THE INTERNATIONAL JOURNAL OF BUSINESS & MANAGEMENT

An Analysis of Domar's Interpretation of Debt Sustainability with Reference to Sub National Government Finances of India

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Abstract:
Public debt and its sustainability is an important issue and the literature envisage that the concept behind public debt and its sustainability has been formulated and developed by many researchers in the context of national and sub national level debt analysis. In the above context, this paper is an attempt to analyze the Domar Model of debt sustainability in details with reference to India. It observes that to achieve, solvency and sustainability the growth rate of public debt should be less then equal to Interest rate and interest rate should be less then Growth rate of GSDP ($k \leq r < g$).

Keywords: *Debt Sustainability, Domar Model, India*
JEL Classification: H6

1. Introduction

Public debt and its sustainability is an important issue and the literature envisage that the concept behind public debt and sustainability has been formulated and developed by many researchers in the context of national and sub national level debt analysis. Though, the analysis of debt of national level and sub national levels are different in the context of calculation of debt, the core concept is same. In the context of India, public debt has been interpreted in terms of Keynesian and Neoclassical approach by many researchers for example Buter and Patel (1992), Patnaik (1996), Lahiri and Kananan (2001), Acharya (2002) and Ahluwalia (2002). Similarly, at the sub national / state level Burnside (2004), Dholakia *et al*. (2004), Rath (2005), Rangarajan and Srivastava (2005), Rajaraman *et al*. (2005), Elena and Nagrajan (2007) and Nayak and Rath (2010) are notable. These studies are based on the neo-classical approach and the basis of sustainability analysis is Domar sustainability conditions[1].In the context of special category states it is argued that since these states are special category states and their developmental expenditure is basically financed with the help of special assistance from the center and the private investment is negligible, the public expenditure plays a crucial role in the growth of Gross State Domestic Product(GSDP) (Sarma and Nayak, 2006) .Thus, growth mainly comes from government spending. In a situation where the primary expenditure is compressed, automatically the victim is the developmental capital outlay, which ultimately results in reduction in growth (Nayak and Rath, 2010). **S**uch condition of special category states conforms to the neoclassical approach and Domar model of debt sustainability conditions.

In the above context, this paper is an attempt to analyze the Domar Model of debt sustainability in details with reference to Sub national government finances of India.

2. The Concept of Public Debt Sustainability

2.1. Public Debt and Deficits

Public debt, D, is defined as the unredeemed face value of accumulated stock of government non-monetary financial liabilities. It is normalized by the GDP (Gross Domestic Product) for the national government debt. In the context of sub national government this can be normalized by GSDP (Gross State Domestic Product).
Thus, in year t:

$$d_t = \frac{D_t}{GSDP_t}$$

The special (and simplifying) features of sub national public debt are that there are no possibilities of seignorage or external

[1] See Burnside (2004), Lancchovichina, Lili Liu and Mohan Nagrajan (2007), Nayak and Rath (2010)

borrowing, and by extension, no need to obtain that subset of public debt that corresponds to non-monetized debt. The dynamics of debt in the context of state governments in India can thus be stated very simply. The debt stock is added to by the fiscal deficit incurred in each year, thus:

$$d_t = \frac{D_{t-1}}{GSDP_{t-1}(1+n)} + f_t \ldots\ldots\ldots\ldots\ldots\ldots Eq-(1)$$

Where in year t,
n = nominal rate of growth
f= fiscal deficit / GSDP

In the above equation two points are important. First, the rate of growth of GSDP (n) is nominal and not the real rate of growth and secondly, the fiscal deficit (f) is the net new borrowing in each period (net of repayment of past debt).

