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Relationship between Logical Reasoning and Performance in Mathematics and Science among Junior High School Students

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Abstract:
The study examined the relationship between logical reasoning abilities and performance in mathematics and science among Junior High School students in the Ga South Municipality. The research design employed was a descriptive survey. Through a Multi-stage sampling procedure, data was obtained from a sample size of 370 JHS two (2) students in the Ga South Municipality. Test of Logical Thinking (TOLT) was adapted to measure the students' logical reasoning abilities and adapted tests in mathematics and science were used to measure students' performance. Frequencies, percentages, means, and standard deviations were used to analyze data for the research questions, while Pearson Product Moment Correlation, multiple linear regression and independent samples t-test were used to test the hypotheses. Results revealed that JHS two (2) students in the Ga South Municipality were low in logical reasoning. A significant positive weak relationship between students' logical reasoning and performance in both mathematics and science was established. Also, combinatorial reasoning was the best predictor of performance in both mathematics and science. It was recommended that the school curriculum in mathematics and science at the JHS level should be deliberately designed to train students in logical reasoning.

Keywords: *Logical reasoning, mathematics, science, junior high school*

1. Introduction

Education exposes individuals to social advancement, and quality education can build significant analytical and social skills which enable young people to make good choices and pursue responsible lifestyles (Basic Education Coalition, 2013). Many educational policies in Ghana, such as the expansion of infrastructure in schools, the removal of schools under trees, the introduction of capitation grants, school feeding programs, free exercise books, and free school uniforms, were introduced at the basic school level, which has led to increase in enrolments (MoE, 2010).

While a number of policy reforms and interventions at the basic school level have improved access to education in Ghana; enhancing the instructional quality and academic achievement of children remain critical challenges especially in mathematics and science (Iddi, 2016).

It is stated in the junior high school teaching syllabus of Ghana for Mathematics (TSM) that:

> *Development in almost all areas of life is based on effective knowledge of science and mathematics. There simply cannot be any meaningful development in virtually any area of life without knowledge of science and mathematics. It is for this reason that the education systems of countries that are concerned about their development put a great deal of emphasis on the study of mathematics. The main rationale for the mathematics syllabus is focused on attaining one crucial goal: to enable all young Ghanaian people to acquire the mathematical skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives (pg ii).*

All areas of life are based on effective knowledge of mathematics and science. Without knowledge of mathematics and science, there will be no development in life. So, any country which is concerned about her development puts a great deal of emphasis on the study of mathematics and science (Forman, 2003).

Mathematics and science at the Junior High School (JHS) in Ghana depend on the knowledge and competencies developed at the primary school level. Pupils are expected at the JHS level to move beyond and apply mathematical ideas in investigating real-life situations (Teaching syllabus for mathematics 2007). The development of strong mathematical competencies at the JHS level is an essential requirement for effective study in mathematics, science, commerce, industry and a variety of other professions and vocations for pupils terminating their education at the JHS level as well as for those continuing into tertiary education and beyond (Teaching syllabus for mathematics 2007).

In 2003, 2007 and 2011, Ghana participated in the Trends in International Mathematics and Science Study (TIMSS) to help compare her educational system and students' achievements with that of other participating countries in an effort to improve science and math education in Ghana. Anamuah-Mensah, Mereku and Asabre-Ameyaw (2004) stated that Ghana's participation in TIMSS was strategic as it enabled the country to find out how the performance of her eighth graders, thus JHS 2 in mathematics and science compared with those of the other countries, because the importance of mathematics and science in today's society provides a significant context for such comparison.

It has long been hypothesized that children's level of cognitive development is an important factor in their ability to learn mathematics and science (Hiebert, 1981). In Ghana, the Science Technology and Mathematics Education (STME) programme seeks to imbibe in the child the skills of innovation, creativity and imagination for better life (Amoah, 2016). Literature points out that among the priorities of mathematics and science education was developing students' logical thinking abilities (Lawson, 1982). It has been suggested that the ability of logical reasoning has an essential function in the academic performance of students and their construction of concepts and knowledge (Atay, 2006; Lawson, 1992). The higher the ability of a person to reason abstractly, the higher the chance the person will effectively function in society (Ongcoy, 2016).

