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Posterior Distribution of Health Insurance: Basis for Determining Credibility Factor and Premium. Evidence from Ghana

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Abstract:
It is always important to determine distribution of insurance claims in order to estimate future expected values. This study seeks to determine the posterior distribution using Bayesian credibility theory. Secondary data collected from Dormaa Municipal Health Insurance Scheme and Dormaa Presbyterian Hospital was analyzed using the Statistical Package for Social Science (SPSS), Excel spreadsheet, and Easy fit by applying Bayesian credibility theory through descriptive analysis and frequency distribution.
It was found that the data submitted by 28 health facilities to Dormaa Municipal Health Insurance Scheme follows Normal Distribution and the sample claims from Dormaa Presbyterian Hospital follows lognormal distribution. The posterior distribution for the municipal health Insurance Scheme was also found to be normal distribution.

Keywords: *Posterior distribution, Bayesian credibility, health insurance, national health insurance*

1. Introduction

A healthy population is a major backbone for the socio-economic development of a nation. A healthy population with the requisite skills and knowledge become the back bone for economic development and transformation. It is in this light that every country in the world places premium on improved health care for her citizens. It has therefore become the state's responsibility to provide health care to the people of Ghana and this responsibility comes with financial challenges in view of the difficult economic issues that confront the country. Financing an efficient and effective health care system is of a major concern to countries all over the world, especially developing economies. In Ghana, healthcare financing has gone through several phases. After independence in 1957, the provision of health care in Ghana was financed by the state through tax revenue. However, it became obvious that this method of financing health care was not sustainable following the economic difficulties the country experienced from the beginning of the 1960s (Osei-Akoto, 2004). As part of efforts to revamp the Ghanaian economy, the state began to reduce expenditure on the provision of social services and the health sector witnessed considerable reduction in state funding. The then government in 1985, introduced user fees for all medical conditions, except certain specified communicable diseases. This system whereby people who access public health facilities pay user fees became known as 'cash and carry' and resulted in several operational challenges as well as people in the country (Asenso-Okyere, Osei-Akoto, Anum, & Appiah, 1997). In order to ameliorate the problems associated with the "cash and carry" system, government introduced the National Health Insurance Law, Act 650 in August, 2003. It sought to provide basic health care services to persons resident in Ghana through mutual and private health insurance schemes, and to establish a National Health Insurance Fund that will provide subsidy to licensed District Mutual Health Insurance Schemes. Health Insurance is an alternative health care financing system which involves resource pooling and risk sharing among members (Ministry of Health, 2003).The Health Insurance Act mandates the creation of district-level Mutual Health Organizations (MHOs) in accordance with national guidelines and the establishment of a National Health Insurance Council (NHIC). The law represents a bold and innovative move by government to provide health insurance coverage to all of its citizens. This is meant to provide financial protection for the entire population and

move away from the "Cash and Carry" system which was creating considerable equity concerns, largely due to the non-functional exemption mechanisms.
In view of the economic importance of National Health Insurance Scheme (NHIS) in developing countries, there is a need to use actuarial analysis to model the distribution of the claim amounts presented to the claim center, which can then be used to estimate the most appropriate credibility factor for the best expected claim for the near future.
The main objective of this article is to determine the posterior distribution of Dormaa Municipal (NHIS) claims and the mean of the posterior distribution. The essence of the posterior distribution and mean is to help insurance companies to determine the credibility factor and premium.

2. Literature Review

2.1. Review of Health Insurance in Ghana

In order to mitigate the negative effects of the "Cash and Carry" system, especially on the poor, the government of Ghana commissioned various studies into alternative health care financing mechanisms. The study proposed that a centralized National Health Insurance Company should be set up to provide a compulsory "Mainstream Social Insurance Scheme". Also, the report recommended pilot "rural-based community financed schemes" for the non-formal sector but gave no further details as to how the Ministry of Health (MOH) was to do this (Aikens, 2005). In 1997, the NHIS pilot project was formally launched in the Eastern Region intended to cover four districts namely; New Juaben, Suhum/Kraboa/Coaltar, Birim South and Kwahu South. As a result, a NHIS Secretariat was established to undertake the preparatory work and carry out the NHIS program. Soon after the implementation of the pilot scheme, there were a lot of debates about the strategic direction of health financing policy generally, and the pilot scheme in particular (Atim & Sock, 2000).
In 2003, the 'National Health Insurance Act' was passed to operationalize the policy decision to move from user fees towards a pre-payment financing mechanism. The Act explicitly requires every Ghanaian to join either a Mutual Health Insurance Scheme (MHIS) or a private mutual or commercial insurance scheme. To mobilize additional funds to support the implementation of the district MHIS, the government of Ghana instituted a National Insurance Levy of 2.5% on specific goods and services. This is intended to subsidize all fully paid contributions to the district health insurance schemes. In addition, 2.5% of social security contributions paid by formal sector employees. Those in the informal sector are expected to make direct contribution based on one's ability to pay. Children under 18 years whose parent(s) or guardian(s) pay their contributions and the elderly are exempted from paying any contribution. Diseases covered include malaria, diarrhea, upper respiratory tract infection, skin diseases, hypertension, diabetics, asthma, and a host of other diseases.
The aim of health insurance is to spread the risks of incurring health care costs over a group of subscribers. Hence, the larger the subscribers the lower the risk burden on the individual and vice versa. Thus the design of the NHIS is based on the following principles; equity, risk equalization, cross-subsidization, quality care, efficiency in premium collection and claims administration, community or subscriber ownership, partnership and reinsurance (Ministry of Health, 2004).

