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# Comparative Analysis of Activity Duration in Project Management

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***Abstract:***
*The paper attempt to compare beta distribution against triangular distribution in order to test the efficacy of each distribution. The distribution that achieve optimum completion time is most beneficial. Project evaluation review technique and critical path method are procedures employed in computing completion time of the entire project activities in project management. Micro soft QM computer software version 2.0 is use to attained these computations. Results were obtained and compared one against another. Network analysis is also utilized in finding the minimal expected completion time of the entire project activities. Findings establishes that triangular distribution has the minimum completion time. Therefore, the conclusion indicates that triangular distribution is the best fit to be selected. To the best of our knowledge, the paper is the first to compare two distinct approach in scheduling project activity duration. The results will pave way to potential users of this tools in future.*

***Keywords:*** *Network analysis, Statistical analysis, project management*

## 1. Introduction

Critical Path Method, commonly abbreviated CPM, is a technique for scheduling a set of project activities in project management. CPM also can be used to determine how long each activity in the project can be delayed without delaying the completion of the project. CPM was developed in the late 1950s by researchers at DuPont and Sperry Rand (Winston, 2004). On the other hand Project Evaluation Review Technique, commonly abbreviated PERT was developed in the late 1950s by consultants working on the development of the Polaris missile. CPM assume activity duration is known with certainty.

However, If the duration of the tasks is not known with certainty, PERT can be utilized to estimate the probability that the project will be completed by a given dateline. PERT is an attempt to correct the shortcoming of CPM by modeling activity duration as a random variable. In PERT the task durations are independent and identically distributed. Hence the central limit theorem (Solberg, 2000), the project duration follows a normal distribution with mean $\mu$ and variance $\sigma 2$. (Winston, 2004) maintained that PERT activity duration may not necessarily follow beta distribution. This research work attempt to compare beta distribution and triangular distribution using a real time data in order to validate these assertions. The significance of this research work is that, it is useful in the planning and scheduling of large complex project that involves many interrelated activities in construction of highways, bridges and building of house for instances.

The paper is organized as follows: Literature is reviewed in section 2. Section 3, presents conceptual framework. Results are presented in section 4. Finally, conclusion is drawn in section 5.

## 2. Literature Review

Triangular distribution originated a century after the discovery of beta distribution (Simpson, 1755). (Johnson, 1997) examine triangular distribution as a proxy for the beta distribution since beta has to increase the weight by four times to that of triangular. This made beta distribution most suitably preferred to project that were executed in the past. The aim of the paper is to find a suitable alternative of the beta distribution. (Njuko, 2016) study the beta distribution along with triangular distribution, applied both distribution on a small firm in order to validate these models. The results obtain from computation shows that beta distribution have the same expected completion time with triangular distribution.

## 3. Conceptual Framework

Project management is the application of skill, knowledge expertise, tools and techniques to project activities that satisfy project requirements (Haseena and Shaheer, 2017). CPM and PERT are project management tools for planning, monitoring, execution, and control of project activity duration in order to ensure timely completion of project. Beta distribution and Triangular distributions are used in project management as tools like the CPM and PERT to estimate project completion time based on maximum and minimum values. However, Beta distribution requires a considerable amount of data to describe a population's distribution (Zou and Normand, 2001).

*3.1. Project Evaluation Review Technique*

PERT is a statistical tool, employed in project management, that was designed to analyze and represent the activities involved in completing a given project, especially the time needed to complete each task and to identify the minimum time needed to complete the total project (Hillier and Lieber-man, 2010).

*3.2. Beta Distribution Density*

Beta distribution has been used to model the behavior of random variable limited to intervals of finite length (Taha, 2011). It is applied extensively in PERT to describe the time to complete a task (Brighton Webs Limited, 2017).

Letting:

min = Minimum time estimate (Optimistic)
mode = Most Likely time estimate
max = Maximum time estimate (Pessimistic)

Beta probability distribution function is given by:

$$f(x) = \frac{(x - min)^{(shape_{a-1})} * (max - x)^{(shape_{b-1})}}{B(shape_a, shape_b) * (max - min)^{(shape_a + shape_{b-1})}}$$

where,

$$shape_a = \Big(\frac{mean - min}{max - min}\Big)\Big(\frac{(mean - min)/(max - mean)}{stdev^2} - 1\Big)$$

$$shape_b = \Big(\frac{max - mean}{mean - min}\Big)shape_a$$

$$E(x) = \frac{(min + 4mode + max)}{6} \quad \text{(expected time estimate)}$$

$$var(x) = \frac{(max - min)^2}{36} \quad \text{(Variance time estimate)}$$

$$\sigma = \frac{(max - min)}{6} \quad \text{(Standard deviation)}$$

*3.3. Triangular Distribution Density*

Triangular distribution, along with the beta distribution, is also widely used in project management to model events that take place within an interval defined by a minimum and maximum value (Carnell, 2016). It is based on estimates of the minimum and maximum and an inspired guess as to what the modal value might be. One way of obtaining the minimum, mode and maximum parameters is to ask the opinion of someone with appropriate experience.

