INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

A Statistical Method for the Prediction of the Bear and Bull Stock Markets Based on the Zeroes of Riemann's Zeta Function

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Abstract:
We define a method for predicting the stochastic behavior of the Bull and Bear periods of the stock market. In this paper, initially, we carry on a comprehensive evaluation of more frequently used statistical methods for evaluating Stock markets. Our work is based on collecting 40 years of data from the Italian stock market. The proposed solution is defined using the statistical analysis of the Bear and Bull Stock markets. We defined a new system to predict the trend of a stock market price, where the trend of the succession of Bull and Bear markets can be described by a probability density function given by a Gaussian distribution. Furthermore, we consider the inverses of the relative time intervals as a measure of the speed with which the phenomenon of the Bear market (or, equivalently, the Bull market) develops in that interval of time. Therefore, this factor can ultimately represent the first statistical weight of the single percentage variation. Again, the time intervals of the individual Bear and Bull market periods are considered, calculated from 01/01/1973. This allows us to consider the hypothesis that a secondary factor of probability is the temporal distance of the event that has already occurred. This work includes a criterion for statistically generating the most probable values of the next Bear and Bull markets and the length of the time intervals corresponding to these market situations. This criterion is based on the following hypothesis:
To obtain the distribution of the predictive points of max and min in the succession of Bull and Bear markets, it is assumed that, in the long period, the random distribution of the successive max and min takes the trend of the distribution of the distance fluctuations between the zeroes of the Riemann's function which, in turn, is approximated by a Unitary Gaussian Distribution (GUE). Our results show that:

- *The linear interpolation of the Variations of the market (positive and negative) relative to different and successive sampling sets for future trends do not show high percentage variations between them,*
- *Above all, the lengths of the single time intervals of the future variations, relative to different and successive sampling sets, are quite similar to each other. Hence, the method appears to be basically stable and promising.*

Keywords: *Gaussian unitary ensemble, linear interpolation, Riemann's zeta function, stock market*

1. Introduction

The stock market is the hub in which people can buy and sell shares systematically. It has an important contribution to the rapidly growing world economy. The fluctuation in the stock market can have a significant influence on people and the entire economy. The Stock market is one of the best alternatives for many business firms for further expansion. Generally, it is well-known that financial markets across all asset classes exhibit trends. These trends have been exploited very successfully by the trading industry over the past decades. Nevertheless, the main objective of investors should be to buy a stock to have capital appreciation.

Various reasons, such as politic-economic situation, natural disasters, poor-corporate governance, and differing policy of the governing company, influence the overall trend of stock markets. In this regard, the return on the investment made by people and corporates in the stock market relies on the decision to purchase stocks. The decision to choose the most beneficial options in the stock market relies on how well people are informed in the stock analysis. This is why it is important to identify the statistical models and their analysis (Sen & Chaudhuri, [1]). These models assist in forecasting the share price movement of stocks. There are various statistical models to study the phenomena of stock behavior. For example, the Brownian motion model predicts the stock market using past information. Initially, models with incomplete information were investigated by Dothan and Feldman [2] using dynamic programming methods in linear Gaussian filtering. Many models are used to predict stock executions, such as "Exponential Moving Average" (EMA) and the "head and shoulders" methods. However, many of the forecast models require a stationary input time series. In reality, financial time series are often non-stationary, and hence, non-stationary time series models are needed. Auto-regression models have been modified by adding time-dependent variables to adapt to the non-stationarity of time series. Some recent papers using these models are [3]-[4]. Otherwise, in the paper [5], the authors developed an interesting method based on a

Neural Network Model. Again, in the paper [6], the authors developed a different method based on the forecast of stock volatility, building a prediction model for the stock volatility price.

An important and interesting line of analysis in the Statistical Models of Financial Analysis relates to the analysis of the Brownian motion in the context of the performance of stock indexes. The interesting studies [7] and [8] belong to this area. Instead, Bahrens et al., in the paper [9], use the Generalized Pareto Distributions (GPD), considering the uncertainty about the threshold explicitly. They introduce a mixture model that combines a parametric form for the center and a GPD for the tail of the distributions and uses all observations for inference about the unknown parameters. Elbahloul [10] applies some statistical analysis methods, such as Exponential smoothing and Mean Squared Error (MSE), to analyze time-series data to make some determinations. Wen et al. [11] developed a method based on the co-integration theory, building Full Graphs (FGs) and Minimum Spanning Trees (MSTs) in terms of the co-integration matrix by using daily stock prices from the Chinese stock market. Again, Shen et al. [12], in their paper, defined a method for Short-term stock market price trend prediction using a comprehensive deep learning system obtained by collecting two years of data from the Chinese stock market.

A different approach to the statistical study of stock markets takes place through the Hidden Markov Model (HMM) method. It can be considered a tool to deal with time series problems. It is not affected by whether the data is linear or not while analyzing market conditions and the transition law between these states. Some very recent works regarding the Hidden Markov Model are [13]-[14]-[15]-[16]. They first divide the market situation into three types: bull market, mixed market, and bear market, and establish a hidden Markov model of state estimation to solve the problem of market situation estimation. Then they propose a Markov estimation trading strategy.

In recent years some authors have tried to introduce a quantum theory for financial markets. Some of the most recent papers that can be cited in this regard are [17]-[18]-[19]-[20]. The stocks have always been traded at certain prices, which, from the point of view of quantum mechanics, present corpuscular property. Meanwhile, the stock prices fluctuate in the market, representing the wave property. Due to this wave-particle dualism, they suppose the micro-scale stock is a quantum system. Unlike the other works, the one by Durmagambetov [17] actually presents an analysis of various mathematical models for stock markets, in addition to the one related to quantum mechanics. In particular, in this work, the possibility of deriving symmetry is also considered between different assets in phase with other assets and vice-versa (change in anti-phase). As a consequence, this work is formally considered a change in trends as a change in symmetries and as a change in conservation laws from the point of view of Noether's theorems. Particularly, this author studied the functional Riemann equations for the Zeta Function, and he solved the problem of constructing solutions of equations with mirror symmetry for the stock market.