2.2. Literature on Actuarial Modeling in Insurance Data

(Boucher, 2014; Hua, 2015; Wright, 2005) used actuarial modeling to fit models to many claim amount drawn from consecutive years. They fitted analytic loss distribution using maximum likelihood estimation for each of the years. They used a lot of models such as Pareto, Burr, Inverse Burr, and Lognormal to fit their data and test to select the best fit one.
(Horg & Klugman, 1983) also used Weibull distribution to fit 35 observations of hurricane loss and they found that Weibull distribution performs as well as the lognormal distribution. (Boucher, 2014; Renshaw, 2004) used actuarial modeling to fit a statistical distribution to some claim amounts, the statistical distribution they tested were Lognormal, Gamma, and Weibull distributions. They used maximum likelihood estimation to fit the distribution, an idea that was extended to their research, only that they based their methodology on the Bayesian solution. They then used the likelihood functions to calculate the posterior distribution.
A typical model for insurance risk has two main component (Cizek, Hardle, & Weron, 2005): one characterizing the frequency of claims (or incidence) of events and analyzed by discrete models such as poison distribution, binomial distributions, and another describing the severity (size or amount) of loss resulting from the occurrence of the event. These are also analyzed by continuous models such as Normal distribution, lognormal distribution, Gamma distribution, Weibull distribution etc.

2.3. Prior and Posterior Distributions

Bayesian method is the process of assigning probabilities to parameters, hypotheses, and models and updating these probabilities on the basis of observed data (Montgomery & Runger, 2003). For example, Bayesians do not treat the mean of a normal population as an unknown constant; they regard it as the realized value of a random variable, say θ ,with a probability density function over the real line.
Probability that describes the extent of our knowledge and ignorance of such non variable entities are usually referred to as *subjective probabilities* and are usually determined using one's intuition and past experience, prior to and independently of any current or future observations (Klugman, 1986). Prior probability distribution of an uncertain quantity is the probability distribution that would express one's belief about this quantity before some evidence is taken into account to estimate this uncertain quantity (parameter). Prior distribution can be:

(1) Informative: This expresses the specific, definite information about a variable; it can be the distribution of past data (population) of which the sample is drawn to estimate the unknown random parameter holding the variance of the population constant (subjective). Two techniques used most frequently to develop informative prior distribution are: (a) Expert opinion: One of the most powerful methods for developing informative prior is to synthesize information from group of experts knowledgeable in the field of study. (b) Data summaries: Here, data for other species and stocks can be used to construct priors for the area for which an assessment is needed.
(2) Uninformative or Non-informative prior: This expresses vague or general information about a variable. The simplest way of determining non-informative prior is assigning equal probabilities to all possibilities. Bayes and Laplace postulate that, when nothing is known about μ in advance, let the prior $f(\mu)$ be a uniform prior distribution, that is, let all possible outcomes of μ have the same probability. They then say that no single probability distribution can model ignorance satisfaction, therefore large classes of distributions are needed.
(Bernardo & Smith, 1994) define non informative prior as a prior which provides little information relative to the experiment. They say that it has minimal effect relative to the data, after finding the posterior mean of insurance loss data. They regard the non-informative prior as mathematical tool.
(Young, 2000) suggested using kernel density estimation to estimate the prior distribution of the parameter of interest. To enhance accuracy, he suggested employing a loss function, which is a linear combination of a squared-error term and another term designed to reduce divergence.
(Landsman & Makov, 2001) used the maximum entropy principle to establish a prior distribution for the dispersion parameter γ of the exponential dispersion model from measures information and later used the criterion to establish a prior distribution for γ in conjunction with knowledge on the probability that a claim exceeds a certain threshold, thus allowing for information on tail behavior to affect a premium.

2.4. Application of Bayesian Method in Actuarial Science

Bayesian ideas and techniques were introduced into actuarial science and it laid down the foundation to the empirical Bayes credibility approach, which is still being used extensively in the insurance industry.
(O' Hagan, Stevens, & Montmartin, 2001) in their article developed a Bayesian computation of the incremental cost-effectiveness acceptability curve for assessing the relative cost-effectiveness of two treatments in health economics, where data on both costs and efficacy are available from a clinical trial.
Wang (2000) focused on quantifying the cumulative risk associated with false-positive results on repeated screening procedures for medical conditions, both at the population and the individual level. They developed actuarial models for life table data and added a Cox regression to enable individual level modeling.
Bayesian Statistics offers a rationalist theory of personality beliefs in contexts of uncertainty, with the central aim of characterizing how an individual should act in order to avoid certain kinds of undesirable behavioral inconsistencies (Bernardo & Smith, 1994). The theory establishes that, expected utility maximization provides the key to the ways in which beliefs should fit together in the light of changing evidence.
Inference is only of value if it can be used, so the extension to decision analysis, incorporating utility, is related to risk and to the use of statistics in science and law" (Linley, 2000). The point here is that Bayesian statistics is not merely a collection of methods for the analysis of data. It is a theory for making decisions under uncertainty, where both components—probability and utility—are equally important and a full Bayesian analysis includes data analysis, construction of probability models, assessment of the prior information and the utility function, and, finally, making decisions.
In the financial world, making decisions under uncertainty has been recognized to be one of the most important activities (Foccardi & Jonas, 1998). They stated "Risk measurement implies that there is a model of the market that, applied to data, measures risk. But risk management is not limited to the more or less scientific process of measuring risk. Once measured, subjective judgment is used to evaluate and make decisions upon the measurement"
(Mendoza, Madrigal, & Gutie'rrez-Pena, 2001; Shi, 2016) fitted a linear regression model to a set of transformed death rates and, using Bayesian predictive arguments, proposed a margin-loaded table. Mendoza, et al derived the joint posterior predictive distribution for the mortality rates to be observed in the future for a specific insured population and, on that basis, proposed a strategy to select an appropriate mortality table which is mandated for reserving purposes by the current Mexican insurance regulations.

