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Evaluation of Computational Analysis for Stiffened Slab Using Finite Strip Method

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Abstract:
A rectangular slab stiffened by ribs oriented perpendicular to the support of the slab as structurally equivalent to an orthotropic plate has been analyzed using the finite strip method. The analytic formulation was obtained from the governing equation of motion of the plate using the classical plate theory to an arbitrary boundary and initial condition. The stiffness formulation from total internal energy was considered. The force-displacement equation technique by applying stiffness and uniformly distributed loading was used to obtain the modal solution and hence, displacement at an arbitrary point of the plate for the assumed harmonic function. Approximate stresses following the displacement are also obtained. The comparative solution of the equilibrium differential equation is obtained by means of convergent series expansions of levy plates and the application of Timoshenko and Woinowsky-Krieger averaging of bending rigidity, utilizing suitable orthogonal displacement functions. A computer procedure based on the finite strip method is given through the application of numerical examples, which shows that the procedure provides good results, especially with low discretization and computational effort.

Keywords: *Stiffened slab, finite element method, finite strip method, numerical analysis*

1. Introduction

Numerical methods over the years have developed to become versatile tools applicable in solving and analyzing structural problems such as the FEM, finite strip method and constrained finite strip approach [i, ii]. Lately, the Finite Element Method (FEM) dominated in this field of engineering because of how simple and powerful it is in modelling, which is frequently a labour-intensive process [iii-v]. Notwithstanding, the Finite Strip Method (FSM) is applicable to some structural problems as an alternative method that requires fewer degrees of freedom and consequently less time through the introduction of strips which breaks up the domain into a number of longitudinal elements. The finite strip method uses strip-shaped elements that extend along the structure. This method can be used for many structures that do not have complex geometry and can be solved with a relatively low degree of freedom [vi]. The finite strip method (FSM) is a powerful and effective contemporary technique of structural analysis used mainly for general linear static, dynamic and stability analysis of structures with a constant geometry and stiffnesses in one direction and simple boundary conditions at the cross ends [vii].

The Finite Strip Method, pioneered in 1968 by Cheung Y.K., is an efficient tool for analyzing structures with regular geometric platforms and simple boundary conditions. According to Cheung [viii], the main benefit of the traditional Finite Element Method lies in the significantly smaller size and bandwidth of the matrix that needs to be solved, making it ideal for programming on small and medium-sized computers. The finite strip method was originally designed for rectangular plate problems similar to Levy's solution by Timoshenko and Woinowsky-Krieger in 1971 [ix]. The plates are bi-dimensional structural elements that have dimensions from the mid-plane larger than the thickness. Unlike in a shell that makes use of thickness to the radius of curvature ratios [x], depending on the ratio between the minimum dimension from the plane and the thickness of plates can be classified for the calculation into two classes [xi]. Namely:

- Thin plates, at which lmin > 5h,
- Thick plates, at which lmin < 5h.

The minimum dimension of the plate (lmin) is significantly larger than the plate thickness (h) for thin plate while in thick plate it is a reverse.

Thin plates accept the hypothesis of Kirchhoff that despises the deformation due to shear strains. Governing equation of the thin plates can be obtained using vector mechanics or variational and energetic principles. This research work proposes models for the analysis of stiffened slabs using Finite Strip Method.

2. Numerical Formulation and Modelling Procedures

2.1. Degree of Freedom and Shape Functions

In the finite strip method, the structure is only discretized in the cross-section represented using a trigonometrical shape function.