In our new work, we will start as a basis from our recent paper [21]. We believe that a fundamental point in the study of stock markets is given by the ability to predict when either a Bear market or a bull market will occur. In other words, we will define market phases characterized either by a progressive decrease in the prices of financial assets and by pessimistic expectations or by a more or less rapid increase in prices. Being able to establish in advance, with a good approximation, the beginning of a Bear market period (or Bull market period) is clearly the fundamental factor for selling (buying) the stocks in advance. We started with the statistical method described in the previous work on this argument [21] and partially derived from the method defined in two previous works ([22] and [23]) by the author. We consider a financial market that will define the successive max and min points of the statistical curve of the market analyzed (FTSE MIB Italia), corresponding to price variations of more than 20% (generally accepted definition for Bear and Bull markets). That is, to be defined as Bear or Bull markets, depending on whether prices are falling or rising. We assume that the unobservable and future processes are modeled by a stochastic process. Precisely, we define a Gaussian trend of the probability distribution to obtain the values of the predictor variables. Based on this method, a criterion is established for statistically generating the most probable values of the next Bear and Bull markets and, above all, the lengths of the time intervals corresponding to these market situations. Nevertheless, herein lies the fundamental novelty of this work, in order to obtain the distribution of future max and min points in the sequence of Bull and Bear markets, we make the fundamental assumption that, in the long period, the random distribution of successive max and min has the behavior of the Distribution of the distance fluctuations between the zeroes of Riemann function, which, in turn, is approximated by a Unitary Gaussian Distribution (GUE). Based on this assumption, we predictively calculate the distribution of future max and min points, i.e., the individual percentage changes and the corresponding time intervals.

The paper is organized as follows:

- In Section 2, we describe the Statistical method used.
- In Section 3, we describe the Zeroes of Riemann's Zeta Function and the GUE's Random Matrices.
- In Section 4, the application of Riemann's function Zeroes to the statistical method for generating the trend of the Bull and Bear markets is defined.
- In Section 5, some results and relative discussion are developed.
- In the end, in Section 6, the concluding remarks are defined.

2. Description of the Statistical Method

In this section, we will refer particularly to the previous work of the author [21]. Our goal is to describe the statistical method for obtaining the values of the predictive variables relating to the peaks corresponding to the Bear market and the Bull market and the time intervals in which these transitions take place. A Bull market can be defined as a period in financial markets when the price of an asset or security rises. The commonly accepted definition of a Bull market is when stock prices rise by 20% or more. On the contrary, Bear markets occur when stock prices fall 20% or more for a sustained period. Bull markets are generally powered by economic strength, whereas bear markets often occur during the economic slowdown and higher unemployment.

The statistical method for predicting Bull and Bear markets is based on the following steps:

- We generated the data of the Bull and Bear markets using the historical Global Comit Index for the Italy Stock Market, which has been available since the beginning of 1973, thus resulting in the longest index available on the Italian stock market. The fundamental inferred data for the Bull and Bear markets have been included in table 1, shown below.
- We hypothesize that the trend of the succession of bull and bear markets can be described by a probability density function given by a Gaussian distribution defined by eq. (1).

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2} \right) \qquad (1)$$

where μ is the expected value, and σ is the variance of the Gaussian function.

Pos.	Peak Date (mm/dd/yyyy)	Trough Date (mm/dd/yyyy)	Initial Price	Final Price	Percent. Variation (ΔV_i)	Number of working Days (Δt_i)
1	01/02/1973	06/19/1973	111.35	162.21	45,6	121
2	06/19/1973	08/07/1973	162.21	117.51	-27,5	14
3	08/07/1973	04/18/1974	117.51	154.25	31,2	183
4	04/18/1974	12/20/1974	154.25	85.97	-44,2	177
5	12/20/1974	03/11/1975	85.97	107.63	25,2	58
6	03/11/1975	10/17/1975	107.63	75.41	-29,8	159
7	10/17/1975	07/20/1976	75.41	88.12	16,8	198
8	07/20/1976	11/10/1976	88.12	64.93	-26,3	82
9	11/10/1976	12/07/1976	64.93	75.76	16,6	20
10	12/07/1976	06/16/1977	75.76	58.55	-22,5	138
11	06/16/1977	09/21/1978	58.55	84.03	43,5	331
12	09/21/1978	12/19/1978	84.03	67.50	-19,6	64
13	12/19/1978	06/03/1981	67.50	292.03	332,6	642
14	06/03/1981	10/20/1981	292.03	172.22	-41	100
15	10/20/1981	03/19/1982	172.22	212.66	23,4	109
16	03/19/1982	07/22/1982	212.66	147.23	-30,7	90
17	07/22/1982	05/20/1986	147.23	908.19	516,8	999
18	05/20/1986	06/20/1986	908.19	653.83	-28	24
19	06/20/1986	06/16/1987	653.83	719.03	9,9	258
20	06/16/1987	02/10/1988	719.03	427.51	-40,5	172
21	02/10/1988	06/14/1990	427.51	763.53	78,6	612
22	06/14/1990	01/29/1991	763.53	486.25	-36,3	164
23	01/29/1991	06/20/1991	486.25	612.32	25,9	103
24	06/20/1991	12/10/1991	612.32	482.89	-21,1	124
25	12/10/1991	02/25/1992	482.89	545.03	12,8	56
26	02/25/1992	09/07/1992	545.03	361.52	-33,6	140
27	09/07/1992	05/11/1994	361.52	817.17	126	438
28	05/11/1994	12/13/1994	817.17	581.64	-28,8	155
29	12/13/1994	07/20/1998	581.64	1623.52	179,1	940
30	07/20/1998	10/09/1998	1623.52	1063.50	-34,5	60
31	10/09/1998	11/15/2000	1063.50	2095.95	97	549
32	11/15/2000	09/21/2001	2095.95	1082.91	-48,3	223
33	09/21/2001	04/17/2002	1082.91	1513.03	39,7	149
34	04/17/2002	07/24/2002	1513.03	1077.76	-28,7	71
35	07/24/2002	08/27/2002	1077.76	1226.88	13,8	25
36	08/27/2002	10/09/2002	1226.88	974.37	-20,5	11
37	10/09/2002	05/18/2007	974.37	2149.12	120,5	1203
38	05/18/2007	03/09/2009	2149.12	655.07	-69,5	472
39	03/09/2009	05/02/2011	655.07	1153.75	76,1	561
40	05/02/2011	09/12/2011	1153.75	744.54	-35,4	96
41	09/12/2011	12/01/2015	744.54	1273.81	71	1102
42	12/01/2015	06/27/2016	1273.81	916.83	-28	150
43	06/27/2016	01/29/2018	916.83	1394.58	52,1	416
44	01/29/2018	12/27/2018	1394.58	1073.50	-23	239
45	12/27/2018	02/19/2020	1073.50	1479.33	37,8	300
46	02/19/2020	03/23/2020	1479.33	917.29	-38	24
47	03/23/2020	01/05/2022	917.29	1654.74	80,3	468
48	01/05/2022	09/29/2022	1654.74	1200.69	-27,4	192