3. Methodology

3.1. Basic Concepts and Definitions

In general insurance the severity and frequency of claims analysis is important because it helps in pricing and product development. The general insurance is based on the principle of pooling risk. Insurance involves a combination of risk pooling and risk transfer which reduces risk physically and monetarily (Baranoff, 2008). The continuous probability distributions considered are Normal, Log-normal, Pareto, Gamma, Weibull, and Exponential.

<u>3.1.1. The Normal Distribution</u>

Normal distribution is the type of continuous distribution that is most extensively used. The normal distribution is continuous and has 2 parameters, μ and σ; they determine the location and scale, respectively (Casella & Berger, 2002). It is, therefore, described by its mean and variance. The mean of the distribution is referred to as the location parameter, and the (standard deviation) measures how the distribution is spread out (known as the scale parameter). Normal distribution is important in actuarial science and has a single peak at the center of the distribution. Generally, finance related random variables follow normal distribution, so knowledge about normal distribution is vital in understanding portfolio theory. The normal curve falls off smoothly in either direction from the central value which is the mean of the distribution. So the probability density function is bell-shaped and symmetrical about the mean. Its formula is given by:

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where $-\infty < x < \infty$ and $-\infty < \mu < \infty$ $\sigma > 0$

Its Mean and Variance are given as $E(X) = \mu$ and $VarX=\sigma^2$

3.1.2. The Lognormal Distribution

Log-normal distribution is normally used to determine the claim size distribution as it is positively skew; the mean of the data is greater than the mode and the random variable does not take negative values, which is a feature of claim size distribution. It is often used in modeling claim size (Boland, 2007; Czado, Kastenmeier, Brechmann, & Min, 2012). It has 2 parameters, mean μ which is the location parameter and standard deviation σ, the scale parameter (Hossack, Pollard, & Zehnwirth, 1983) . Its probability density function is given by:

$$f(x,\mu,\sigma^2) = \frac{1}{\sigma x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{logx-\mu}{\sigma}\right)^2\right]$$

Where, $0 \le x < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$

The mean and variance are given by;

E(X) = $e^{\mu+\left(\frac{\sigma^2}{2}\right)}$ and Var(X) =$e^{2(\mu+\sigma^2)}$-$e^{2\mu+\sigma^2}$. x stands for loss or claim amount

3.1.3. The Pareto Distribution

Pareto distribution is a positively skew, heavily –tailed distribution which is used to model claim size. It has two parameters, α, which is the shape parameter and β, the scale parameter. For the mean and variance to exist in Pareto distribution, α and β, must be greater than 1 and 2 respectively,(Cizek et al., 2005). Pareto is mostly used to model income distribution and insurance claims size. The probability density function of Pareto distribution is given by:

$$f(x/\alpha,\beta) = \frac{\beta\alpha^\beta}{(\alpha+x)^{\beta+1}}, x > 0 \alpha > 0$$

The mean and variance of Pareto distribution is given by;

E(X) = $\frac{\alpha}{\beta-1}$, $where \beta > 1$ $and Var(X) = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$, $where \beta > 2$

3.1.4. The Gamma Distribution

Gamma distribution is used in the study of claim size and the analysis of heterogeneity of risk. It has two parameters, α which is the shape parameter and β the scale parameter.

The probability density function for gamma distribution is given by;

$$f(x/\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x},$$

$$where\ x > 0, \alpha,\beta > 0$$

Γ (α) is a number which depends on *α*. The Gamma distribution is not symmetrical; instead it is positively skew. The mean and variance are given by;

E(X) = $\frac{\alpha}{\beta}$and *Var(X)* =$\frac{\alpha}{\beta^2}$.

3.1.5. The Exponential Distribution

The exponential distribution is another type of continuous distribution used in actuarial modeling of claims size in general insurance. The distribution has one parameter. The probability density function for exponential distributions is given by**;**

$$f(x/\lambda) = \lambda e^{-\lambda x}, where x > 0$$

The mean and variance of exponential are E(X) = $\frac{1}{\lambda} and Var(X) = \frac{1}{\lambda^2}$

3.1.6. The Weibull Distribution

The random variable x with Weibull distribution has a probability density function given by;

$$fx/\alpha,\beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, where x > 0, \alpha > 0, \beta > 0$$

It has two parameters α, which is the location parameter and β, the scale parameter. (Kleiber & Kotz, 2003) explain that the Weibull distribution has received maximum attention for the last ten years and is still growing strong, and (Horg & Klugman, 1983) noticed its potential in insurance loss.