Piaget (1969) defines logical reasoning as mental operations used by individuals when they encounter specific problems. Piaget created a model for cognitive development that has found widespread use in education and psychology (Etzler & Madden, 2017). He conceptualized four different stages in the cognitive development of a person. These, he outlined, as: sensorimotor (0-2 years), preoperational (2-7 years), concrete operational (7-11 years) and formal operational (11-16 years). The main difference among these stages of cognitive development is the mode of thinking. Children at the formal operational stage can think logically about abstract propositions and test hypotheses systematically compared to children at the concrete stage, who are limited to reasoning in concrete forms. Inhelder and Piaget (1958) revealed that the development of logical reasoning occurred between the ages of nine and fifteen years. And as the child increases in age (from 9 years upwards), the quality of logical reasoning ability also increases. Inhelder and Piaget (1958) advanced the need for the development of propositional logic, of formal operational, and the integration of these operational schemata and propositional logic as essential logical reasoning operations.

1.1. Modes of Logical Reasoning

Researchers such as Capie, Newton, and Tobin (1981), DeCarcer, Gabel, and Stever (1978), Lawson (1985) and Demirel (2003), through the work of Inhelder and Piaget, identified five different modes of formal operational reasoning. They identified the modes as proportional reasoning, controlling variables, probabilistic reasoning, correlational reasoning and combinatorial reasoning.

Proportional reasoning is the ability to realize equal proportions of two quantities and logic to understand and solve quantitative relations. Controlling variables involves identifying all the variables in a given condition, formulating a hypothesis for the role of variables, and systematically controlling variables to verify the hypothesis to derive the conclusion.

Probabilistic reasoning is the ratio of expected probability for all possible probabilities. Correlational reasoning is the ability to realize relationships between variables. Combinatorial reasoning refers to the count of all the possible cases for solving problems without duplications. According to Capie, Newton, and Tobin (1981), all five modes of formal reasoning are determinants of students' success in science and mathematics.

The evidence most often mentioned for the apparent relationship between the logical reasoning abilities recognized by Piaget and children's mathematics learning is the frequent correlation between performance on various Piagetian tasks (the five modes) and mathematics achievement (Carpenter, 1980; Hiebert, 1981). Demirel (2003) stated that logical thinking includes the ability to use numbers effectively, provide scientific solutions to problems, detect the separations between concepts, classify, generalize, represent with a mathematical formula, compute, provide a hypothesis, test and simulate. It is assumed that students improve their logical thinking when they can judge through hypothesis (Tuna, Çağrı Bibe & İncikapı, 2013). For example, a student who can prove a hypothesis in the form of "If …, then …" can be categorized to be in the period of abstract operations. This is a proof for the student's improvement in his or her logical thinking.

1.2. Logical Reasoning and Performance in Mathematics and Science

Mathematics is an excellent tool for the development and improvement of a person's intellectual ability in logical reasoning, spatial visualization, abstract thought and analysis (Adegoke, 2013). Zaman (2011) mentioned that mathematical reasoning developed logical reasoning, which helped students understand the realities around them and make good decisions. Relatively, a number of studies in mathematics education have reported significant relationships between formal reasoning (logical reasoning) and students' achievement in the subject (Etzler & Madden, 2017). Research has determined that the ability to reason formally is the strongest predictor of successful achievement in mathematics and science. Some studies, such as Heng-Yuku and Sullivan (2000), Choudhury and Das (2012), and Nunes, Byrant, Barnes, and Sylva (2012), revealed a significant relationship between logical reasoning ability and attainment in mathematics.

Science education enables students to apply scientific concepts and methods to address problems in research, professional practice, and daily life (Abd-El-Khalick, BouJaoude, Duschl, Lederman, Mamlok-Naaman, Hofstein, & Tuan, 2004). Inhelder and Piaget (1958) argued that formal operational reasoning includes an important aspect of scientific reasoning in which learners are supposedly able to use evidence to evaluate hypotheses. Lawson, Banks and Logvin (2007) stressed that students' reasoning abilities have been established as an important factor for science achievement. A study conducted by Sungur and Tekkaya (2003) reported a significant effect of reasoning abilities on achievement in biology. Johnson and Lawson (1998) conducted a study to show the effects of reasoning ability and prior knowledge on biology achievement in two teaching classes: expository and inquiry. Students' ability to reason logically has been found to be the strongest predictor of meaningful understanding in science (Lawson, Alkhoury, Benford, Clark & Falconer, 2000).