The fiscal deficit is the excess of total expenditures over total non-debt receipts. Non debt receipts are the sum of revenue receipts on the current account and receipts if any from sale of government assets on the capital account. So from Equation (1), for constant values of 'f' and 'n', the debt ratio will stabilize at the level food', thus

$$d = \frac{f(1+n)}{n}$$

To illustrate, if there is a 3 percent limit on the fiscal deficit as a percent of GDP, following the Maastricht Treaty imposed by consensus on (EU) member countries, and if 'n' is 10 percent, d will stabilize at 33 percent of GSDP. States with higher starting values of'd' will decline to this level, but the decline will be asymptotic, stretching up to infinity.
A state with debt at 40 percent of GSDP will reach 34 percent after 20 years. This is important to remember. Stabilizing the fiscal deficit at some percent of GSDP will not immediately stabilize debt/GSDP at the corresponding level.
There is no theoretical basis for designating any particular debt/GSDP value as superior or preferable to any other. The bound on debt/GSDP is imposed by the fact that tax revenues are costly to raise, as tax administration officials the world over will testify, and always politically difficult. Thus, sustainability is closely related to revenue-raising capacity, which again is a state-specific matter, and could in principle be crossed for a state with weak revenue-raising capability even at very low levels of debt/GSDP.

2.2. Solvency, Stability and Sustainability

"Solvency" refers to positive net worth, while sustainability refers more to the sufficiency of liquid assets to meet current or committed obligations. In the public debt context, the term sustain ability embodies concern about the ability of the government to service its debt. A government which does not generate enough current revenues for debt service must either default on its obligations, or borrow more in order to service past debt as well as to cover its ongoing imbalances. Continual borrowing of this kind is known as a Ponzi game, and will show up in the time path of the debt/GSDP ratio. Clearly solvency and sustainability are closely related, in that an unsustainable time path will ultimately threaten the solvency of the state
It follows from the above that the time path of debt/GSDP is an important indicator of the sustainability status of the state. Where debt/GSDP shows signs of stabilizing at some particular level, the debt path for that reason is seen as more sustainable independently of the level at which the debt ratio has been stabilized, because stabilization is in itself an indicator of fiscal control. What matters critically from a sustainability perspective is whether debt is being added to over time. Therefore, in what follows, the conditions that lead to stabilization of debt/GSDP will be defined. It should be remembered that even where the debt is stable as a percent of GSDP, it is still increasing in absolute terms over time in proportion to GSDP. Stabilization is always understood to mean a constant percent of the gross domestic product, which serves as a measure of the debt carrying capacity of the economy. Where the debt stock is stable in absolute terms, it will decline over time as a proportion to GSDP.

2.3. Interpretation on the Basis of Domar Model of Debt Sustainability[2]

Domar's analysis is the basis of studying debt sustainability by many researchers[3].Continuing the discussion here we are following the same approach of Domar (1944) to analyze the concept of solvency, stability and sustainability.

2.3.1. Condition for Solvency and Sustainability of Public Debt

According to Domar analysis (1944), the inter-temporal financing constraint of the sub-national government can be written as:

$$Def_t^p = D_{t-1} - r(D_{t-1}) \quad \text{Eq. (2)}$$

Where,
Def_t^p = Primary deficit in time *"t"*
D_t= Debt stock in time *"t"*
D_{t-1}= Debt stock in time *"t-1"*

Here, in order to attain solvency, present outstanding stock of public debt (D_0) must be equal to the sum of the discounted primary

[2] Please see Nayak and Rath (2010) for details
[3] Please see Burnside (2004), Ianchovichina, Lili Liu and Mohan Nagrajan (2007) , Nayak and Rath (2010) and Pattnaik (1996)

surplus of future years. This can be interpreted as:

$$D_0 = -\sum_{t=1}^{\infty} \frac{Def_t^p}{(1+r)^t} \quad \text{..... (Eq-3)}$$

Simplifying Equation (3), we can write

$$D_t - D_{t-1} = Def_t^p + r(D_{t-1})$$

But $D_t = (1+k)D_{t-1}$ (Where, "k" is the growth rate of public debt)