Its mean and variance are given by;

E(X) = $\alpha^{-\frac{1}{\beta}}\Gamma\left(1+\frac{1}{\beta}\right), Var(X) = \alpha^{-\frac{2}{\beta}}\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$

3.2. Descriptive Statistics

The descriptive statistics of the aggregate monthly claim amount was used in pointing out some features of the data in order to determine the best distribution for the sample data. The descriptive statistics obtained are the mean, mode, median, skewness, kurtosis etc. These are obtained by using SPSS on the 36 data points from the Dormaa Presbyterian Hospital. The idea behind the descriptive statistics results is to identify distribution family that the aggregate claim from the sample will follow. The summary statistics is presented in table 2.

3.3. Research Design

Secondary data was used from Dormaa Municipal Health Insurance Scheme, regarding their claims presented from the health centers (January 2008-December 2014). Two assumptions were made on the data:

1. All the claims came from the same distribution (they are independent and identically distributed). This means that there is no correlation between reported claims.
2. All future claims were to be generated from the same distribution, that is, the future claims are to follow the same distributions as the past claims.

3.4. Population and Sample Size

The targeted population for the study was made up of all the 28 accredited health care providers in Dormaa Municipal that submit claims to the Municipal Health Insurance Scheme. However, the sample size used for the study was Dormaa Presbyterian Hospital because their claim amounts are sent directly to the head office of national health insurance Scheme in Accra

3.4.1. Sample Size Determination and Sampling

The sample size that was used to determine the likelihood function of claim amount was based on the formula below:

$$n = \frac{Z_{\frac{\alpha}{2}}^2 \sigma^2}{\varepsilon^2}$$

Where, ε = how close should the sample estimates be to the unknown parameter (bound)

Z= the confidence level

σ= the estimates of the standard deviation of the population.

We needed the sample estimate to be 0.065% away from the unknown parameter, with 95% confidence level. The sample was drawn from a population with a random mean of 393120 and variance 632970 as shown in table 1. The ε margin was determined as;

$$\varepsilon = \frac{0.065}{100} \times 393120 = 258\ (bound).$$

We will like to be able to determine the sample estimate, the mean or average claim amount of Dormaa Municipal Health Insurance Scheme to be within$\pm$ 258 cedis, with 95% (Z = 1.96) confidence level. The sample size was then determined as;

$$n = \frac{1.96^2 \times 632970}{(258)^2} = 36$$

3.4.2. Simple Random Sampling

Simple random sampling techniques were employed to select sample of claim amount from Dormaa Presbyterian Hospital given that its parameter is the same as the unknown population parameter, for determining the likelihood function. 36 monthly aggregate claim amounts were selected from the secondary data provided by the hospital to form the sample size, such as;

$(x_1, x_2, x_3, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots\ x_n)$

3.5. The Likelihood Function

After the sample data such as $x_1, x_2, x_3, \dots \dots \dots \dots \dots, x_n$,has been obtained, the likelihood function is obtained by estimating the parameters of four positively skew distribution, Exponential, Lognormal, Gamma and Weibull using Easy fit, the above distributions were used because table 2 shows that the mode of the sample data is less than the mean, hence the sample data is positively skew. The chi square values of each distribution is determined and the one with the minimum chi square value is used as the best function to fit the sample data to use in estimating the population unknown parameter.

If an unknown parameter μ from a population is to be estimated from a sample of independent and identical random variables $X_1 = x_1, X_2 = x_2, X_3 = x_3 \dots \dots \dots X_n = x_n$, which distribution $f(X_i/\mu)$ depends on the parameter μ that needs to be estimated, while other parameters in the same population are constant, then the likelihood function of the sample that can be used for the estimation of parameter μ is given by;

$$Lik(\mu) = f(x_1/\mu).f(x_2/\mu).f(x_3/\mu) \dots \dots \dots \dots f(x_n/\mu)$$

$$= \prod_{i=1}^{n} f(x_i/\mu)$$

Where, $f(x_1/\mu).f(x_2/\mu).f(x_3/\mu) \ldots \ldots \ldots \ldots f(x_n/\mu)$is a joint density if the $X_{i's}$ are continuous.

3.6. Chi-square Goodness of Fit Tests

The chi-square goodness-of-fit test is used to test how well the distribution fits a given data set. (Boland, 2007) explains that when testing the fit of a continuous distribution, the data is usually first binned (or grouped) into k intervals. We then calculate the number of expected observationsE_i based on a grouped data and compare it with the actual observed numbersO_i for each interval.

Where: $E_i = \int_a^b (probability\ density\ of sample)\ dx \times samplesize$

$E_i = \int_a^b f(x) \times samplesize.$

We then measure the fit of the distribution which is obtained from the test statistic:

$$X^2 = \frac{(O_i - E_i)^2}{E_i}$$

If the value of the calculated chi-square test statistic is large, we will reject the distribution being considered since it signifies a lack of fit between the observed and expected values (Montgomery & Runger, 2003).

