A^*} + \dfrac{\delta - \beta}{4\beta\eta B^*}\right)e^{(\gamma + \eta)x} - \dfrac{\delta - \beta}{2\beta B^*}xe^{(\delta + \beta)x}\right], & x < 0 \end{cases}$$

Where

$$\gamma = \frac{v}{2D_x}, \qquad \eta = \frac{\sqrt{v^2 + \dfrac{4\alpha D_x}{A}}}{2D_x}, \qquad A^* = 2AD_x(\delta - \beta) - vA, \qquad B^* = 2AD_x(\delta + \beta) - vA.$$

We have required that $P_s(\infty) < \infty$ and $P_s(-\infty) < \infty$. There are no point sources of toxin (only distributed sources), and so $P_s(\infty)$ is assumed to be continuous at x=o. Since the dispersive flux $DP_s'(x) - vP_s(x)$ is also continuous this further implies that $P_s'(x)$ is continuous. We use the boundary conditions $X_s(\infty) < \infty$ and $X_s(-\infty) = S$. Also we require $X_s(x)$ and $X_s'(x)$ to be continuous where the input of toxin starts at x=0. The resulting toxin concentration $P_s(x)$ is relatively smooth with a discontinuity of $q/D_p A$ in the second derivative at x=0. The dissolved oxygen concentration $X_s(x)$ has a discontinuity in the fourth derivative at x=0.

4.Numerical Approach For Finding The Steady-State Solution When K Is Non-Zero

In the general case, the boundary concentrations for toxin and dissolved oxygen are still $P_s(-\infty) = 0$ and $X_s(-\infty) = S$ far upstream and far downstream, respectively.

$$P_s(\infty) = \frac{q}{K_1 A}\left(1 + \frac{k}{X_s(\infty)}\right) \text{ and } X_s(\infty) = S - \frac{K_2 q}{K_1 \alpha}.$$

Furthermore, we can obtain flux conditions directly by mathematical analysis: $P_0'(0) \le q/Av$ and $X_0'(0) \le X_k'(0) \le 0$, where $P_0(x)$ and $X_0(x)$ are the steady-state solutions for k= 0, and $P_k(x)$ and $X_k(x)$ are the corresponding cases for non-zero k.

A preliminary numerical routine has been developed to search for the solution. We integrate from a grid of initial valuesat x D 0 and progressively refine this grid to find the solution to the boundary conditions. The Euclidean norm is used formeasuring the agreement of trial initial conditions with toxin and dissolved oxygen concentration far upstream and downstream. The numerical routine has been used in an exploratory investigation with the calibrated parameters given inTable 1 for the The krishna River. For initial testing of the algorithm the case where k is negligible is used. The results agreewith the analytical solution under the conditions of no pollution and saturated dissolved oxygen far upstream, tending to asteady state far downstream for a long (considered infinite) river.

5.Concluding Remarks

We have presented a mathematical model for river pollution comprising a coupled pair of nonlinear equations and have investigated the effect of freshening on the degradation of toxin In some simplified cases we have obtained analytic steady-state solutions. The preliminary numerical approach agrees with these analytical solutions under relevant conditions. This model and its solutions could aid in decision support on restrictions to be imposed on farming and urban practices, e.g. it can enable scenarios to be tested for fish survival. Further features can be readily included in this simplified model such as variable toxin input, tidal flow and the like. The model can also be used to illustrate the effect of freshening processes toincrease dissolved oxygen to the water. The extension to the transient spatial model is relatively straightforward.

For the model presented here, it is observed that a pollution insertion rate of q =0.06 kg m^{-1} (as for the The Krishna River) would result in an ecologically dead river at fardownstream distances when dissolved oxygen falls below the ideallevel required for

fish to live on [5]. The toxin being inserted into the river is about 4 times too high and this implies that the total biological oxygen demand (BOD) rate for the river should have a maximum insertion of 5000 kg BOD day[-1].However, fortunately, further investigation shows that for a river of the length in this study (the lower 25 km of the The krishna River), the dissolved oxygen level remains above the critical value of 30% of the saturated value in agreement with previous conclusions [8]. The critical value is not reached in the The krishna River because of the finite length over which pollution is actually discharged so that the river reaches the sea before this environmental catastrophe can occur. Further details of the model are given elsewhere [9].

6.Acknowledgements

7.Reference

1. G. Hang , F.L. Burton, "Wastewater Engineering: Treatment," Dispo Reuse, 3 ed., McGraw-Hill, New York, 1999.

2. R.L. Knight, V.. Payne Jr., R.E. Borer, R.A. Clarke Jr., J.H. Pries, Constr ucted wetlands livestock wastewater treatment, for Ecol. Engin. 15 (2000) 41_55.

3. H.W. Streeter, E.B. Phelps, A study of the pollution and nutrial purification of the Ohio river, in: US Public Health Service, Public Health Bulletin,

4. A. DAVID, in: M.B. Beck (Ed.), Mathematical Models in Water Pollution Control, Modelling of Dissolved Oxygen in a Non-tidal Stream, John Wiley and Sons Ltd, Southern gate, Chichester, 1987, pp. 137_163.

5. S. Murphy, General information on dissolved Oxygen, www.bcn.boulder.co.us/basin. University of Colorado at Boulder, Colorado, 2007 (retrieved 27.09.07).

6. W.E. Dobbins, BOD and Oxygen relationships in streams, Journal Sanitary Engineering Division, Proc. ASCE 90 (1964) 53_78.

7. S.C. Chapra, Surface Water quality Modeling, McGraw-Hill, New York, 1997.

8. Pollution Control Department (PCD), Water quality standards & criteria in India (in chennai), 4 ed., Ministry of Science, Technology and Environment, India, 2000.

9. B. Pimpunchat, W.L. Sweatman, W. Triampo, G.C. Wake, A. Parshotam, Modelling river pollution and removal by aeration, in: L. Oxley, D. Kulasiri (Eds.).