Table 1: Global Comit Index - Bear and Bull Markets and Corresponding Corrections Since 1973
**Variations with a Negative Sign Correspond to Bear Markets, with a Positive Sign to Bull Markets*

- We calculated the time distance t_i from the origin of the times by adding the working time intervals Δt_i of the various periods reported in table 1, which is set on 01/01/1973.
- The inverses $(1/\Delta t_i)$ of the working time intervals Δt_i are also calculated. These inverses can be considered a measure of the speed (and, therefore, the strength) with which the phenomenon of the Bear market (or, equivalently, the Bull market) develops in that interval of time. Therefore, in deep analysis, it can represent the first statistical weight of the single percentage variation (ΔV_i) in that time interval with respect to the entire Gaussian probability distribution.
- After that, we calculated the normalized values ϕ_i of the primary weights described above. They are defined as follows:

$$\phi_i \equiv \left(1/\Delta t_i\right)_{norm} \triangleq \left(1/\Delta t_i\right) / \sum_i \left(1/\Delta t_i\right) \tag{2}$$

- Now, we consider the time intervals of the individual Bear and Bull market periods, calculated from the origin of the time axis, i.e., from 01/01/1973. In this regard, we make a second hypothesis: that a secondary factor of probability is the temporal distance of the event that has already occurred. In other words, the more distant the phenomenon, the more likely it will happen. Therefore, with this assumption, we make the chain of events non-Markovian. However, to avoid making this secondary factor too important, we consider it a logarithmic power, thus smoothing out the difference between the extreme values. In this way, starting from the time distances t_i from the origin of the times of the single events, we define the normalized coefficients Υ_i of the secondary weights as:

$$\gamma_i \equiv \left[1/(\ln t_i)^n\right]_{norm} \triangleq \left[1/(\ln t_i)^n\right] / \sum_i \left[1/(\ln t_i)^n\right] \tag{3}$$

- As a consequence of the previous considerations, the overall normalized coefficients $[\phi_i \Upsilon_i]$ are defined, for each time interval Δt_i, in the following way:

$$\left[\phi_i \gamma_i\right]_{norm} \triangleq \phi_i\, \gamma_i / \sum_i \phi_i \gamma_i \tag{4}$$

- Based on overall normalized coefficients $[\phi_i \Upsilon_i]$, it is possible to define the probability density given by a Gaussian distribution. For this purpose, being ΔV_i the single percentage variation in the time interval Δt_i, we have to determine based on (1) the expected value μ and the variance σ of the Gaussian function:

$$\mu \triangleq \frac{\sum_i \left[\phi_i \gamma_i\right]_{norm} \Delta V_i}{n} \tag{5}$$

$$\sigma^2 \triangleq \frac{\sum_i \left\{\left[\left(\phi_i \gamma_i\right)_{norm} \left(\Delta V_i\right)\right] - \mu\right\}^2}{n} \tag{6}$$

- We define two Gaussian distributions, relating respectively to the Bull and Bear markets, i.e., in practice relating to the positive percentage changes $(\Delta V_i)_p$ and negative percentage changes $(\Delta V_i)_n$ in subsequent time periods. So we will have, relatively to the expected values μ and to the variances σ of the two Gaussian functions:

$$\mu_p \triangleq \frac{\sum_{i,p} \left\{\left[\phi_i \gamma_i\right]_{norm,p} \Delta V_{i,p}\right\}}{n_p} \tag{7}$$

$$\sigma_p^2 \triangleq \frac{\sum_{i,p} \left\{\left[\left(\phi_i \gamma_i\right)_{norm,p} \left(\Delta V_{i,p}\right)\right] - \mu_p\right\}^2}{n_p} \tag{8}$$

$$\mu_n \triangleq \frac{\sum_{i,n} \left\{\left[\phi_i \gamma_i\right]_{norm,n} \Delta V_{i,n}\right\}}{n_n} \tag{9}$$

$$\sigma_n^2 \triangleq \frac{\sum_{i,n} \left\{\left[\left(\phi_i \gamma_i\right)_{norm,n} \left(\Delta V_{i,n}\right)\right] - \mu_n\right\}^2}{n_n} \tag{10}$$