Letting:

a = Minimum time estimate
c = Most Likely time estimate
b = Maximum time estimate

Triangular probability distribution function is given by:

$$f(x) = \begin{cases} 0, & \text{for} \quad x < a \\ \frac{2(x-a)}{(b-a)(c-a)}, & \text{for} \quad a \le x \le c \\ \frac{2(b-x)}{(b-a)(b-c)}, & \text{for} \quad c \le x \le b \\ 0, & \text{for} \quad x > b \end{cases}$$

$$E(x) = \frac{(a+b+c)}{3} \quad \text{(expected project completion time)}$$

$$var(x) = \frac{(a^2 + b^2 - ab - ac - bc)}{18} \quad \text{(Variance time estimate)}$$

$$\sigma = \sqrt{var(x)} \quad \text{(Standard deviation)}$$

**4. Results and Discussion**

A computer software known as QM for windows version 2.0 is utilized to solve the formulated program on a I7 Intel core processor personal computer.

The project is expected to be completed in 59.08 weeks with 50% chance.

The probability that the project will be completed four weeks sooner than expected is 3.44%, while the probability that the project will be completed three weeks later than expected is 91.31%.

The possibility that the project will be completed between 56.88 weeks and 61.28 weeks is 68.26%, while the probability that the project will be completed between 54.68 weeks and 63.48 weeks is 95.44% and the possibility that the project will be completed between 52.48 weeks and 65.68 weeks is 99.74%. The amount of variation in the project duration is 2.2 weeks.

Table 1 depict the relative analysis between beta distribution and triangular distribution. The total durations of individual critical activities is equal to the sum up expected completion time of the entire project. This will be equal to 59.52 weeks for Beta and for Triangular is 59.08 weeks. The difference is 0.44 weeks which translate to more than 3 days. This means that

| S/N | Description | Beta | Triangular |
|---|---|---|---|
| 1 | Expected project completion time | 59.52 | 59.08 |
| 2 | Variance | 4.84 | 8.65 |
| 3 | Standard Deviation | 2.20 | 2.94 |
| 4 | Previous history of similar project | Data available | No prior data |
| 5 | Execution of similar project | Many time | First time |

*Table 1: Comparative analysis between beta and triangular*

the project will be completed more than three days behind schedule when triangular is used for computation. The variation in the duration shows that triangular is 8.65, while beta is 4.84, from these figures we can infer that triangular has more variation than beta. Standard deviation measure the amount of variation in the duration, we see from beta distribution has 2.2 standard deviation, while triangular has 2.94. This clearly demonstrate that triangular has more amount of variation than beta distribution.

Figure 1 show a visual representation of the relationship between beta and triangular distributions. The outer curve is the beta plot while the inner curve is the triangular plot. The shape of a triangle is clearly seen.

Table 2 was extracted from (Aliyu, 2013) it show the data used in the comparison in addition to that fifteen comparison were made with different data.

