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Covered Interest Rate Arbitrage Parity (CIRAP) and 'Efficiency' of Foreign Exchange (Rupee/Dollar) Market in India – Time Series Econometric Study with Forward Rates and ARIMA Forecasts

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Abstract:
The study is devoted to examining the 'efficiency' of Indian foreign exchange (rupee/dollar) market and the relevance of Covered Interest Rate Arbitrage Parity (CIRAP) doctrine therein over the period 11th November, 2011- 27th February, 2015. ARIMA (4, 1, 0) stochastic structure of monthly spot rate (S_t) has been used to generate one – period ahead forecast (S_{t+1}^e) series. These forecasts are MMSE forecasts and 'Rational' by nature. Forward rates (tF_{t+1}) served as the 'Unbiased predictor' of the spot rate (S_{t+4}) implying that CIRAP did hold well in the market. Again absence of 'risk premium' testifies for the 'efficiency' of the Indian foreign exchange (rupee / dollar) market over the period of study.

Keywords: *ARIMA Forecasts, MMSE Forecasts, 'Rational' Forecasts, CIRAP Doctrine, Risk Premium, Forward Exchange Rate, Efficient Market*

1. Introduction

The decade of 1970s brought in a turning point in the realm of international economics and finance. The Britton Woods System broke down and flexible exchange rate system replaced the fixed exchange rate system in 1970s. Determination of exchange rates became the centre-price of deliberations in international economics while the management of balance of payments became almost a non-entity. Consequently, over the last three decades a large number of theories on exchange rate grew up. On the other hand, the issues of dynamic adjustment of balance of payments were relegated to the background.

The most exciting feature of this period is the growth of renewed interest of economists in the 'Interest Arbitrage Parity Doctrine 'and thus the '*Interest Rate Arbitrage Parity Theory*' has emerged as an influential theory of the determination of the exchange rate since 1970s.

The '*Interest Rate Arbitrage Parity Theory*' is theoretically attractive but the empirical support for the theory is mixed. Yet the research on this subject is extensive. Which indicates that these exists a reluctance for rejecting the theory, at least in the short run. The present study is an attempt in this direction with an objective of examining how far the *Rupee/Dollar* exchange rates conformed to the '*Interest Rate Arbitrage Parity Doctrine*' over the period (*11th November, 2011 to 27th February, 2015*).

2. Review of Literature

Fama, Eugene F. Journal (1984), reported empirical evidence in favor of Covered Interest Parity and the existence of risk premium in the forward market. Frankel, Jeffrey A, Froot, Kenneth (1990), found some role of UIRP in the dynamics of the foreign exchange market. Guy Meredith, Menzie D. Chinn, (1991) rejected role of UIRP in exchange rate determination, although there is little consensus on why it fails. David Gruen, Gordon Menzies (1991) stressed upon failure of UIRP in the foreign exchange market even when the investor makes unsystematic mistakes while forming expectation of exchange rate changes. Bhatti, Razzaque H, Moosa, Imad, a (1995) have shown the supportive evidences of UIRP hypothesis through a cointegration analysis. The authors compare the Treasury bill rate denominated in eleven currencies to the US dollar and find a long-run relationship in all cases. Malliaropulos, D (1997) employs a multivariate GACRCH model of Risk Premium and reproduces the existence of time-varying risk-premium in deviations from UIRP. Burton Hollifield, R Uppal (1997) examine the effect of segmented commodity markets on the relation between forward and future spot exchange rates in a dynamic economy and find the presence of risk premium. McCallum, J. Monet (2000) extends the dataset used by McCallum to include the recent eight years. In most cases UIRP gets supported by the data as well as it passes all the conventional econometric tests. Jose Olmo, K. Pilbeam (2009) proposes two new profitability based tests for CIAP doctrine. UIPRP conditions are found to hold good to three of the four currencies studied.