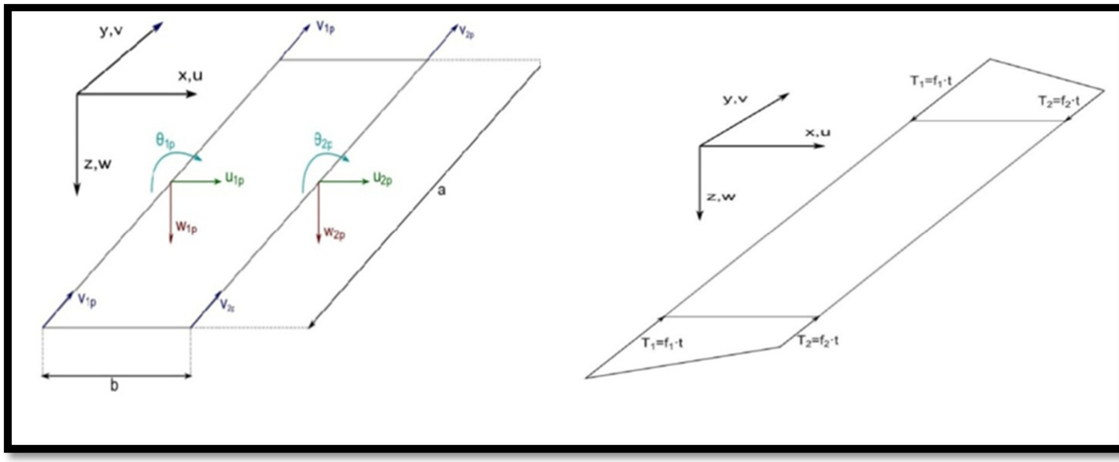

Figure 1: Finite Strip Degrees of Freedom Definition (Left) and Strip Stress Distribution (Right)[viii]

In figure 1, the local coordinates are named with small letters (x-y-z) and will always be associated with the strip element. Displacements are represented with the translation U-V-W and the rotation θ for global displacements and u-v-w and ϕ for local displacements. The subscript p refers to the half-wave number (number of longitudinal terms). The shape functions for the transverse direction are assumed to be the same polynomial function for every boundary condition.
In polynomial form, the x-displacement component of a plate problem in bending [xii] in eqn (1):

$$w(x) = C_1 + C_2x + C_3x^2 + C_4x^3 \qquad (1)$$

Where: $C1$-$C4$ represent generalized displacements.

Using the condition: φ = dw/dx, after writing the polynomial for the nodal lines 1 and 2 with the coordinates x = 0 and x = b, respectively, we obtain Eq. 2:

$$\begin{bmatrix} w_0 \\ \varphi_0 \\ w_b \\ \varphi_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & b & b^2 & b^3 \\ 0 & 1 & 2b & 3b^2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \qquad (2)$$

That is$[\delta] = [A][C]$, and the polynomial coefficients as:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-3}{b^2} & \frac{-2}{b} & \frac{3}{b^2} & \frac{-1}{b} \\ \frac{2}{b^3} & \frac{1}{b^2} & \frac{-2}{b^3} & \frac{1}{b^2} \end{bmatrix} \begin{bmatrix} w_0 \\ \varphi_0 \\ w_b \\ \varphi_b \end{bmatrix} \qquad (3)$$

The general form of y-displacement function, as given in Timoshenko and Woinowsky-krieger [xi], can be written as:

$$Y_m = A \sin\frac{ky}{a} + B \cos\frac{ky}{a} + C \sinh\frac{ky}{a} + D \cosh\frac{ky}{a} \qquad (4a)$$

K is a parameter Case: Simply supported, boundary conditions are expressed as $(w(0) = w''(0) = 0, w(a) = w''(a) = 0)$

$$Y_m = \sin\frac{m\pi y}{a} \qquad (4b)$$

Therefore, the expressions for general displacements are as follows:

$$u = \sum_{m=1}^{r} \left[\left(1 - \frac{x}{b}\right) \ \frac{x}{b}\right] \begin{Bmatrix} u_{im} \\ u_{2m} \end{Bmatrix} * \ Y_m \qquad (5a)$$

$$v = \sum_{m=1}^{r} \left[\left(1 - \frac{x}{b}\right) \ \frac{x}{b}\right] \begin{Bmatrix} v_{im} \\ v_{2m} \end{Bmatrix} * \ Y_m * \frac{a}{\mu_m} \qquad (5b)$$

$$w = \sum_{m=1}^{r} \left[1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3} \quad x\left(1 - \frac{2x}{b} + \frac{x^2}{b^2}\right) \quad \frac{3x^2}{b^2} - \frac{2x^3}{b^3} \quad x\left(\frac{x^2}{b^2} - \frac{x}{b}\right)\right] \begin{Bmatrix} w_{im} \\ \varphi_{im} \\ w_{jm} \\ \varphi_{jm} \end{Bmatrix} * \ Y_m \qquad (5c)$$

Where $\mu_p = m * \pi$ and m is the half-wave number and Y_m is the function for the longitudinal direction. It refers to the shape of the sinus function, which varies depending on the boundary conditions.