$$\Rightarrow D_{t-1} = (1+k)^{-1} D_t$$

$$\Rightarrow (1+k)D_{t-1} - D_{t-1} = Def_t^p + r(D_{t-1})$$

$$\Rightarrow D_{t-1}(1+k-1) = Def_t^p + r(D_{t-1})$$

$$\Rightarrow kD_{t-1} - r(D_{t-1}) = Def_t^p$$

$$Def_t^p = (k-r)\,D_{t-1} \quad \text{.....(Eq-4)}$$

Substituting Equation-4 in Equation-3 we can rewrite as

$$D_0 = -\sum_{t=1}^{\infty} \frac{(k-r)D_{t-1}}{(1+r)^t}$$

$$\Rightarrow D_o = (r-k)\sum_{t=1}^{\infty} \frac{D_{t-1}}{(1+r)^t} \quad \text{.....(Eq-5)}$$

Equation-5 can be deduced for debt stock Do= 0[the necessary condition is, $r = k$ (i.e. interest rate must be equal to growth rate of debt stock)]

From Equation-2 we can write that

$$D_t = Def_t^p + D_{t-1} + r(D_{t-1})$$

$$\Rightarrow D_t = Def_t^p + (1+r)D_{t-1}$$

$$\Rightarrow D_t = (1+r)D_{t-1} + Def_t^p \quad \text{.....(Eq-6)}$$

By dividing both sides of the equation by $GSDP_t$ (Gross Domestic Product in time, t), Equation (6) can be written as:

$$\frac{D_t}{GSDP_t} = \frac{(1+r)D_{t-1}}{GSDP_t} + \frac{Def_t^p}{GSDP_t} \quad \text{.....(Eq-7)}$$

But, $GDP_t = (1+g)GSDP_{t-1}$, (Where g is the growth rate of GSDP)

Thus Equation-7 can be written as:

$$\frac{D_t}{GSDP_t} = \frac{(1+r)D_{t-1}}{(1+g)GSDP_{t-1}} + \frac{Def_t^p}{GSDP_t}$$

$$\Rightarrow d_t = \left(\frac{1+r}{1+g}\right) d_{t-1} + P_t^p \quad \text{.....(Eq-8)}$$

$$Where, d_t = \frac{D_t}{GSDP_t}, d_{t-1} = \frac{D_{t-1}}{GSDP_{t-1}}, P_t^p = \frac{Def_t^p}{GSDP_t}$$

Since the targeted value of P_t^p is a constant term, Equation-8 is a first order difference equation. By solving equation-8 can be written as:

$$d_t = \left(\frac{1+r}{1+g}\right)^t d_0 - p_t^p \left(\frac{1+r}{1+g}\right)^t \left(\frac{1+g}{g-r}\right) + p_t^p \left(\frac{1+g}{g-r}\right) \quad \text{.....(Eq-9)}$$

Thus in the long run when $t \to \infty$, d_t will tend to $P_t^p\left(\frac{1+g}{g-r}\right)$ only when

$$P_t^p \left(\frac{1+r}{1+g}\right)^t \to 0$$

$$\Rightarrow 0 \le \frac{1+r}{1+g} < 1$$

$$\Rightarrow 1+r<1+g$$

$$\Rightarrow r<g$$

In other words, interest rate must be less then annual growth rate of GSDP. Therefore, solvency and sustainability of public debt $k \le r < g$ .It means when a state government is running by accumulating primary deficit, Growth rate of public debt $\le$ Interest rate < Growth rate of GSDP

To determine the impact of inflation on the above process Equation-9 can be used after calculating "r" and "g" in real terms. " g" is computed after dividing the GSDP at current price with GSDP deflator.

$$\text{The real "r"} = \left(\frac{R+1}{\wedge+1}\right)-1$$

Where "r" stands for real interest rate, "g" stands for real growth rate and "^" is inflation rate. Thus by using Equation-8 and by using the real growth and interest rate we can forecast the future debt-GSDP.