2.1. Objective of Study

The present research study is an attempt to enquire into the tenets of the UIRP Doctrine and the theoretical resolutions which follow from the UIRP doctrine. More specifically, we seek to examine in the context of Indian foreign exchange market if

i. the expected future spot rate is in parity with the corresponding future spot rate
ii. the officially quoted forward exchange rate is the unbiased predictor of the future spot rate.
iii. the ARIMA (p, d, q) forecast for future exchange rates is in conformity with the actual future spot rate.
iv. CIRP holds for rupee-dollar exchange rates over the period of study.

2.2. Data: Nature, Period of Dataset, Transformation and Sources

The study is based on time series datasets on Rupee-dollar exchange rates and Forward Rate for Dollar quoted by RBI. The study involves monthly dataset on rupee-dollar exchange rates over the period (*11th November, 2011 - 27th February, 2015*) and the corresponding one month forward rates. Forward Rates were derived on the basis of the forward premium (*in annualized percent*) quoted by the RBI. RBI Bulletins constitute the main source of these time-series datasets. The study uses the logarithmic transformation of the level datasets on the spot rate and the corresponding quoted one-month forward rate.

2.3. Theoretical Issues

In 'Interest Rate Parity Arbitrage' analysis we deal with four macroeconomic variables like S_t= spot exchange rate (rupee/dollar) at time t.

S_{t+n} = Spot exchange rate which prevails at period (t + n)

$tE(S_{t+n}) = tS^e_{t+n}$ = Expected spot exchange rate to prevail at period (t + n), when expectation is formed at time t by the market agents like speculative & hedgers.

tF_{t+n} = Forward exchange for the period (t + n) quoted at time t i.e., the official expected spot rate for the period (t + n) quoted at time t.

2.3.1. Relation between F_{t+n} and tS^e_{t+n}:

Covered interest rate parity arbitrage doctrine holds that

$$tF_{t+n} = tE(S_{t+n}) = tS^e_{t+n} \qquad (1.1)$$

i.e, Forward Exchange Rate is the Unbiased Predictor of Future Spot Exchange Rate.

2.3.2. Relation between tS^e_{t+n} and S_{t+n}:

In this analysis

$$tE(S_{t+n}) = tS^e_{t+n} = E[S_{t+n}/\Omega] \qquad (1.2)$$

where $\Omega = [S_{t-i};\ i = 1,2,3 \ldots\ldots]$

tS^e_{t+n} is the minimum mean square error (MMSE) forecast for S_{t+n} by the market agent such that

$$S_{t+n} = tS^e_{t+n} + \epsilon_{t+n};\ \epsilon_{t+n} \sim \text{iid N } (0,\sigma^2_\epsilon) \qquad (1.2a)$$

ThustS^e_{t+n}, an ARIMA (p, d, q) forecast for S_{t+n}, virtually emerges as a 'Rational Expectation Forecasts forS_{t+n}.

2.3.3. Risk Premium & Forward Market Efficiency:

If economic agents are risk average, they need some return for the risk which they bear while holding the risky foreign exchange. This additional return is the 'risk premium'. In the event of the presence of risk premium, the equation (1.1) is modified to yield S^e_{t+n} = E $(S_{t+n}) = tF_{t+n} + RP_t$; where, RP_t = risk premium on foreign currency. Thus the forward rates may systematically over – or under predict the future actual exchange rate not because of exchange market efficiency but because of the presence of risk – premium.

2.3.4. Forward Rate and ARIMA (p, d, q) Forecasting:

The MMSE forecasts for $s_t$can be obtained through ARIMA (p, d, q) forecast. In many cases, forecasts obtained by this method are more reliable than those obtained from the traditional econometric modeling, particularly for short term forecast.