- The two Gaussian distributions, relating to the Positive Variations (Bull market) and the Negative Variations (Bear market), will therefore be given, respectively, based on (1) by:

$$g_p(x_{i,p}) = \frac{1}{\sigma_p \sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(x_{i,p} - \mu_p)^2}{\sigma_p^2}\right) \tag{11}$$

$$g_n(x_{i,n}) = \frac{1}{\sigma_n\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(x_{i,n}-\mu_n)^2}{\sigma_n^2}\right) \quad (12)$$

being:

$$x_{i,p} \triangleq \left(\phi_i\gamma_i\right)_{norm,p}\left(\Delta V_{i,p}\right) \quad (13)$$

$$x_{i,n} \triangleq \left(\phi_i\gamma_i\right)_{norm,n}\left(\Delta V_{i,n}\right) \quad (14)$$

3. Zeroes of Riemann's Zeta Function and GUE's Random Matrices

In this section, we will base ourselves on the work of Berry [24]. The well-celebrated Hypothesis of Riemann can be defined by saying that all the complex zeros of his function (z) have real part ½. In this way, the quantities $\{E_j\}$ are defined by:

$$\zeta\left(\frac{1}{2}-iE_i\right)=0 \quad (15)$$

are all real. That is supported by a fact: the first few millions $\{E_j\}$ have been calculated and are all-real.

As well-known, (z) can be defined by the following expressions:

$$\zeta(z)=\prod_p\left(1-p^{-z}\right)^{-1} \qquad (\text{Re } z > 1) \quad (16a)$$

$$\zeta(z)=\sum_{n=1}^{\infty}n^{-z} \qquad (\text{Re } z > 1) \quad (16b)$$

Nevertheless, neither representation converges on the line Re(z)=1/2, where the zeroes are based on Riemann's hypothesis and the experiment. Many representations are valid for Re(z)=1/2, such as the following, derived by eq. (16b):

$$\zeta(z)=\frac{1}{1-2^{1-z}}\sum_{n=1}^{\infty}(-1)^{n+1}n^{-z} \qquad (\text{Re } z > 0) \quad (17)$$

The symmetry of the eq. (15) relating (z) to (1-z) involves that each of the zeroes (15) with real E_j has a counterpart with $-E_j$.

An optimal separation of the $\{E_j\}$ into an average part and a fluctuating part can be obtained by the Riemann staircase. This is defined as:

$$N_R(E)\equiv\sum_{j=1}^{\infty}\Theta\left(E-E_j\right) \quad (18)$$

where: Θ represents the unit step function. In fact, $N_R(E)$ is the number of zeroes with $E_j < E$. The average $\langle N_R(E)\rangle$ is a smooth approximation of the staircase function, whose form is given by:

$$\langle N_R(E)\rangle=\frac{E}{2\pi}\left(\ln\left\{\frac{E}{2\pi}\right\}-1\right)+\frac{7}{8} \quad (19)$$

The deviations from the average $\langle N_R(E)\rangle$ represent the fluctuations in $\{E_j\}$. The statistics of these fluctuations can be analyzed numerically. The researchers found that, with high accuracy, these fluctuations have the statistics of the eigenvalues of a typical Gaussian Unitary Ensemble (GUE) relative to complex Hermitian matrices. The elements of these matrices are distributed according to a Gaussian function in an invariant way under unitary transformations.

One such statistic is the probability distribution of the normalized spacing $\{S_j\}$ between adjacent zeroes. These are defined as:

$$S_j=\left(E_{j+1}-E_j\right)\Big/\left\langle d_R\left[\left(E_j+E_{j+1}\right)/2\right]\right\rangle \quad (20)$$

where $\langle d_R(E)\rangle$ is the average density of zeroes, obtained by eq.(19) as:

$$\langle d_R(E)\rangle=\frac{d}{dE}\langle N_R(E)\rangle=\frac{1}{2\pi}\ln\left\{\frac{E}{2\pi}\right\} \quad (21)$$

The spacing Gaussian distribution $P_{GUE}(S)$ is given using the random-matrix theory [25] with the following approximation:

$$P_{GUE}(S)=\frac{32}{\pi^2}S^2\exp\left\{-4S^2/\pi\right\} \quad (22)$$

In figure 1, we represented the Gaussian distribution $P_{GUE}(S)$ that approximates the Spacing distribution P(S) relative to the first 10100 Zeroes of the Riemann's Function, obtained by the table of A. Odlyzko [26] relative to the zeroes of the Riemann Zeta Function.

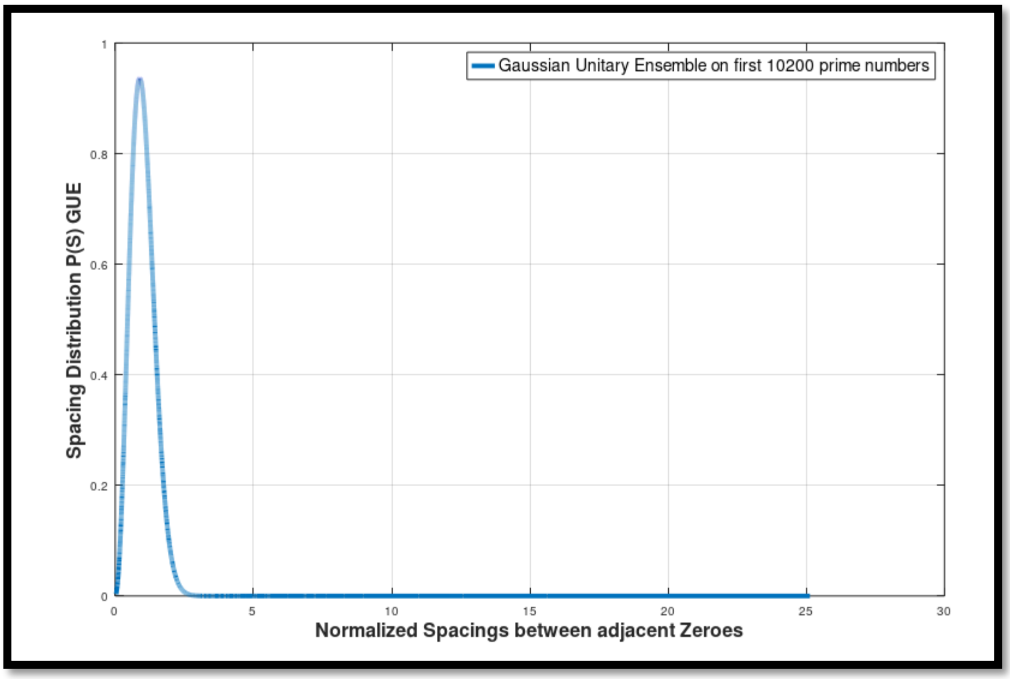

Figure 1: Spacing Distribution P(S) Approximation: Gaussian Unitary Ensemble $P_{GUE}(S)$.