1.3. Statement of the Problem

Lee (2011) stated that students face obstacles in their reasoning with logical implications that negatively influence their performance in mathematics and science. Their inability to engage in deductive reasoning of logical implications hampered their abilities to construct or validate proofs for mathematical and scientific statements (Lee, 2011).

The current state of teaching and learning science and mathematics in Ghana is poor (Azure, 2015). Studies have shown that many Ghanaian students tend to learn science and mathematics by rote and hence lack the understanding of concepts since no meaningful learning occurs (Anamuah-Mensah & Benneh, 2010; Jones, 2008; O'Connor, 2002). Ampiah (2016) stated that the basic school science syllabus is concentrated heavily on content knowledge, scientific concepts and theories, with little or no attention being given to scientific literacy that enables students to make informed judgments about scientific issues affecting their daily lives through scientific applications. Ampiah (2016) revealed that, from his investigations, questions set for Primary, Junior and Senior High Schools were predominantly focused on knowing, with a few on applying and reasoning. He, however, recommended that examination bodies, schools, and educators raise the level of their questions so that the students can incorporate higher-order questions.

Ghanaian JHS 2 students' performance in the Trends in International Mathematics and Science study was less than the international average in TIMSS 2011, 2007 and 2003. Ghana occupied 44th place out of the 45 participating countries in 2003 (Adetunde, 2009), and in 2011, Ghana was 42nd out of 42 countries that participated. Anamuah-Mensah, Mereku and Asabere-Ameyaw (2004) revealed that the poor performance of the Ghanaian JHS 2 students in TIMSS could be attributed largely to the lack of congruence between what is emphasized in the mathematics and science curriculum in Ghana and what is currently valued globally in mathematics and science, thus logical reasoning, applying, and knowing. The Ghanaian curriculum, textbooks, syllabi and assessment place a great deal of emphasis on number work and knowledge of facts and procedures but less on logical reasoning (Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2004).

The development of logical reasoning is a goal widely held by most educators. However, reported research offers little of a prescriptive nature concerning how to nurture and enhance children's ability to use logic. Therefore, the purpose of this study was to examine the relationship between logical reasoning abilities and JHS 2 students' performance in mathematics and science.

1.4. Purpose of the Study

The main purpose of this study was to examine the relationship between logical reasoning abilities and JHS 2 students' performance in mathematics and science. Specifically, the study sought to find out;

- The level of logical reasoning abilities of JHS 2 students
- The relationship between logical reasoning and JHS 2 students' performance in mathematics.
- Which of the five modes of logical reasoning can predict performance in mathematics?
- The relationship between logical reasoning and JHS 2 students' performance in science.
- Which of the five modes of logical reasoning can predict performance in science?

1.5. Research Question and Hypothesis

In order to further assess logical reasoning ability among selected junior high school students, the following question and hypothesis were developed to guide the study.

- What is the level of logical reasoning abilities of students?

1.6. Hypothesis

- H_0: There would be no significant relationship between logical reasoning abilities and JHS 2 students' performance in mathematics.
- H_1: There would be a significant relationship between logical reasoning abilities and JHS 2 students' performance in mathematics.
- H_0: The five modes of logical reasoning will not predict JHS 2 students' performance in mathematics.
- H_1: The five modes of logical reasoning will predict JHS 2 students' performance in mathematics.
- H_0: There would be no significant relationship between logical reasoning abilities and JHS 2 students' performance in science.
- H_1: There would be a significant relationship between logical reasoning abilities and JHS 2 students' performance in science.
- H_0: The five modes of logical reasoning will not predict JHS 2 students' performance in science.
- H_1: The five modes of logical reasoning will predict JHS 2 students' performance in science.