3.7. Determining the Posterior Distribution

After the goodness-of-fit test, the Posterior distribution was determined by multiplying the likelihood function of the sample distribution by the Prior distribution of the population mean, this is denoted as;

$$f(\mu/x) \propto \prod_{i=1}^{n} f(x_i/\mu) \times f(\mu)$$

3.8. Bayes Theorem

(Aczel., 1999) states that, the theorem allows us to reverse the conditionality of events, that is we can obtain the probability of event B given event A from the probability of event A given event B. This Theorem can be extended to a partition of more than two sets in which event A can occur. This is given by;

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A)},$$

$$where\ P(A) is\ the\ total\ probability$$

$P(A) = \sum_{i=1}^{n} P(A/B_i)\,P(B_i)$

$=P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + \cdots + P(A/B_n)P(B_n)$

$P(B_1)$ is the prior probability of event B_1

$P(B_1/A)$ is the posterior probability of B_1

Where event A can occur in $B_1, B_2, \ldots \ldots \ldots \ldots B_n$

4. Results

4.1. Descriptive Statistics of the Population Data

The summary statistics of the aggregate monthly claim amount was used in pointing out the salient features of the data in order to determine the distribution for the population data. The descriptive statistics obtained are the mean, mode, median, skewness, etc. These are obtained by using SPSS and Excel spreadsheets on the 84 data points from the scheme office. The logic behind the best model selection criteria will be based on the results of summary statistics and the test statistics.

N	Mean	Median	Mode	Std deviation	Variance	Skewness	Range	Sum
84	39.312	38.469	36.456	7.9557	63.297	0.028	32.414	3302.2

Table 1: Summary Statistics of population (× 0000)

Source: Dormaa Municipal Health Insurance, computed by Authors

The 95% confidence level for the mean is$37.586 \leq \mu \leq 41.039$.

The normal probability density function (pdf) is given by:

$$f(x/\mu, \sigma_1{}^2) = \frac{1}{\sigma_1\sqrt{2\pi}} exp^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma_1}\right)^2}$$

Where: $\mu = E(x) = 39.312\ and \sigma_1{}^2 = Var(x) = 63.297$. These are the parameters of the population of the study. But the expected aggregate claim amount calculated from the population is not constant but unknown, because each monthly claim amount are not deterministic, but are also in an expectation, therefore, μ is treated as a random variable, but the population variance $\sigma_1{}^2$ is assumed to be constant, because the expected claim will lie between $\pm\sqrt{Var(x)}$ from the population mean.

Since μ is unknown random variable, which need to be estimated again by using sample data, it distribution would be an informative one from the population (Dormaa Municipal health Insurance Scheme). Therefore the prior distribution of μ is an informative prior from the population distribution, which follows normal distribution with new mean θ and variance $\sigma_1{}^2$ that is the variance of the population such as $\mu \sim N(\theta, \sigma_1{}^2)$. The probability density function (pdf) is given by:

$$f(\mu) = \frac{1}{\sigma_{1\sqrt{2\pi}}} exp^{\frac{-1}{2}\left(\frac{\mu-\theta}{\sigma_1}\right)^2}$$

This is the prior distribution of the population mean

Where: $\theta = E(\mu) = unknown\ quantity$, $and\ the\ Var(\mu) = \sigma_1{}^2 = 63.297$.

4.2. Descriptive Statistics of the Sample Data

The descriptive statistics of the aggregate monthly claim amount was used in pointing out some features of the data in order to determine the best distribution for the sample data. The descriptive statistics obtained are the mean, mode, median, skewness, kurtosis etc. These are obtained by using SPSS on the 36 data points from the Dormaa Presbyterian Hospital. The idea behind the descriptive statistics results is to identify distribution family that the aggregate claim from the sample will follow. The summary statistics is presented in table 2.

N	Mean	Median	Mode	Std deviation	Variance	Skewness	Range	Sum
36	21.2090	20.3900	15.3100	2.9328	8.601	0.121	11.549	763.530

Table 2: Descriptive statistics$(\times \mathbf{0000})$

Source: Dormaa Presbyterian Hospital, computed by Authors

The 95% confidence level for the mean is $20.0300 \leq \mu \leq 21.9430$.

The descriptive statistics of the sample data shows that the mode (15.310)< mean (21.2090). This shows that the distribution families that can be used to model the sample claims amount from the Dormaa Presbyterian Hospital are Exponential, Log-normal, Pareto Weibull, Gamma etc. The main distributions that were considered under this study to model the sample claims are the following four: Exponential, Log-normal, Gamma and Weibull.

4.3. Parameter Estimation

Given any model, there exists a great deal of theories for making estimates of the model parameters based on the empirical data. In this case the sample claim data was used to compute the parameters of the selected distributions. Whenever the parameters of any distribution are obtained using sample data, it implies that, the statistical distribution has been fitted to the claims data. The parameters were computed by using Easy fit and SPSS. In this regard table 3 below shows the parameters of the distributions. Where α was taken to be the value of the first parameter (shape parameter) and β was taken to be the second parameter (Scale parameter)

Model	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$
Exponential	0.04715	
Log-normal	3.0451	0.137
Gamma	52.297	0.4055
Weibull	8.215	22.283

Table 3: Parameters estimates

Source: Dormaa Presbyterian Hospital, computed by the Authors

The parameters obtained were used to derive the probability density function (pdf) and the expectation of each distribution for the sample data. The probability density function of each distribution is then used in the computation of probabilities from bin or group ranges in the sample data.