The above Equation-8 can be expressed as:

$D_t = (r\text{-}g)\, d_{t\text{-}1} + P_t^p$

This can be presented through the following diagram

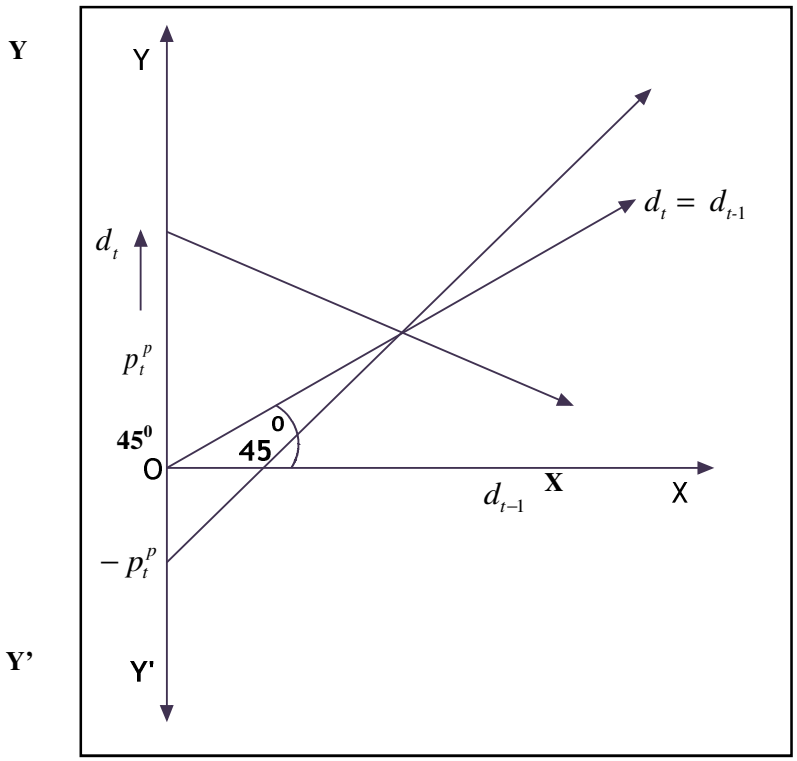

Figure 1: Debt Sustainability

The above diagram explains the condition of public debt sustainability by the following two conditions:

i. When Primary deficits exists, r< g for sustainability of public debt.
ii. When there is primary surplus even if r>g there will be public debt sustainability.

2.3.2. Relationship between Gross Fiscal Deficit and Debt to GSDP Ratio

Introducing Fiscal deficit (FD), the inter-temporal financing constraint of the sub-national government can be written as:

$$Def_t^p = D_t - D_{t-1} - r(D_{t-1})$$

$$FD_t = D_t - D_{t-1} \quad \text{.....(Eq-10)}$$

Where,

$FD_t \rightarrow$ Fiscal Deficit in the year "t"

D_t And $D_{t-1} \rightarrow$ Outstanding Debt in the year "t"

$$\Rightarrow D_t = D_{t-1} + FD_t$$

$$\Rightarrow \frac{D_t}{GSDP_t} = \frac{D_{t-1}}{GSDP_t} + \frac{FD_t}{GSDP_t}$$

$$\Rightarrow \frac{D_t}{GSDP_t} = \frac{D_{t-1}}{(1+g)GSDP_{t-1}} + \frac{FD_t}{GSDPt} \quad \text{.....(Eq-11)}$$

This can be generalized in a static form as:

$$\frac{D}{GSDP} - \frac{D}{(1+g)GSDP} = \frac{FD}{GSDP}$$

$$\frac{D}{Y}\left[1 - \frac{D}{(1+g)GSDP}\right] = \frac{D}{Y}\left[\frac{g}{1+g}\right] = \frac{FD}{Y}$$

$$\Rightarrow \frac{Debt}{GSDP} = \left(\frac{1+g}{g}\right) * \left(\frac{FiscalDeficit}{GSDP}\right) \text{.....} (Eq-12)$$