4. Application of Riemann's Function Zeroes to a Statistical Method for Generating the Trend of the Bull and Bear Markets

In order to determine the trend of the Bull and Bear markets through statistical analysis, the case study related to the Global Comit Index for the Italy Stock Market, which has been available since the beginning of 1973, is used. From the data of this Index, we obtained those defined in table 1, i.e., the succession of Bear and Bull Markets, the corresponding positive and negative Corrections since 1973, and the corresponding working time intervals.

Using the method and process described in detail in Section 2 for this data, we generate two Gaussian distributions, relating respectively to the Bull and Bear markets, i.e., in practice relating to the positive percentage changes $(\Delta V_i)_p$ and negative percentage changes $(\Delta V_i)_n$ in subsequent time periods. At this point, we implement the following procedure:

- The method used is based on the following assertion. Since both the distribution of the distance fluctuations between the Zeros of the Riemann function and the trend of the sequences of the Bull and Bear markets can be approximated by Gaussian probability Distributions, then we hypothesize that the trend of the future Bull and Bear markets can be predicted pseudo-randomly, generating a succession of points on the Gaussian curve PGUE(S) relating to the Distribution of the distance fluctuations between the Riemann Zeroes. It is evident that, by increasing the number of points and increasing the sequence of tests carried out, the results will tend to be, on average, best fitted to the future trends. This hypothesis is based on the assumption defined by us, in a more general way, that as the number of points considered tends to infinity, each random probability distribution should be generated by a PGUE(S), approximating the Distribution of the fluctuations of the distances between the zeroes of the Riemann function.
- Now, we practically define the procedure for obtaining the predictive values of the variables considered. With this aim, first of all, we define a pseudo-random number generator using an algorithm based on Linear Congruential Generators (LCG).
- Using the LCG algorithm, we generate four pseudo-random positions on the Gaussian distribution PGUE(S). These pseudo-random positions are relative to future trends for Bull markets. In the same way, after that, we generate four other pseudo-random positions on the PGUE(S) for negative market changes (Bear markets). Obviously, the procedure can be repeated with a greater number of generated positions.
- At each generated pseudo-random position, we compute the corresponding percentile on the Gaussian PGUE(S).
- Subsequently, we determine the position of the next future Positive (Bull markets) and negative (Bear markets) values on the corresponding Gaussian Distributions given by eq. (11) and from eq. (12), calculating on these distributions the percentiles corresponding to those of the individual pseudo-random positions generated on the PGUE(S), as described in step 3. This allows us to determine both the amount of time necessary to reach the single point of the Bull (Bear) market and the value of the corresponding percentage change.
- To obtain the predictive value of Δti the following expression is used:

$$\Delta t_i \triangleq \left(\Delta t_{psrand} + \Delta t_m\right) / 2 \tag{23}$$

Where: Δt_{psrand} is the Δt_i of the determinate element in table 1 by the pseudo-random algorithm LCG, and Δt_m is the determinate medium Δt, respectively, for the Bull and Bear market intervals. To calculate the Peak values of the Bull and Bear markets, respectively peak values of Max and Min, we will use the following expression:

$$\Delta V_i \triangleq \left[\left(\phi_i \gamma_i \right)_{norm} \left(\Delta V_{psrand} \right) \right] - \sigma_m \tag{24}$$

Where: ΔV_{psrand} is the ΔV_i of the element in table 1 determined by the LCG pseudo-random algorithm, and σ_i is the standard deviation obtained by (8) and (10), respectively, for the Bull and Bear markets.

- At this point, we generate a linear interpolation of the sequence of points relating to the max and min over the time interval taken into consideration, i.e., the sequence of Bull and Bear markets, also inserting the max and min points, generated with the method described above, relatively to future trends.

5. Results and Discussion

In order to describe the method defined in the previous Sections, we have generated some explanatory graphics using GNU Octave version 5.1.0.

- Figure 2 and figure 3 give the Random Positions of the points obtained with the pseudo-random procedure in Section 4. In Fig. 2a, four points are represented relative to the Bull markets, that is to the positive variations of the stock market, while in Fig. 2b, four points are represented relative to the Bear markets, that is, to the negative variations of the stock market. These points are defined on the profile of the Gaussian distribution $P_{GUE}(S)$, which approximates the Distribution of the distance fluctuations between the zeroes of the Riemann function.
- Figure 3 gives the linear interpolation of the subsequent Bull and Bear markets, both for the past time and for the forecast markets. In particular, the predictive trend of the next Bull and Bear markets, obtained as described in Sections 2-3-4, calculating the four points of max (Bull markets) and the four points of min (Bear markets), is highlighted with a red line.
- Finally, in figure 4, we represent the linear interpolation of the percentage Variation of the market (positive and negative) in a shorter time interval than that of figure 3. Specifically, in figure 4a and figure 4b, we describe two different linear interpolations for the future trend obtained by different series of pseudo-random positions. What is important to highlight by looking at these two figures is that there is no big difference in the trends of the red lines, i.e., in the interpolating lines relating to future trends. Of course, there are different percentage variations, but these differences are not particularly large.
- Above all, the lengths of the individual time intervals of future changes are quite similar.

A further refinement of the method could be obtained by averaging several successive extractions of the pseudo-random sequences.