4.4. Determining Probability Density Function (PDF) and Expectation

The probability density functions of the above distributions are paramount in estimating probabilities of each distribution at a given interval. The pdf and expectation of the distribution are determined based on the parameter values calculated in table 3. The 36 sample aggregate monthly claim amounts from Presbyterian hospital are random variable which can assume values such as, $x_1, x_2, x_3, \dots\dots\dots\dots\dots, x_{36}$ .The pdf for each distribution were determined to help the researchers to calculate the probability of each distribution after the sample data have been binned or grouped. The probabilities obtained are used to calculate the expected claims in group interval to compare to actual claims. The probability density functions are shown in table 4 below.

Model	PDF	Expectation
Exponential	$\alpha e^{-\alpha x}, x > 0$	21.209
Log-normal	$\frac{1}{\sigma\sqrt{2\pi}}\frac{1}{x}e^{\left\{-\frac{1}{2}\left(\frac{logx-\mu}{\sigma}\right)^2\right\}}$, $x > 0, \mu > 0, \sigma > 0$	22.502
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x, \alpha, \beta > 0$	128.969
Weibull	$\alpha\beta x^{\beta-1}e^{-\alpha x^\beta}, x > 0$	0.9299

Table 4: Probability density function and expectation estimates
Source: Dormaa Presbyterian Hospital, computed by Authors

The pdf of the models were determined for calculating probabilities of all the distributions at the given intervals in table 4.

4.5. Determining Probabilities of Distributions

The sample aggregate claims from the Municipal hospital were grouped or binned to six classes to know the actual claims for each interval. The probabilities for each interval were then calculated for every individual distribution considered in the study. The probabilities were computed using cumulative distribution function of the distributions and excel spreadsheet.

The probability of each range is essential in determining the expected claims for each distribution. Table 5 shows all the actual aggregate claims for each interval and their corresponding probabilities after the sample data have been binned or grouped

Amount	Actual (frequency)	Exponential	Log-normal	Gamma	Weibull
< 17	2	0.5502	0.06097	.06733	0.1026
17 – 19	8	0.0404	0.17037	0.1640	0.1340
19 – 21	10	0.0367	0.2670	0.2585	0.2224
21 – 23	7	0.0390	0.2469	0.2491	0.2676
23 – 25	5	0.0304	0.15227	0.1585	0.1970
25 <	4	0.3080	0.1024	0.1024	0.0762
Total	36	1	1	1	1

Table 5: Probability estimates
Source: Dormaa Presbyterian Hospital, computed by authors

4.6. Determining Expected Aggregate Claims

In other to be able to determine the best fit distribution for the aggregate claims from the municipal hospital, the expected claims for each bin or group intervals were estimated from each distribution. This enables the application of chi-square for determining the best fit model for the aggregate claim, using the minimum chi-square value. The sample size was multiplied by the probabilities of each interval to estimate the expected claims for the distributions using the formula below. Table 7 shows the expected claims of the distributions

$$E_i = \int_a^b f(x)dx \times samplesize$$

Amount	Actual O_i	Exponential E_i	Lognormal E_i	Gamma E_i	Weibull E_i
< 17	2	20	2	2	4
17 – 19	8	2	6	6	5
19 – 21	10	1	10	9	8
21 – 23	7	1	9	9	9
23 – 25	5	1	5	6	7
25 <	4	11	4	4	3
Total	36	36	36	36	36

Table 6: Expected claim and actual claims
Source: Dormaa Presbyterian Hospital, computed by the authors

4.7. Determining Chi-Square Values

The chi-square goodness-of-fit test is used to test how well the distribution fits a given data set. We calculate the number of expected observations E_i based on a grouped data and compare it with the actual observed numbersO_i for each interval.We then measure the fit of the distribution which is obtained from the test statistic;

$$X^2 = \frac{(O_i - E_i)^2}{E_i}$$

If the value of the chi-square test statistic is large, we will reject the distribution being considered since it signifies a lack of fit between the observed and expected values. Table 7 shows the chi-square values computed from table 7. In addition, goodness of fit test and p-p plot will be run to ensure that we draw the most accurate conclusion.

Model	Chi-square values
Exponential distribution	171.655
Log-normal distribution	1.111
Gamma distribution	1.333
Weibull distribution	4.640

Table 7: Chi-square values of distributions
Source: Dormaa Presbyterian Hospital, computed by the researchers

Lognormal					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value	36 0.16015 0.28297				
Significant level	0.2	0.1	0.05	0.02	0.01
Critical Value	0.17418	0.1991	0.22119	0.24732	0.26532
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	36 0.83146				
Significant level	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Table 8: Goodness of fit test
Source: Dormaa Presbyterian Hospital