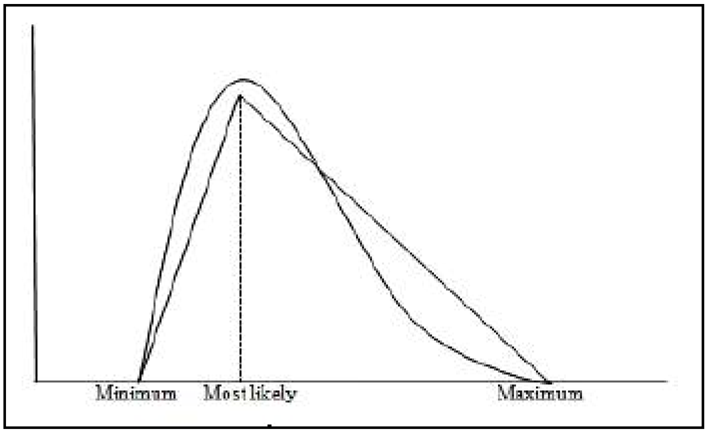


*Figure 1: Comparison between Beta and Triangular distribution*

| Serial Number | Arc (i,j) | Minimum (a) | Likeliest (c) | Maximum (b) |
|---|---|---|---|---|
| 1. | (1,2) | 1 | 2 | 3 |
| 2. | (2,3) | 2.1 | 3.4 | 4 |
| 3. | (2,4) | 2 | 4 | 6.1 |
| 4. | (3,4) | 0 | 0 | 0 |
| 5. | (4,5) | 1.9 | 5 | 6 |
| 6. | (5,6) | 1.3 | 2 | 3 |
| 7. | (5,7) | 1.4 | 4 | 7 |
| 8. | (6,7) | 0 | 0 | 0 |
| 9. | (7,8) | 3 | 4.1 | 5 |
| 10. | (8,9) | 2 | 4.3 | 6 |
| 11. | (8,10) | 3 | 5 | 7 |
| 12. | (9,10) | 0 | 0 | 0 |
| 13. | (10,11) | 2.1 | 4.2 | 6 |
| 14. | (11,12) | 3 | 4 | 5 |
| 15. | (11,14) | 3 | 4 | 8 |
| 16. | (12,13) | 1.1 | 3 | 5 |
| 17. | (13,14) | 1.3 | 2 | 3 |
| 18. | (14,15) | 2 | 3 | 4 |
| 19. | (15,16) | 1 | 3 | 5 |
| 20. | (15,17) | 3 | 4 | 5 |
| 21. | (16,17) | 0 | 0 | 0 |
| 22. | (17,18) | 1.1 | 2.2 | 3.3 |
| 23. | (18,20) | 3 | 4 | 5 |
| 24. | (18,19) | 2 | 3 | 4 |
| 25. | (19,20) | 0 | 0 | 0 |
| 26. | (20,21) | 2 | 4 | 6 |
| 27. | (21,22) | 1.2 | 2.9 | 3.8 |
| 28. | (22,23) | 1.8 | 2.2 | 3.6 |

*Table 2: Time Estimate in weekly basis*

**5. Conclusion**

Triangular distribution and beta distribution were compared one against another subsequently, triangular distribution has the optimum completion time. Hence it was selected as the best distribution, with 50% probability of completion. The expected completion time is 59.08 weeks since the activity duration is in weekly basis. In future delphi method can be used to estimate activity duration in triangular distribution, employing this method could fetch a better results. Preferably be applied to a large complex project that has activities in the range of thirty and above.

**6. References**


i. Aliyu, A. M. (2013). Project management using cpm. a pragmatic application. Global Journal of Pure and Applied Sciences. 18(3): 197–206.
ii. Brighton Webs Limited (2017). Statistics for energy and the environment. Retrieved on May 19, 2017 from http://www.brighton-webs.co.uk/distribution/Beta.htm.
iii. Carnell, R. (2016). Provides the Standard Distribution Functions for the Triangle Distribution. Retrieved on January 21, 2016 from http://www.brighton-webs.co.uk/distribution/triangular.htm.
iv. Haseena, V. and Shaheer, K. (2017). A literature review on project management system. International journal of science and research. 6(5): 2349–2352.
v. Hillier, F. S. and Lieberman, G. J. (2010). Introduction to Operations Research. 9th Edition. Boston: McGraw-Hill.
vi. Johnson, D. (1997). The triangular distribution as a proxy for the beta distribution in risk analysis. Statistician. 46(1): 387–398.
vii. Njuko, A. E. (2016). Application of Project Evaluation Review Technique: A Case Study of Gessadaddo farms, Yola. Modibbo Adama University of Technology, Yola: Master's Thesis.
viii. Simpson, T. (1755). A novel method for fittingunimodal continuous distributions. Technical Report. John Hopkins University Press.
ix. Solberg, R. P. (2000). Operations Research: Principle and Practice. 2nd Edition. New York: John Wiley and Sons, Inc.
x. Taha, H. A. (2011). Operations Research an Introduction. 9th Edition. New Jersey: Upper Saddle River.
xi. Winston, W. L. (2004). Operations Research: Applications and Algorithms. 4th Edition. U.S.A: Thomson.
xii. Zou, K. H. and Normand, S. L. T. (2001). On determination of sample size in hierarchical binomial models. Statistician. 20(1): 2163–2182.