2. Methodology

The study adopted the descriptive survey research design. Descriptive design was adopted because its purpose is to observe, describe and document the situation as it exists in its current state and interpret the relationship (correlation) between variables (Williams, 2007).

2.1. Population, Sample and Sampling Procedure

The target population for the study was selected from 7030 second-year students in Public and Private Junior High Schools in the Ga South Municipality. Ten (10) schools were selected randomly: four (4) public schools and six (6) Private schools. Based on Krejcie and Morgan's (1970) table of sample size determination, 370 students were sampled for the study.

2.2. Instruments

The instruments used for the study were in three (3) sections. Section A was a test to measure logical reasoning, section B involved a test to measure performance in mathematics and section C measured performance in science.

The Test of Logical Thinking (TOLT) instrument developed by Tobin and Capie (1981) was adopted in this study to measure the logical reasoning ability of students. The TOLT instrument consists of 10 items that measure the five modes of formal reasoning abilities of students. The items were distributed into the following:

- Items 1 and 2 measured proportional reasoning;
- Items 3 and 4 measured controlling variables;
- Items 5 and 6 measured probabilistic reasoning;
- Items 7 and 8 measured correlational reasoning and
- Items 9 and 10 measured combinatorial reasoning.

The Cronbach alpha coefficient of the original instrument is .85. Test scores from 0-3, 4-7, and 8-10 were used as the basis for categorizing the participants according to their levels of logical reasoning as low-level, medium-level and high-level, respectively, as done by Oliva (2003) in his study.

Sections B and C of the instrument consisted of adapted standardized tests in mathematics and science from the Institute of Educational Development and Outreach (University of Cape Coast) to measure performance in both subjects. The adapted mathematics test contained 15 items with a Cronbach's alpha of .80 and 11 items for science with a Cronbach's alpha of .77.

2.3. Data Collection Procedures

Before administering the instruments, formal permission was sought from the head teachers of the schools involved. Permission from the public schools was sought through the Ga South Education Office. After permission was granted, two days were dedicated to the collection of data. During the first day of the data collection, all the JHS 2 students in each school were gathered in a class and were briefed about the purpose of the study and how to respond to the instruments. Test of Logical Thinking (Section A) was administered on the first day of data collection. The students were given 35 minutes for TOLT to respond to the items. The respondents were also informed that the results of the test would not be used for their daily classroom assessment but only for research purposes. Each student was given a pseudonym and was asked to identify their instruments with the pseudonyms given to them to ensure anonymity and confidentiality. Another day was used to collect data on their performance in mathematics (Section B) and science (Section C). The students were given 35 minutes to respond to the test items; that is, 20 minutes for the mathematics test and 15 minutes for the science test.

2.4. Data Analysis

After the assumptions of using parametric tests were met, frequency counts and percentages, means and standard deviations were used to analyze research question one. Pearson Moment Correlation Coefficient was used to analyze hypothesis 1 and 3. Multiple linear regression was used to analyze hypotheses 2 and 4. All the hypotheses were tested at a .05 significant level of confidence.

3. Results and Discussion

The results and discussion have been presented based on the research question and hypotheses that guided the study.

Modes of Logical Reasoning	N	Minimum	Maximum	Mean	Std. Deviation
Proportional	370	0	4	.22	.496
Controlling	370	0	4	.20	.469
Probabilistic	370	0	4	.10	.392
Correlational	370	0	2	.11	.344
Combinatorial	370	0	2	1.71	.547
Logical Reasoning	370	0	5	2.29	.912

Table 1: Descriptive Statistics on JHS Students' Level of Logical Reasoning

Results from table 1 are descriptive statistics on JHS students' performance in logical reasoning. From the table, it is revealed that the minimum score of the students' logical reasoning was 0, which signifies low-level logical reasoning and the same across all the five components of logical reasoning. Moreover, the maximum score for students' logical reasoning was 5, which also signifies medium-level logical reasoning ability. However, the mean score for JHS school students' logical reasoning ability was 2.29 (SD = .912), which apparently falls within the low-level logical reasoning ability.