We can put the displacement equations in the form of a general vector such that:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{m=1}^{r} [N_{uv}] * \begin{Bmatrix} u_{im} \\ v_{im} \\ u_{jm} \\ v_{jm} \end{Bmatrix} = \sum_{m=1}^{r} [N_{uv}] * d_{uv}^{m} \quad (6)$$

$$w = \sum_{m=1}^{r} [N_w] * \begin{Bmatrix} w_{im} \\ \varphi_{im} \\ w_{jm} \\ \varphi_{jm} \end{Bmatrix} = \sum_{m=1}^{r} [N_w] * d_{w}^{m} \quad (7)$$

2.2. Bending Matrix

The bending strains follow the Kirchhoff plate theory hypothesis [xi, xiii]. The hypothesis which Kirchhoff's theory is based on four conditions stated below:

- Points in the middle plain only move vertically (u=v=0)
- All the points contained in a normal to the middle plain have the same vertical displacement
- σ_z is not considered
- Points on the normal lines to the plain stay in the same orthogonal lines to the middle plain after the deformation

From the previous hypothesis, we can write our displacements as:

$$\left.\begin{aligned} u(x,y,z) &= -z\,\theta_x\,(x,y) \quad (8) \\ v(x,y,z) &= -z\,\theta_y\,(x,y) \quad (9) \end{aligned}\right\} \boxed{1^{st} \text{ and } 4^{th} \text{ hypothesis}}$$

$$w(x,y,z) = w(x,y) \longrightarrow \boxed{2^{nd} \text{ hypothesis}} \quad (10)$$

Where:

w is the vertical displacement, θ_x and θ_y are the angles that define the turn of the normal line to the middle plain.

Taking figure 1 into consideration, we obtain the following:

From *xz* plain: $\theta_x = \frac{\partial w}{\partial x}$ (11)

From *yz* plain: $\theta_y = \frac{\partial w}{\partial y}$ (12)

Using Eqns. 8 to 12, we can conclude:

$$u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} \quad (13)$$

$$v(x,y,z) = -z\frac{\partial w(x,y)}{\partial y} \quad (14)$$

$$w(x,y,z) = w(x,y) \quad (15)$$

Taking the last expression in Equations 13 to 15, we can define the deformation as:

$$\{\varepsilon_B\} = \begin{Bmatrix} -z\frac{\delta^2 w}{\delta x^2} \\ -z\frac{\delta^2 w}{\delta y^2} \\ 2z\frac{\delta^2 w}{\delta x \delta y} \end{Bmatrix} = \sum_{m=1}^{r} z\,[N_w] \begin{Bmatrix} w_{1m} \\ \theta_{1m} \\ w_{2m} \\ \theta_{2m} \end{Bmatrix} = \sum_{m=1}^{r} z\,[B_B^m]\,d_w^m \quad (16)$$

As in the case of the membrane stress, we can write the bending term of the internal strain energy during buckling as:

$$U_B = \frac{1}{2}\int \{\varepsilon_B\}^T \{\sigma_B\} dA \quad (17)$$

Where; U_B is the internal strain energy, ε_B is the strain tensor, σ_B is the stress tensor, ∫ denotes the integral over the area (A) of the plate. The superscript "T" represents the transpose operation, which is applied to the strain tensor (ε_B) to ensure compatibility with the stress tensor (σ_B)

From Eqn 17, we can substitute:

$$\{\sigma_B\} = [D_B]\{\varepsilon_B\} \quad (18)$$

$$[D_B] = \begin{bmatrix} D_x & v_x D_y & 0 \\ v_y D_x & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \quad (19)$$

$$D_x = D_y = D = \frac{E_y t^3}{12(1 - v_x v_y)} \quad D = vD \quad D_{xy} = \frac{Et^3}{24(1+v)} \quad (20)$$

We can then write the expression for the strain energy as:

$$U_B = \frac{1}{2}\int_0^a \int_0^b \{\varepsilon_B\}^T [D_B]\,\{\varepsilon_B\} dxdy \quad (21)$$

The elastic stiffness matrix for membrane stress can be extracted from the statement for internal energy, such as:

$$U_B = \sum_{m=1}^{r}\sum_{n=1}^{r}\frac{1}{2}(d_w^m)^T\left(\int_0^a\int_0^b\{B_B\}^T[D_B]\{B_B\}dxdy\right)d_w^n \quad (22)$$