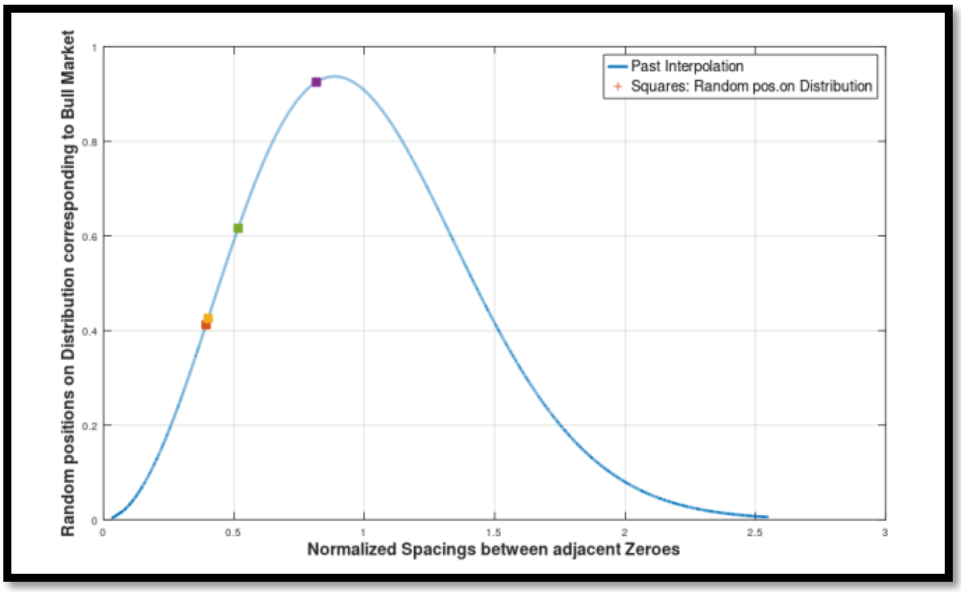

Figure 2: Random Positions Distribution for Bull Markets on the Gaussian Distribution Regarding the Normalized Spacing between Adjacent Zeroes

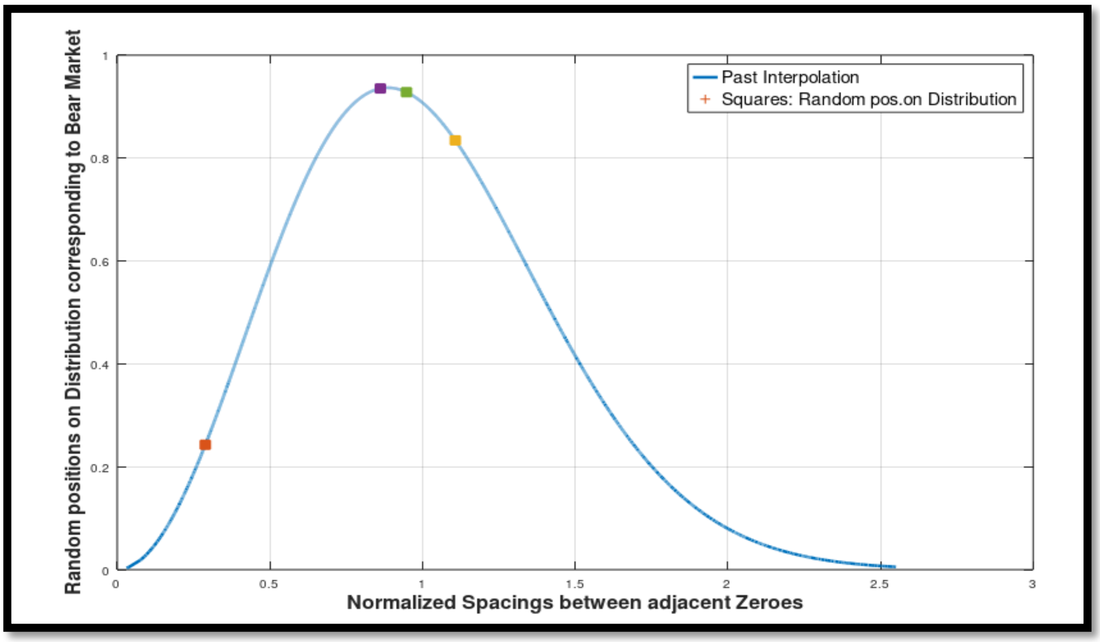

Figure 3: Random Positions Distribution for Bear Markets on the Gaussian Distribution Regarding the Normalized Spacing Between Adjacent Zeroes