2.4. Methodological Issues

2.4.1. Forecast Model Estimation: Further Elaboration:

Let the ARIMA (p, d, q) model be for the time series y_t

$$\varphi(\beta)\, \Delta^d\, y_t = \varphi(\beta)\, \omega_t = \theta(\beta)\, \omega_t \qquad (1.3)$$

with $\varphi(\beta) = 1 - \varphi_1 \beta - \varphi_2 \beta^2 - \ldots\ldots\ldots - \varphi_p \beta^p$

and $\theta(\beta) = 1 - \theta_1 \beta - \theta_2 \beta^2 - \ldots\ldots\ldots - , \theta_q \beta^q$

Eqn.(1.3) can be expressed in terms of error term series ε_t such that

$$\varepsilon_t = \theta^{-1}(\beta)\, \varphi(\beta)\, \omega_t \qquad (1.4)$$

where $\omega_t = \Delta^d y_t$

The objective in estimation is to find a set of auto-regressive parameters ($\varphi_1, \varphi_2 \ldots, \varphi_p$) and a set of moving average parameters ($\theta_1, \theta_2, \ldots\ldots, \theta_q$) which minimize the sum of squared errors

$$S(\varphi_1, \ldots\ldots\ldots, \varphi_p, \theta_1, \ldots\ldots\ldots, \theta_q) = \Sigma\varepsilon^2_t \qquad (1.5)$$

Now let us assume that the error terms ($\varepsilon_1,\ldots, \varepsilon_t$) are all normally distributed and independent with mean 0 and variance σ^2_ϵ. Then the conditional log-likelihood function associated with parameter values ($\varphi_1, \varphi_2, \ldots, \varphi_p, \theta_1, \theta_2, \ldots, \theta_q, \sigma_\epsilon$) is given by

$$L = -T \log \sigma_\epsilon - S(\varphi_1, \ldots, \varphi_p, \theta_1, \ldots, \theta_q)/2\sigma^2_\epsilon \quad (1.6)$$

Here L is the *conditional logarithmic likelihood function*. Consequently,

$$\varepsilon_1 = \omega_1 - \varphi_1\omega_0 - \varphi_2\omega_{-1} - \ldots - \varphi_p\omega_{-p+1} + \theta_1\varepsilon_0 + \ldots + \theta_q\varepsilon_{-q+1} \quad (1.7)$$

Equation (1.7) shows that the maximum-likelihood estimate of the model's parameters is given by the minimization of the sum of squared residuals. Thus, under the assumption of normally distributed errors, the maximum-likelihood estimate is the same as least-square-estimate.

2.4.2. Minimum Mean-Square-Error Forecasts:

Optimum forecasts are forecasts with '*minimum mean-square forecast error*'**.** Thus the forecast $\hat{y}_T(1)$ will be so chosen that $E[e^2(1)] = E[\{y_{T+e} - \hat{y}_T(1)\}^2]$ is minimized. This forecast is the conditional expectation of y_{T+1} such that

$$\hat{y}_{T+1} = E[y_{T+1}/y_T, y_{T-1}, \ldots, y_1] \quad (1.8)$$

Eqn. (1.8) gives the minimum mean-square-error forecast.
Eqn. (1.8) can be written as

$$\varphi(\beta)(1-\beta)^d y_t = \theta(\beta)\varepsilon_t \quad (1.9)$$

since $\Delta = 1-\beta$. Therefore,

$$y_t = \varphi^{-1}(\beta)(1-\beta)^{-d}\theta(\beta)\varepsilon_t = \Psi(\beta)\varepsilon_t = \sum_{j=0}\Psi j\varepsilon_{t-j} \quad (1.10)$$

Eqn. (1.10) expresses the ARIMA model as a purely moving average process of infinite order. Then

$$y_{t+1} = \Psi_0\varepsilon_{T+1} + \Psi_1\varepsilon_{T+1-1} + \ldots + \Psi_1\varepsilon_T + \Psi_{1+1}\varepsilon_{T+1} + \ldots$$
$$= \Psi_0\varepsilon_{T+1} + \Psi_1\varepsilon_{T+1-1} + \ldots + \Psi_{1-1}\varepsilon_{T+1} + \sum_{j=0}\Psi 1 + j\varepsilon_{t-j} \quad (1.11)$$

In eqn. (1.11) the infinite sum has been divided into two parts. The second part begins with the term $\Psi_{j\varepsilon T}$ and thus describing information up to and including time period T.