Where our stiffness matrix is:

$$K_B^{mn} = \int_0^a\int_0^b(B_B)^T[D_B]B_B dxdy \quad (23)$$

The potential energy in terms of external surface loads is:

$$U_w = -\int_0^a\int_0^b(d_w^m)^T(C)^T q(x,y)\sin k_m y\, dxdy \quad (24)$$

$$[P] = \int_0^a\int_0^b\{C\}^T q(x,y)\sin k_m y\, dxdy \quad (25)$$

Hence, total potential energy is the sum of the elastic strain energy and the work potential of each strip; thus:

$$W = U_B + U_w \quad (26)$$

By minimizing the total potential energy, we set:

$$\left\{\frac{\partial W}{\partial\{d\}}\right\} = \{0\} \quad (27)$$

Thus:

$$\left\{\frac{\partial W}{\partial\{d\}}\right\} = \int_0^a\int_0^b\{B\}^T[D]\{B\}\, d\, dxdy - \int_0^a\int_0^b\{C\}^T q(x,y,t)\sin k_m y\, dxdy = \{0\} \quad (28)$$

And

$$[K]\{\delta\} - [P] = \{0\} \quad (29)$$

Furthermore:

$$\{B\}_m = \begin{bmatrix} -C''_{m1}\sin k_m y & -C''_{m2}\sin k_m y & -C''_{m3}\sin k_m y & -C''_{m4}\sin k_m y \\ k_m^2 C_{m1}\sin k_m y & k_m^2 C_{m2}\sin k_m y & k_m^2 C_{m3}\sin k_m y & k_m^2 C_{m4}\sin k_m y \\ 2k_m C'_{m1}\cos k_m y & 2k_m C'_{m2}\cos k_m y & 2k_m C'_{m3}\cos k_m y & 2k_m C'_{m4}\cos k_m y \end{bmatrix} \quad (30)$$

2.3. Stiffness Matrix Formulation

The explicit form of the stiffness matrix for a single strip in the m and n-th harmonic function is given as:

$$[K]_m = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} \quad (31)$$

Where,

$$K_{11} = \frac{13ab}{70}k_m^4 D + \frac{12a}{5b}k_m^2 D_{xy} + \frac{6a}{5b}k_m^2 D_1 + \frac{6a}{b^3}k_m^2 D$$
$$K_{12} = \frac{3a}{5}k_m^2 D_1 + \frac{a}{5}k_m^2 D_{xy} + \frac{3a}{b^2}D + \frac{11ab^2}{420}k_m^2 D$$
$$K_{22} = \frac{ab^3}{210}k_m^4 D + \frac{4ab}{15}k_m^2 D_{xy} + \frac{2ab}{15}k_m^2 D_1 + \frac{2a}{b}D$$
$$K_{13} = \frac{9ab}{140}k_m^4 D - \frac{12a}{5b}k_m^2 D_{xy} - \frac{6a}{5b}k_m^2 D_1 - \frac{6a}{b^2}D$$
$$K_{23} = \frac{13ab^2}{840}k_m^4 D - \frac{a}{5}k_m^2 D_{xy} - \frac{a}{10}k_m^2 D_1 - \frac{3a}{b^2}D$$
$$K_{33} = \frac{13ab}{70}k_m^4 D + \frac{12a}{5b}k_m^2 D_{xy} + \frac{6a}{5b}k_m^2 D_1 + \frac{6a}{b^2}D$$
$$K_{14} = -\frac{13ab^2}{840}k_m^4 D + \frac{a}{5}k_m^2 D_{xy} - \frac{a}{10}k_m^2 D_1 + \frac{3a}{b^2}D$$
$$K_{24} = -\frac{3ab^3}{840}k_m^4 D - \frac{ab}{15}k_m^2 D_{xy} - \frac{ab}{30}k_m^2 D_1 + \frac{a}{b}D$$
$$K_{34} = -\frac{11ab^2}{420}k_m^4 D - \frac{a}{5}k_m^2 D_{xy} - \frac{3a}{5}k_m^2 D_1 - \frac{3a}{b^2}D$$
$$K_{44} = \frac{ab^3}{210}k_m^4 D + \frac{4ab}{15}k_m^2 D_{xy} + \frac{2ab}{15}k_m^2 D_1 + \frac{2a}{b}D$$

(32)

These expressions above correspond to the value after the integration. The finite strip which has two degrees of freedom per nodal line in bending, under uniform external loading as:

$$[P]_m = \int_0^a\int_0^b\{N\}^T q(x,y)\sin k_m y\, dxdy \quad (33)$$

Also, to dispense with the integration:

$$[P]_m = q(x,y)\begin{Bmatrix} \frac{b}{2} \\ \frac{b^2}{12} \\ \frac{b}{2} \\ \frac{-b^2}{12} \end{Bmatrix} \frac{1}{k_m}(1-\cos k_m a) \quad (34)$$