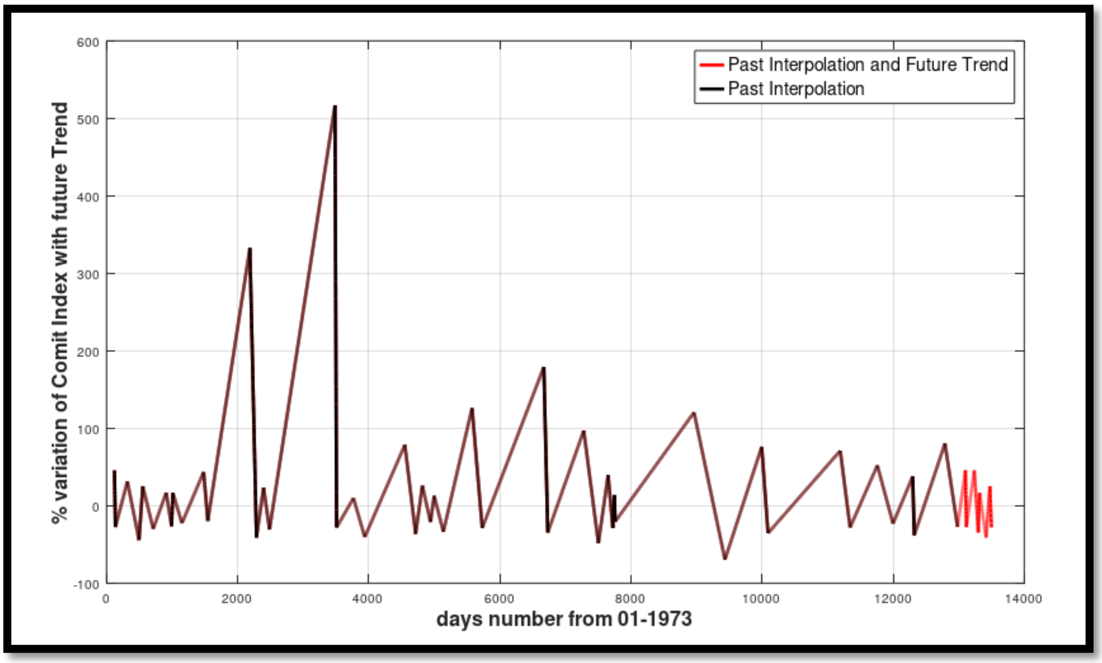

Figure 4: Linear Interpolation of the % Variations of the Comit Index and Future Trend (Red Line)

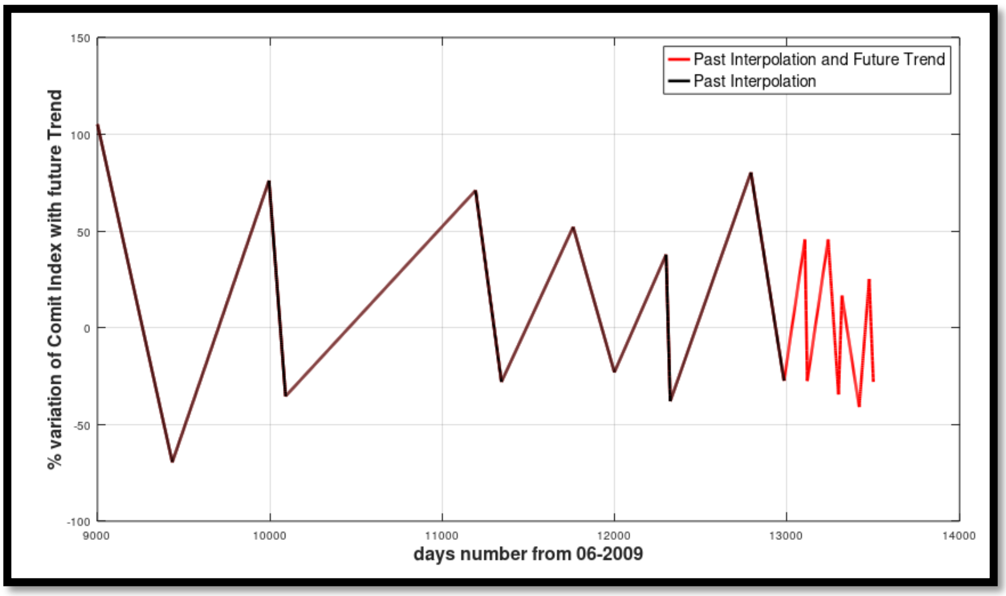

Figure 5: Linear Interpolation of the % Variations of the Index and Future Trend (Red Line) Limited to the Last Years, Obtained by a First Set of Pseudo-Random Positions

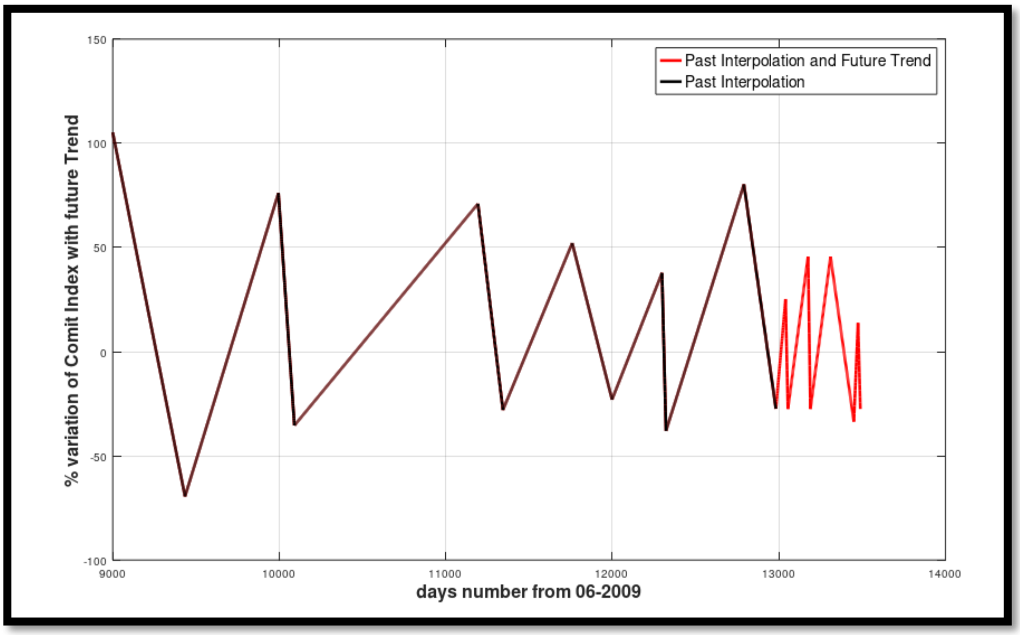

Figure 6: Linear Interpolation of the % Variations of the Index and Future Trend (Red Line) Limited to the Last Years, Obtained by a Second Set of Pseudo-Random Positions

6. Conclusion

In this paper, we investigated the Bear and Bull Stock markets and proposed a statistical method to generate the most probable variations of the next future Bear and Bull markets. Furthermore, with our statistical method, we generated predictive values of the lengths of the time intervals corresponding to these market situations. We relied on 40 years of data from the Italian stock market. The fundamental criterion applied in this work is based on the following hypothesis: in order to obtain the distribution of future max and min points in the succession of Bull and Bear markets in the long period, the random distribution of successive max and min points must have the distribution of the distance fluctuations between the zeroes of the Riemann function which, in turn, is approximated by a Unitary Gaussian Distribution (GUE). An implementation of the method and the most relevant results are described.