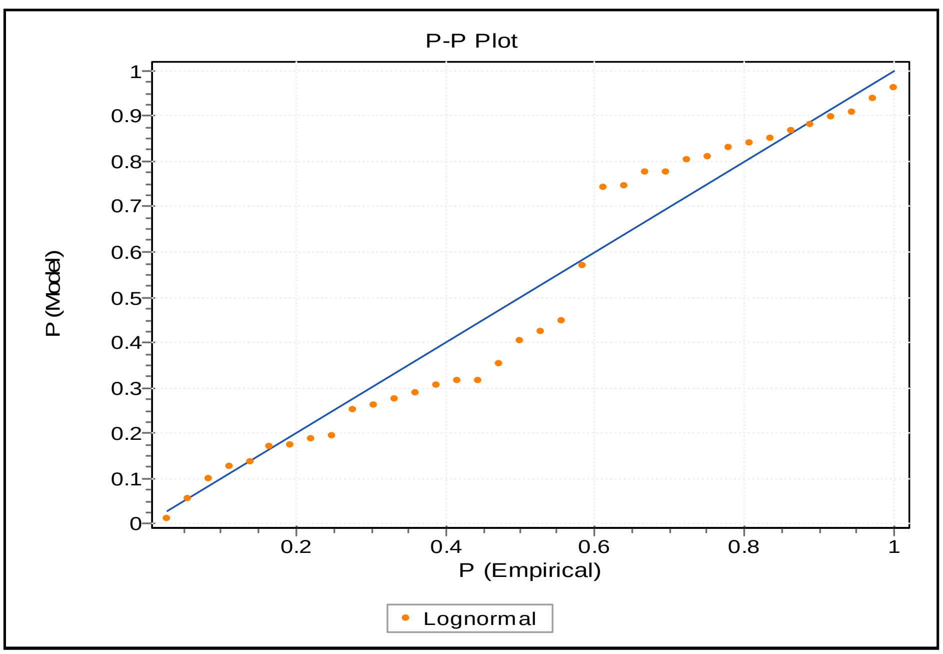

Figure 1: lognormal p-plot of Dormaa Presbyterian Hospital data

4.8. Determining the Likelihood Function

The minimum chi-square value from all the distributions is 1.111.The Kolmogorov Smirnov and Anderson Darling test were also perform and they are shown in table 4.9, it clearly shows that the Lognormal distribution could be used as the best fit model for the aggregate claim amount of Dormaa Presbyterian hospital, which constitutes the sample of this study. Therefore if $x_1, x_2, x_3, \ldots x_n$ constitute the sample data, then the distribution of the sample data follow \lognormal distribution given the prior mean μ (the unknown population mean) and variance $\sigma_2^2 = 8.601\ (sample\ variance)$, sample variance is assumed to be constant from,$x \sim LN(\mu, \sigma_2^2)$. It probability density function (pdf) is given by:

$$f(x/\mu, {\sigma_2}^2) = \frac{1}{\sigma_2\sqrt{2\pi}}\frac{1}{x}exp^{-\frac{1}{2}\left(\frac{logx-\mu}{\sigma_2}\right)^2}$$

Therefore to estimate an unknown parameter μ from a sample of independent and identical random variables, $X_1 = x_1, X_2 = x_2, X_3 = x_3 \ldots\ldots\ldots X_n = x_n$, with distribution $f(x/\mu)$.The likelihood function of the sample that can be used for the estimation of parameter μ is given by:

$$Lik(\mu) = f(x_1/\mu).f(x_2/\mu).f(x_3/\mu)\ldots\ldots\ldots\ldots f(x_n/\mu)$$
$$= \prod_{i=1}^{n} f(x_i/\mu)$$

Where$f(x_1/\mu).f(x_2/\mu).f(x_3/\mu)\ldots\ldots\ldots\ldots f(x_n/\mu)$is a joint density if$X_{i's}$ are continuous
The likelihood function of the Log-normal distribution that sample data follows was determined below:

$$lik(\mu) = \frac{1}{\sigma_{2\sqrt{2\pi}}}\frac{1}{x_1}exp^{-\frac{1}{2}\left(\frac{logx_1-\mu}{\sigma_2}\right)^2} \times \ldots \times \frac{1}{\sigma_{2\sqrt{2\pi}}}\frac{1}{x_n}exp^{-\frac{1}{2}\left(\frac{logx_n-\mu}{\sigma_2}\right)^2}$$
$$\left(\frac{1}{\sigma_{2\sqrt{2\pi}}}\right)^n \frac{1}{\prod_1^n x_i}exp^{-\frac{1}{2}\sum_{i=1}^{i=n}\left\{\left(\frac{logx_i-\mu}{\sigma_2}\right)^2\right\}}$$

But $\left(\frac{1}{\sigma_{2\sqrt{2\pi}}}\right)^n and \frac{1}{\prod_1^n x_i}$ are constants because they do not contain the estimated parameter, therefore the likelihood function is;

$$lik(\mu) \propto exp^{-\frac{1}{2}\sum_{i=1}^{i=n}\left\{\left(\frac{logx_i-\mu}{\sigma_2}\right)^2\right\}}$$

4.9. Determining the Posterior Distribution

The posterior distribution, which is the current probability or the probability distribution function of aggregate monthly claims at Dormaa Municipal Health Insurance Scheme. The posterior distribution was determined by multiplying the likelihood function of the sample distribution by the prior distribution of the population mean, this is denoted as;

$$f(\mu/x) \propto \prod_{i=1}^{n} f(x_i/\mu) \times f(\mu)$$
$$f(\mu/x) \propto exp^{-\frac{1}{2}\sum_{i=1}^{i=n}\left\{\left(\frac{logx_i-\mu}{\sigma_2}\right)^2\right\}} \times exp^{-\frac{1}{2}\left(\frac{\mu-\theta}{\sigma_1}\right)^2}$$
$$f(\mu/x) \propto exp^{-\frac{1}{2}\left\{\sum_{i=1}^{i=n}\left\{\left(\frac{logx_i-\mu}{\sigma_2}\right)^2\right\}+\left(\frac{\mu-\theta}{\sigma_1}\right)^2\right\}}$$