The bending and twisting moments for a strip are:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [D]\sum_{m=1}^{r}[B]_m\{\delta\}_m \quad (35)$$

2.4. Beam Stiffness Matrix

The stiffness matrix of a beam is formulated in the same way as that for a slab strip, and for compatibility, both plate strip and beam should have the same variation of deflection at the connecting nodal line [6]. A beam is treated as a one-dimensional member, the stiffness of which is assumed to be concentrated along its centre line. For compatibility, the variation of the beam vertical deflection and rotation should be the same as the slab strips, which have common nodal lines. For this reason, we express the beam displacements by the same used for the slab strip.

$$\begin{Bmatrix} \overline{w}^* \\ \overline{\theta}^* \end{Bmatrix} = Y_r \begin{Bmatrix} \overline{w} \\ \overline{\theta} \end{Bmatrix} \quad (36)$$

The internal virtual work is given as:

$$U_I = \int_0^a \frac{d^2\overline{w}^*}{dy^2}\left(EI\frac{d^2w^*}{dy^2}\right)dy + \int_0^a \frac{d^2\overline{\theta}^*}{dy^2}\left(GJ\frac{d^2\theta^*}{dy}\right)d \quad (37)$$

and the corresponding external virtual work as:

$$U_E = \int_0^a \overline{w}^*\, q^* dy + \int_0^a \overline{\theta}^*\, m^* dy \quad (38)$$

Therefore:

$$w^* = \sum Y_m\, w_m \text{ and } \theta^* = \sum Y_m\, \theta_m \quad (39)$$

Where w_m and θ_m are the nodal displacement parameter, and the external applied nodal line forces of intensity q* and m* are expressed in the form of infinite series.

$$q^* = \sum Y_m\, q_m \text{ and } m^* = \sum Y_m\, m_m \quad (40)$$

Where Y_m is the basic function, and q_m and m_m are the nodal force parameter.

By equating the internal and external virtual work from Eqns. 37 and 38, we obtain:

$$\overline{w}_r \sum_{m=1,2,..} w_m \int_0^a EI\frac{d^2Y_r}{dy^2}\frac{d^2Y_m}{dy^2}dy + \overline{\theta}_r \sum_{m=1,2,..} \theta_m \int_0^a GJ\frac{d^2Y_r}{dy^2}\frac{d^2Y_m}{dy^2}dy = \overline{w}_r \sum_{m=1,2,..} q_m \int_0^a Y_r\, Y_m dy + \overline{\theta}_r \sum_{m=1,2,..} m_m \int_0^a Y_r\, Y_m dy \quad (41)$$

Similarly, we define a generalized beam force vector as:

$$[P]_m = q(x,y)\sum_{m=1,2,..}\begin{Bmatrix} q \\ m \end{Bmatrix}\left(\int_0^a Y_m = \frac{1}{k_m}(1-\cos k_m a)\right) \quad (42)$$

Putting each of the virtual displacement parameters $\overline{w}_r$ and $\overline{\theta}_r$ equal to zero separately, Eqn (42) gives

$$[P]_m = \sum_{m=1,2,..}\begin{bmatrix} K_{rm}^f & 0 \\ 0 & K_{rm}^t \end{bmatrix}\begin{Bmatrix} w \\ \theta \end{Bmatrix} \quad (43)$$

Or

$$[P]_m = \sum_{m=1,2,..}[K\,]_{rm}\{d\,\}_m \quad (44)$$

$[K\,]_{rm}$ is a diagonal submatrix of the stiffness matrix of the beam, which is of order 2s x 2s, where s is the number of series terms considered. The elements K_{rm}^f and K_{rm}^t are flexural and torsional stiffness coefficients given by:

$$K_{rm}^f = \int_0^a EI\frac{d^2Y_r}{dy^2}\frac{d^2Y_m}{dy^2}dy \quad (45)$$

$$K_{rm}^t = \int_0^a GJ\frac{d^2Y_r}{dy^2}\frac{d^2Y_m}{dy^2}dy \quad (46)$$

If r is taken equal to 1, 2,.., n, in Eqns (32) through (42). For a simply-supported beam, we use the same basic function as for a slab strip. With this equation, we assume that the twisting is prevented at the two ends of the beams. Because of the orthogonality of the function $Y_m\, Y_r$, the stiffness coefficients K_{rm} in Eqns (41) and (42) vanish when $r \neq m$ and when *E1* and *GJ* are constant, we have:

$$K_{rm}^f = K_m^f = \frac{(n\pi)^4}{2}\frac{EI}{a^3} \quad (47)$$

$$\text{and } K_{rm}^t = K_m^t = \frac{(n\pi)^2}{2}\frac{GJ}{a} \quad (48)$$

Further, the beam force vector and the displacements parameters term of the series can be related where: $[K\,]_{rm} = [K\,]_{mm}$

Where: b^3 and β can be obtained from the Torsion constant table.

J=βa

3. Problem Formulation

It is common knowledge that the flexural motion of stiffened slab is governed by the following differential equation:

$D\nabla^4 w = q(x,y)$ (49)

Where:
w is the flexural deflection of the plate,
D is the bending rigidity of the plate in both directions,
x and y are the spatial coordinates,
q is the intensity of loading.

The plate is simply supported on two parallel edges; the other two edges are arbitrary or on elastic support.

The displacement field of the finite strip with slab and beam behaviour, as illustrated in figure 2 below, is described by the following equation:

$w(x,y) = \sum_{m=1}^{r} w(x)\sin\frac{m\pi y}{a}$ (50)

$k_m = \frac{m\pi}{a},\ m = 1,2,\dots,r$ (51)

The approximate displacement function for the points on a single strip combines the sine harmonic series in the longitudinal direction, y (analytical aspect) and the polynomial function $F_m(x)$ in the transverse direction, x (numerical aspect).

The deflection of the strip may be written in the transform of two uncoupled function:

$w(x,y) = \sum_{m=1}^{r} F_m(x)\sin\frac{m\pi y}{a}$ (52)

$$F_m(x) = C(x)\delta_m(x) = \left[1-\frac{3x^2}{b^2}+\frac{2x^3}{b^3} \quad x\left(1-\frac{2x}{b}+\frac{x^2}{b^2}\right) \quad \frac{3x^2}{b^2}-\frac{2x^3}{b^3} \quad x\left(\frac{x^2}{b^2}-\frac{x}{b}\right)\right]\begin{Bmatrix} w_{im} \\ \varphi_{im} \\ w_{jm} \\ \varphi_{jm} \end{Bmatrix} \quad (53)$$

Since opposite ends of the finite strip are simply supported, the basic function and its first and second derivatives are involved in the evaluation of the stiffness matrix. By considering the mid-span displacement for a strip, we substitute x as equal to 0.5.

3.1. Timoshenko's Approach

The theory established by the set of ribs can only give a rough idea of the actual state of stress and strain of the slab [6]. Figure 2 shows the Timoshenko ribbed slab. This averaging of bending rigidity is used in validating the new model.

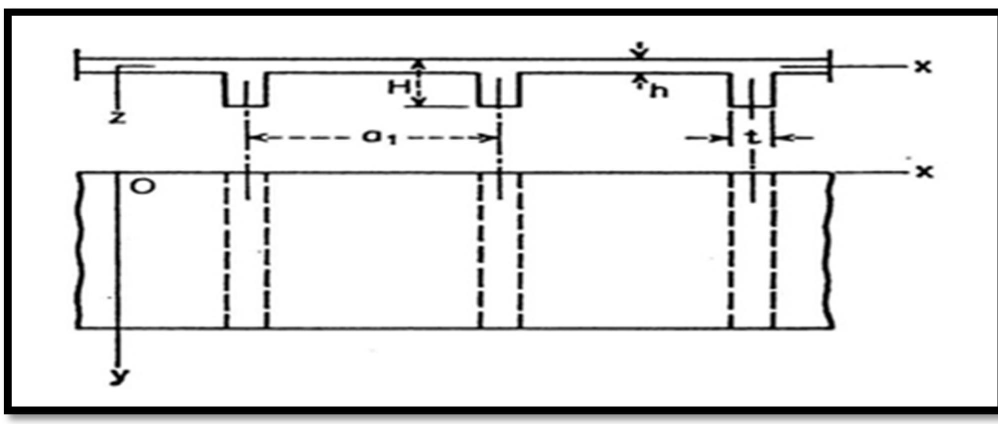

Figure 2: Stiffened Slab [xi]