Further, equation-10 can be manipulated for Debt for different time like D1, D2, and D3 as:

$$FD_t = D_t - D_{t-1}$$

$$D_t = D_{t-1} + FD_t$$

$$D_t = D_{t-1} + FD_t$$

$$\Rightarrow D_1 = D_0 + FD_1$$

$$\Rightarrow D_2 = D_1 + FD_1 \text{ [When Fiscal Deficit (FD) remains constant]}$$

$$\Rightarrow D_2 = D0 + FD_1 + FD_1$$

$$\Rightarrow D_2 = D_0 + 2FD_1$$

So in general, if the Fiscal deficit is constant term over the entire period it can be written as:

$$\Rightarrow D_t = D_0 + t(FD_0)$$

By dividing both side of the above equation by $GSDP_0$

$$\frac{D_t}{GSDP_0} = \frac{D_0}{GSDP_0} + \frac{t(FD_0)}{GSDP_0}$$

$$\Rightarrow \frac{D_t}{GSDP_0}(1+g)^t = \frac{D_0}{GSDP_0} + \frac{t(FD_0)}{GSDP_0} \text{.....} (Eq-13)$$

Where 'g' is the growth rate of GSDP and "t" is the time.

2.3.3. Incremental Debt-GSDP Ratio

As it is mentioned above in Equation-2, the inter-temporal financing constraint of the sub-national government can be written as:

$$Def_t^p = D_t - D_{t-1} - r(D_{t-1})$$

$$\Rightarrow D_t - D_{t-1} = Def_t^p + r(D_{t-1})$$

By dividing both side by $GSDP_t$

$$\Rightarrow \frac{D_t}{GSDP_t} - \frac{D_{t-1}}{GSDP_t} = \frac{Def_t^p}{GSDP_t} + \frac{rD_{t-1}}{GSDP_t}$$

But, $GDP_t = (1+g)GSDP_{t-1}$, (Where g is the growth rate of GSDP)

So substituting $GDP_t = (1+g)GSDP_{t-1}$, the above equation can be written as

$$\Rightarrow \frac{D_t}{GSDP_t} - \frac{D_{t-1}}{(1+g)GSDP_{t-1}} = \frac{Def_t^p}{GSDP_t} + \frac{rD_{t-1}}{(1+g)GSDP_{t-1}}$$

$$\Rightarrow \frac{D_t}{GSDP_t} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{(1+g)GSDP_{t-1}} + \frac{rD_{t-1}}{(1+g)GSDP_{t-1}}$$

$$\Rightarrow \frac{D_t}{GSDP_t} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{GSDP_{t-1}}\left(\frac{1}{1+g} + \frac{r}{1+g}\right)$$

$$\Rightarrow \frac{D_t}{GSDP_t} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{GSDP_{t-1}}\left(\frac{1+r}{1+g}\right)$$

Now by subtracting $\frac{D_{t-1}}{GSDP_{t-1}}$ form both side of above equation

$$\Rightarrow \frac{D_t}{GSDP_t} - \frac{D_{t-1}}{GSDP_{t-1}} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{GSDP_{t-1}}\left(\frac{1+r}{1+g}\right) - \frac{D_{t-1}}{GSDP_{t-1}}$$

$$\Rightarrow \frac{D_t}{GSDP_t} - \frac{D_{t-1}}{GSDP_{t-1}} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{GSDP_{t-1}}\left(\frac{1+r}{1+g} - 1\right)$$

$$\Rightarrow \frac{D_t}{GSDP_t} - \frac{D_{t-1}}{GSDP_{t-1}} = \frac{Def_t^p}{GSDP_t} + \frac{D_{t-1}}{GSDP_{t-1}}\left(\frac{r-g}{1+g}\right)$$

This equation shows that the left hand side gives us the incremental Debt- GSDP ratio.

5. Conclusion

From the above discussion, it can be concluded that the concept of debt sustainability refers to the ability of the government to service its debt obligations and solvency requires that with a finite time horizon, public debt in the last period becomes non-positive which implies that present value of future debt should be zero. To achieve, solvency and sustainability the growth rate of public debt should be less then equal to Interest rate and interest rate should be less then Growth rate of GSDP ($k \leq r < g$).

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