Expanding the expression in the curly bracket in equation 4.7 gives;

$$\sum_{i=1}^{i=n}\left\{\left(\frac{logx_i-\mu}{\sigma_2}\right)^2\right\} + \left(\frac{\mu-\theta}{\sigma_1}\right)^2$$

$$= \frac{\sum_1^n(logx_i)^2 - 2\mu\sum_1^n logx_i + n\mu^2}{\sigma_1^2} + \frac{\mu^2 - 2\mu\theta + \theta^2}{\sigma_2^2}$$

Simplifying the right hand side further and grouping the parameter to be estimated

$$= \frac{\mu^2(n\sigma_2^2 + \sigma_1^2)}{(\sigma_1\sigma_2)^2} - 2\mu\left(\frac{\sigma_2^2\sum_1^n logx_i + \sigma_1^2\theta}{(\sigma_1\sigma_2)^2}\right) + \frac{\sigma_2^2\sum_1^n(logx_i)^2 + (\sigma_1\theta)^2}{(\sigma_1\sigma_2)^2}$$

The third term of the expression does not contain the estimated parameter μ, hence it is considered constant. Putting back the remaining terms in to equation 4.7 gives the posterior distribution of the form;

$$f(\mu/x) \propto exp^{-\frac{1}{2}\left\{\frac{\mu^2(n\sigma_2^2+\sigma_1^2)}{(\sigma_1\sigma_2)^2} - 2\mu\left(\frac{\sigma_2^2\sum_{i=1}^{i=n}logx_i+\sigma_1^2\theta}{(\sigma_1\sigma_2)^2}\right)\right\}}$$

The posterior distribution in the above equation is in the form of Normal/Gaussian distribution.

4.10. Determining Mean and Variance of Posterior Distribution

Let μ_* and σ_*^2 be the mean and variance of the posterior distribution respectively. It can then be express in the form

$$f(\mu/x) = exp^{-\frac{1}{2}\left(\frac{\mu-\mu_*}{\sigma_*}\right)^2}$$

Expanding the exponential part of equation 4.9 gives$\frac{\mu^2-2\mu\mu_*+\mu_*^2}{\sigma_*^2}$, comparing the expression to the exponential part of equation 4.8 gives the following equations

$$\sigma_*^2 = \frac{(\sigma_1\sigma_2)^2}{n\sigma_2^2 + \sigma_1^2} and \mu_* = \frac{\sigma_2^2\sum_1^n logx_i + \sigma_1^2\theta}{(\sigma_1\sigma_2)^2} \times \sigma_*^2$$

Therefore equation 4.11 can be evaluated from equation 4.10 by simple substitution

$$\mu_* = \frac{\sigma_2^2 \sum_1^n logx_i + \sigma_1^2\theta}{(\sigma_1\sigma_2)^2} \times \frac{(\sigma_1\sigma_2)^2}{n\sigma_2^2 + \sigma_1^2}$$

$$\mu_* = \frac{\sigma_2^2 \sum_1^n logx_i + \sigma_1^2\theta}{n\sigma_2^2 + \sigma_1^2}$$

5. Discussion

The posterior distribution of the Dormaa Municipal Health Insurance scheme aggregate monthly claims is a normal distribution. The mean of the posterior distribution contains the sum of the natural log of the sample data, the variance of the prior distribution, the variance of the sample data and the mean of the prior distribution.

After estimating each distribution probability in table 5 and their expected claims in table 6, it was found that Log-normal distribution is the best fit for the sample data, based on its minimum chi-square value of 1.111. Gamma and weibull distributions were the second and third with chi-square values of 1.333 and 4.640 respectively. The exponential distribution was the fourth with a chi-square value of 171.655, which indicates non good fit. Therefore it is found that the likelihood function for determining the posterior distribution is Log-normal distribution.

It also found that the posterior distribution of Dormaa Municipal Health Insurance Scheme is a normal distribution with mean $\frac{\sigma_2^2 \sum_1^n logx_i + \sigma_1^2\theta}{n\sigma_2^2 + \sigma_1^2}$ and variance $\frac{(\sigma_1\sigma_2)^2}{n\sigma_2^2 + \sigma_1^2}$. The posterior distribution was determined after multiplying the prior distribution which is normal distribution by the likelihood function which is Log-normal distribution.

6. Conclusion

The main objective of this article was to determine the posterior distribution, using Bayesian approach to credibility theory to know the actual posterior distribution of national Health Insurance Scheme. The study adopted a case study approach. The research was carried out in the Dormaa Municipality. The study analysed 84 aggregate monthly claims as a population submitted by all 28 health facilities under the National Health Insurance Scheme in the municipality and 36 aggregate monthly claims as a sample from Dormaa Presbyterian Hospital. Dormaa Presbyterian Hospital was used as a sample because their claims do not pass through the Municipal Health Insurance Scheme.

The study found out that the posterior distribution which is the current distribution of the municipal health insurance scheme is normal distribution.

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