We define the following terms as:

$D_x = \frac{E_x a_1 h^3}{12(a_1 - t + \alpha^3 t)},\ \alpha = \frac{h}{H}$ (54)

$D_y = \frac{E_y I}{a_1},\ D_1 = 0,$ (55)

$I = \frac{1}{3}[ty^3 + a_1(H-y)^3 - (a_1 - t)(H-y-h)^3]$ (56)

$y = H - \frac{H^2 t + h^2(a_1 - t)}{2(a_1 h + (H-h)t)}$ (57)

$D_{xy} = D'_{xy} + \frac{C}{2a_1}$, and $D'_{xy} = \frac{1-v}{2}\frac{E_x(h/2)^3}{12(1-\vartheta_x\vartheta_y)}$ $C = \frac{1-v}{2}\frac{E_x(H-h/2)^3}{12(1-\vartheta_x\vartheta_y)}$ (58)

Where D'_{xy} is the torsional rigidity of the slab without the ribs, C is the torsional rigidity of the rib, E, ϑ, and I is the moment of inertia of a T section of width a_1, taken with respect to the middle axis of the cross-section of the plate.

From the differential equation, we solve for the deflection surface of stiffened slab by applying Levy's solution satisfying a simply-supported boundary condition under uniform loading as:

$$w(x,y) = \frac{qa^4}{0.5\pi^5(D_x+D_y)}\sum_{m=1,3,5..}^{\infty}\left(A_m\cosh\frac{2\alpha_m y}{b} + B_m\frac{2\alpha_m y}{b}\sinh\frac{2\alpha_m y}{b} + G_m\right)\sin\frac{m\pi x}{a} \quad (59)$$

Where,

$$A_m = -\frac{2(\alpha_m\tanh\alpha_m+2)}{\pi^5 m^5\cosh\alpha_m} \text{ and } B_m = \frac{2}{\pi^5 m^5\cosh\alpha_m} \quad (60)$$

$G_m = \frac{4}{\pi^5 m^5}$ and, $\alpha_m = \frac{m\pi b}{2a}$ and $M_x = q\pi^2a^2\sum_{m=1,3,5..}^{\infty} m^2\left(((\vartheta-1)A_m + 2\vartheta B_m)\cosh\frac{2\alpha_m y}{b} + (\vartheta-1)B_m\frac{2\alpha_m y}{b}\sinh\frac{2\alpha_m y}{b} + G_m\right)\sin\frac{m\pi x}{a}$,

$$M_y = -q\pi^2a^2\sum_{m=1,3,5..}^{\infty} m^2\left(((1-\vartheta)A_m + 2B_m)\cosh\frac{2\alpha_m y}{b} + (1-\vartheta)B_m\frac{2\alpha_m y}{b}\sinh\frac{2\alpha_m y}{b} - \vartheta G_m\right)\sin\frac{m\pi x}{a} \quad (61)$$

4. Results and Discussion

Consider figure 3, a simply supported ribbed slab used for numerical illustrations. In the experimental program, the following input was used for the FSM stiffened slab model and compared to Timoshenko model:

- E=35.83*10^6kN/m^2
- G=21.0*10^6kN/m^2
- UDL=25kN/m2; % Uniformly distributed load
- a=4.5m; % Length of stiffened plate
- b=12.75m; % Total width of stiffened plate under s-s
- m=1; % We analyze the first half-wave mode, v=0.3
- h_1=0.15m; % Height of slab strips
- b_1=3m; % Width of slab strips
- h_2=0.5m; % Height of beam strips
- b_2=0.25m; % Width of beam strips