7. References

i. Sen, J., & Chaudhuri, T. D., "A Framework for Predictive Analysis of Stock Market Indices: A Study of the Indian Auto Sector," 2016, *arXiv preprint,* Available: arXiv:1604.04044.

ii. Dothan M.U., & Feldman D., "Equilibrium Interest Rates and Multiperiod Bonds in a Partially Observable Economy," *The Journal of Finance*, 1986, vol. 41, pp. 369–382. http://dx.doi.org/10.2307/2328441.

iii. Geweke J., & Terui N.. "Bayesian Threshold Autoregressive Models for Non-linear Time Series," *Journal of Time Series Analysis,* 1993, vol. 14, pp. 441–454.

iv. Schmidhuber C., “Trends, Reversion, and Critical Phenomena in Financial Markets”, 2020, *ArXiv*: 2006:07847v32 [q-fin.ST]. Preprint.

v. Liu S., Zhang C. & Ma J., “CNN-LSTM Neural Network Model for Quantitative Strategy Analysis in Stock Markets," *Proceedings of International Conference On Neural Information Processing (ICONIP)* 2017, Part II, LNCS 10635, pp. 198–206. Available: DOI:10.1007/978-3-319-70096-0_21.

vi. Zhang X. & Zhang L., “Forecasting Method of Stock Market Volatility Based on Multidimensional Data Fusion," *Hindawi*, 2022, Article ID 6344064, Available: https://doi.org/10.1155/2022/6344064.

vii. Rajpal R., "Mathematical Modeling in Finance," *International Journal of Scientific & Engineering Research*, 2018, vol. 9, Issue 11, pp. 1202–1222.

viii. Ibrahim D. & Abergel F., “Non-linear filtering and optimal investment under partial information for stochastic volatility models," 2015, *ArXiv*: 1407.1595v2 [q-fin.PM].

ix. Behrens C.N., Lopes H. F. & D. Gamerman, "Bayesian analysis of extreme events with threshold estimation," *Statistical Modelling*, 2004, 4, 227–244.

x. Elbahloul K., “Stock Market Prediction Using Various Statistical Methods Volume I," *Preprint*, July 2019. Available: doi:10.13140/RG.2.2.13235.17446.

xi. Wen D., Ma C., & Wang G.J., "Investigating the features of pairs trading strategy: A network perspective on the Chinese stock market," *Phys. A Stat. Mech. Appl.* 2018, 505, 903–918. Available: https://doi.org/10.1016/j.physa.2018.04.021.

xii. Shen J. & Shafiq M., “Short-term stock market price trend prediction using a comprehensive deep learning system," *J Big Data*, 2020, Available: https://doi.org/10.1186/s40537-020-00333-6.

xiii. Chen P., Yi D. & Zhao C., “Trading Strategy for Market Situation Estimation Based on Hidden Markov Model," *Mathematics*, 2020, vol. 8, pp. 1126–1138. Available: doi:10.3390/math8071126.

xiv. Liu M., Huo J., Wu Y. & Wu J., “Stock Market Trend Analysis Using Hidden Markov Model and Long Short Term Memory," *ArXiv*: 2104.09700v1 [q-fin.PR], 2021. Preprint.

xv. Nguyen N., “Hidden Markov Model for Stock Trading," *Int. J. Financial Stud*. 2018, 6, 36; Available: doi:10.3390/ijfs6020036.

xvi. Hassan M.R. "A combination of hidden markov model and fuzzy model for stock market forecasting." *Journal of Neurocomputing*, pp. 3439–3446, 2009.

xvii. Durmagambetov A., “Stock market mathematical models," Sept. 2019. Research Proposal. Available: doi:10.13140/RG.2.2.10435.14882.

xviii. Chaiboonsri C. & Wannapan S., “Applying Quantum Mechanics for Extreme Value Prediction of VaR and ES in the ASEAN Stock Exchange”, *Economies* 2021, Vol. 9, 13. Available: https://doi.org/10.3390/economies9010013.

xix. Zhang C. & Huang L., “A quantum model for the stock market” *ArXiv*:1009.4843v2 [q-fin.ST], Sept. 2010. Preprint.

xx. Bagarello F., “A quantum statistical approach to simplified stock markets." https://arxiv.org/abs/0907.2531, Jul. 2009. Preprint.

xxi. Caporali R.P.L., “A Statistic Method for the Prediction of the Succession of Bear and Bull Stock Market," *International Journal of Basic Sciences and Applied Computing*, 2023, Vol.9, 6, pp. 1–7. Available: doi: 10.35940/ijbsac.F0482.029623.

xxii. Caporali R.P.L., "Overhead Crane: an adaptive Particle Filter for anti-Sway closed-loop and collision detection using Computer Vision," *International Journal of Innovative Computing, Information, and Control*, 2019, Vol. 15, 4, pp. 1223–1241. Available: doi: 10.24507/ijicic.15.04.1223.

xxiii. Caporali R.P.L., "Classical and Quantum Kalman Filter: an application to an Under-actuated Non-linear System as a Gantry crane," *International Journal of Recent Engineering Research and Development*, 2022, Vol. 20.

xxiv. Berry M.V., “Riemann’s Zeta Function: a model for Quantum Chaos?”, *Springer, Berlin*, 2005. Available: doi.org/10.1007/3-540-17171-1_1.

xxv. Porter C.E., "Statistical theories of Spectra: Fluctuations," *Academic Press, New York*, 1965.

xxvi. Odlyzko A., "Tables of zeros of the Riemann zeta function," Available: https://www.users.cse.umn.edu/~odlyzko/zeta_tables/index.html.