		Mathematics
Logical Reasoning	Pearson Correlation	.105*
	Sig. (2-tailed)	.044

Table 2: Relationship between Logical Reasoning Abilities and Performance in Mathematics of JHS 2 Students
**. Correlation is significant at the 0.05 level (2-tailed)*

From the table, there is a positive relationship *r* = .105 between logical reasoning and the performance of students in mathematics. The result is statistically significant because Sig. (2-tailed) p = .044. Therefore, the null hypothesis is rejected because $p < 0.05$.

Model	Unstandardized Coefficients		Standardized Coefficients	T	Sig.	Correlations			Collinearity Statistics	
	B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
(Constant)	8.493	.968		8.770	.000					
Proportion	.495	.606	.052	.817	.415	.075	.050	.049	.893	1.120
Controlling	.611	.691	.061	.884	.377	.095	.054	.054	.767	1.305
Probabilistic	.295	.863	.024	.342	.732	.064	.021	.021	.749	1.336
Correlational	.165	.793	.013	.208	.835	.029	.013	.013	.986	1.014
Combinatorial	**1.232**	**.530**	**.141**	**2.323**	**.021**	**.145**	**.142**	**.141**	**.989**	**1.011**

a. Dependent Variable: Mathematics.

Table 3: Coefficients of Prediction between Mathematics and the Five Modes of Logical Reasoning

Table 3 indicates that combinatorial reasoning, with the highest t-value of 2.323 (Sig= .021), is the only significant predictor of performance in mathematics. The Tolerance and VIF values show that there is no multicollinearity in the multiple linear regression.
Regression Equation: *Mathematics (Y) = 8.493 + 1.232 combinatorial.*

		Science
Logical Reasoning	Pearson Correlation	.130*
	Sig. (2-tailed)	.012

*Table 4: Relationship between Logical Reasoning Abilities and JHS Students' Performance in Science *. Correlation is significant at the 0.05 level (2-tailed)*

From the table, there is a positive relationship *r* = .130 between logical reasoning and the performance of students in science. The result is statistically significant because Sig. (2-tailed) p = .012. Therefore, the null hypothesis is rejected because $p < 0.05$.

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
	B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
(Constant)	8.054	.759		10.608	.000					
Proportion	-.312	.475	-.041	-.657	.512	.000	-.040	-.039	.893	1.120
Controlling	.496	.542	.062	.915	.361	.108	.056	.054	.767	1.305
Probabilistic	.792	.676	.080	1.171	.243	.094	.072	.069	.749	1.336
Correlational	.709	.622	.068	1.140	.255	.091	.070	.067	.986	1.014
Combinatorial	1.647	.416	.236	3.964	.000	.240	.237	.234	.989	1.011

Dependent Variable: science. Source: Field data, (2018)

Table 5: Coefficients of the Prediction between Science and the Five Modes of Logical Reasoning

Table 5 indicates that combinatorial reasoning, with the highest t value of 3.964, is the only significant predictor of performance in science. The Tolerance and VIF values show that there is no multicollinearity in the multiple linear regression.
Regression equation: Science (Y) = 8.054 + 1.647 combinatorial.
RQ1. What is the level of logical reasoning abilities of students?

The purpose of this research question was to examine the level of students' logical reasoning abilities. As done by Oliva (2003), the scores on logical reasoning were categorized into 0-3 (low-level logical reasoning ability), 4-7 (medium-level logical reasoning ability) and 8-10 (high-level logical reasoning ability). The results show that the logical reasoning ability of JHS 2 students in the Ga South Municipality was low-level logical reasoning because their mean score was **2.29** (SD = .912), which apparently falls within the low-level logical reasoning ability. This result is consistent with the work of Healy and Hoyles (2000) and Knuth, Choppin, and Bieda (2009), who identified that students had difficulty in reasoning with logical implications. This result is evident because students were not exposed to logical training to enhance their logical reasoning. This argument corroborates the submission of Anamuah-Mensah, Mereku and Asabere-Ameyaw (2004) that the Ghanaian curriculum – textbooks, syllabus and assessment placed a great deal of emphasis on the number of work and knowledge of facts and procedures but less on logical reasoning. This result is also consistent with Lee (2011), who acknowledged students' difficulty in logical reasoning and recommended that students should be exposed to logical training and counterexamples in mathematics education in order to enhance their logical reasoning abilities.