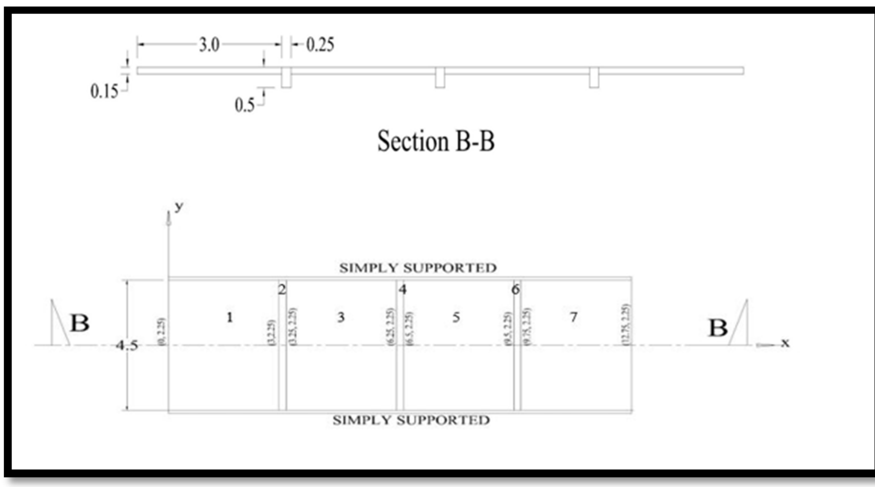

Figure 3: Simply Supported Ribbed Slab

Strip	x-axis	y-axis	FSM			Timoshenko [xi]		
			w(x,y)m	Mx(kNm)	My(kNm)	w(x,y)m	Mx(kNm)	My(kNm)
1	1.50	3	-2.98E-04	1.74E-01	-1.44E+00	1.61E-05	2.79E-02	-2.88E-02
2	3.13	3	-0.0015	-2.78E+01	-1.51E+01	8.79E-05	1.52E-01	-1.57E-01
3	4.75	3	2.54E-05	1.13E-01	1.80E-01	2.10E-04	3.63E-01	-3.76E-01
4	6.38	3	2.24E-04	-6.87E+00	-9.13E-01	3.83E-04	6.62E-01	-6.84E-01
5	8.00	3	2.54E-05	1.13E-01	1.80E-01	2.10E-04	3.63E-01	-3.76E-01
6	9.63	3	-0.0015	-2.75E+01	-1.51E+01	8.79E-05	1.52E-01	-1.57E-01
7	11.25	3	-2.98E-04	-1.72E-02	-1.44E+00	1.61E-05	2.79E-02	-2.88E-02

Table 1: Comparison for FSM and Timoshenko

Table 1 shows that the results obtained from the current FSM are compared with those of the analytical solution. Considering the structural geometry, FSM results show details for w, Mx and My. In figures 4a and 4b, diagrammatical representations are presented. The curves follow a similar pattern as a result of the strips.

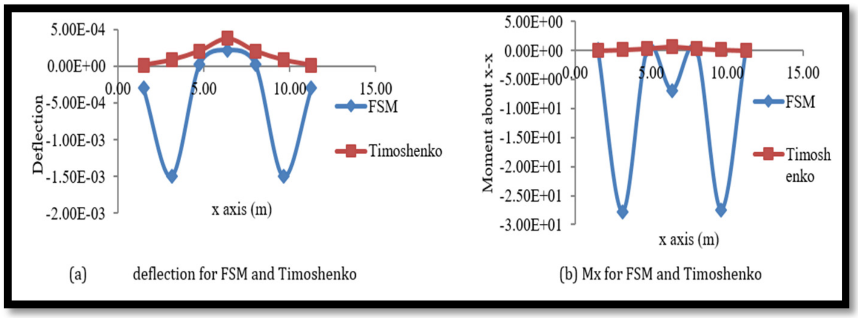

Figure 4 (a & b): Comparison between FSM and Timoshenko Model

The results obtained from the current FSM are compared with those of the analytical solution. The numerical result relevant to the discretization by means of strips of simple shape functions shows that finite strips of different thicknesses preserve the bending rigidities via finite element considerations than the uniformly smeared bending rigidity of Timoshenko.

5. Conclusion

This paper presents the development of a program capable of implementing the FSM theory to solve slab problems. The new formulation of the FSM with the theories of Kirchhoff for the analysis of rectangular structures is implemented in the program. The validity of the combined strips is demonstrated in the illustrative problem, and the results obtained are compared with approximate solutions. Therefore, we can extract some conclusions:

- Although the Finite Strip Method is a semi-analytical theory that utilizes trigonometric Fourier series in the longitudinal direction and FEM in the transversal direction, it can be correctly utilized to solve a rectangular problem with various sections.
- The dimension of the problem is greatly reduced with FSM. It is reduced to a system of two middle nodes with four displacements each. Using the Fourier series, we can define the boundary conditions with trigonometric functions that satisfy them in the loaded edges.
- The boundary conditions are extremely important to determine the stiffness matrices in the FSM; the effect of changing them will result in some coefficients inside the matrix varying.
- The behaviour of our program is acceptable in many cases and much quicker than the FEM solution to the same problem.

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