3.1. Hypothesis 1

- H_0: There would be no significant relationship between logical reasoning abilities and JHS 2 students' performance in mathematics.

The purpose of this hypothesis was to establish the relationship between logical reasoning abilities and JHS 2 students' performance in mathematics. To test this hypothesis, the Pearson Product Moment Correlation Coefficient statistic was adopted. The findings indicate a significant positive weak relationship (r = .105, Sig= .044) between JHS 2 Students and performance in mathematics. Thus, the more a student is able to reason logically, the likelihood that he or she will perform well in mathematics. This result is consistent with the works of Heng–Yuku and Sullivan (2000), Choudhury and Das (2012), and Nunes, Byrant, Barnes, and Sylva (2012), which revealed a significant positive relationship between logical reasoning ability and attainment in mathematics. Cantu and Herron (1978) and Goodstein and Howe (1978) also revealed that formal reasoning ability (logical reasoning) is a reliable indicator of successful achievement in mathematics and science. Su, Ricci, and Mnatsakanian (2016) mentioned that a teacher who attempts to emphasize reasoning, logic, and validity gives students access to mathematics as an effective way of practising critical thinking.

3.2. Hypothesis 2

- H0: The five modes of logical reasoning will not predict JHS 2 students' performance in mathematics.

The aim of this hypothesis was to find out whether the five modes of logical reasoning can predict JHS 2 students' performance in mathematics. To test this hypothesis, multiple linear regression statistic was employed. The results revealed that, among all the modes of logical reasoning, combinatorial reasoning was the best significant predictor (t value of 2.323, Sig= .021) of performance in mathematics. This result is contrary to the findings of Ongcoy (2016), who reported that probabilistic reasoning was the best predictor of students' performance, with proportional reasoning and combinatorial reasoning having less predictability.

3.3. Hypothesis 3

- H_0: There would be no significant relationship between logical reasoning abilities and JHS 2 students' performance in science.

The purpose of this hypothesis was to establish the relationship between logical reasoning abilities and JHS 2 students' performance in science. To test this hypothesis, the Pearson Product Moment Correlation Coefficient statistic was used. The findings indicate a significant positive weak relationship (r = .130, Sig= .012) between JHS 2 Students and performance in science. In other words, the more a student is able to reason logically, the likelihood that he or she will perform well in science. The result agrees with Lawson, Banks and Logvin's (2007) work that the students' reasoning abilities have been established as an important factor of science achievement. Sungur and Tekkaya (2003) also reported a significant effect of reasoning abilities on biology achievement. Even though the relationship identified in this study between logical reasoning and performance in science was a weak positive relationship, Lawson, Alkhoury, Benford, Clark, and Falconer (2000) concluded that students' ability to reason logically has been found to be the strongest predictor of meaningful understanding in science.

3.4. Hypothesis 4

- H_0: The five modes of logical reasoning will not predict JHS 2 students' performance in science.

The purpose of this hypothesis was to find out whether the five modes of logical reasoning can predict students' performance in science through multiple linear regression. From the results, combinatorial reasoning was the significant best predictor of performance in science (t value of 3.964, Sig= .000). This finding is contrary to the results of Bird (2010), who revealed that most students show significant deficiencies in proportional, probabilistic and correlational reasoning and among the logical reasoning modes, probabilistic reasoning is the single best predictor of student performance in general chemistry.

4. Recommendations

Based on the findings of this study, the following recommendations are made to develop the logical reasoning ability of JHS 2 students. The Curriculum Research and Development Division (CRDD) should design the school curriculum in mathematics and science at the JHS level to deliberately train students on logical reasoning because logical reasoning training has its place in developing students' mathematics and science learning. Teachers should consider logical training in mathematics and science classrooms because a number of mathematical and scientific concepts can be related to logical implications. Once students are exposed to logical reasoning training, it will enhance their productive use of deductive inferences, which will support their mathematical and scientific justifications of concepts and relationships in class. Students should be encouraged to discover knowledge on their own and move from rote learning to meaningful learning, which can help improve their thinking skills.

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