However, the forecast $\hat{y}_T(1)$ can be based only on information available up to time T. Now forecast can be written as a weighted sum of those error terms, $\varepsilon_T, \varepsilon_{T-1}, \ldots$ Then the desired forecast is

$$\hat{y}_T(1) = \Sigma_{j=o}\Psi^*_{1+j}\epsilon_{T-j} \quad (1.12)$$

where the weights are chosen optimally to minimize the mean square forecast error. Then using Eqn. (1.11) and (1.12) we get

$$e_T(1) = y_{T+1} - \hat{y}_T(1)$$
$$= \Psi_0\varepsilon_{T+1} + \Psi_1\varepsilon_{T+1-1} + \ldots + \Psi_{1-1}\varepsilon_{T+1} + \Sigma_{j=o}(\Psi_{1+j} - \Psi^*_{1+j})\varepsilon_{T-j} \quad (1.13)$$

Since by assumption $E(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$, the mean-square forecast is

$$E[e^2_T(1)] = (\Psi^2_0 + \Psi^2_1 + \ldots + \Psi^2_{1-1})\sigma^2_\epsilon + \Sigma_{j=o}(\Psi_{1+j} - \Psi^*_{1+j})^2\sigma^2_\epsilon \quad (1.14)$$

Then this expression is minimized by setting the "optimum" weights Ψ^*_{1+j} equal to true weights $\Psi 1_{+j}$, for j = 0,1,.......In that case optimal forecast $\hat{y}_T(1)$ just becomes the conditional expectation of y_{T+1}. Consequently,

$$\hat{y}_T(1) = \Sigma(\Psi_{1+j}\varepsilon_{T-j}) = E[y_{T+1}/y_t, \ldots y_1] \quad (1.15)$$

Eqn. (1.15) provides the basic principle for estimations of forecast from ARIMA models.

2.4.3. Computation of a Forecast:

Let the ARIMA (p, d, q) model be

$$\omega_t = \varphi\omega_{t-1} + \ldots + \varphi_p\omega_{t-p} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \ldots - \theta_q\varepsilon_{t-q} + \delta \quad (1.16)$$

where $y_t = \Sigma^d\omega_t$

The one period forecast of ω_t is $\hat{\omega}_t(1)$. Now from eqn. (1.16) we get

$$\omega_{T+1} = \varphi_1\omega_T + \ldots + \varphi_p\omega_{T-p+1} + \varepsilon_{t+1} - \theta_1\varepsilon_T - \ldots - \theta_q\varepsilon_{T-q} + \delta \quad (1.17)$$

Taking conditional expected value of ω_{T+1} in equation (1.17) we get

$$\hat{\omega}_T(1) = E[\omega_{T+1}/\omega_T, \ldots]$$
$$= \varphi_1\omega_T + \ldots + \varphi_p\omega_{T-p+1} - \theta_1\hat{\omega}_T - \ldots - \theta_q\hat{\varepsilon}_{T-q+1} + \delta \quad (1.18)$$

where $\hat{\omega}_T, \hat{\omega}_{T-1}$ etc. are the observed residuals and the expected value of ε_{T+1} is 0.

Now using the one-period forecast $\hat{\omega}_T(1)$, one can obtain two-period forecast, $\hat{\omega}_T(2)$ such that

$$\hat{\omega}_T(2) = E[\omega_{T+2}/\omega_T, \ldots]$$
$$= \varphi_1\hat{\omega}_T(1) + \varphi_2\omega_T + \ldots + \varphi_p\omega_{T-p+2} - \theta_1\hat{\omega}_T - \ldots - \theta_q\hat{\varepsilon}_{T-q+2} + \delta \quad (1.19)$$

The two-period forecast is then used to produce the three period forecast, and so on until the h-period forecast $\hat{\omega}_T(h)$ is reached:

$$\hat{\omega}_T(h) = \varphi_1\hat{\omega}_T(h-1) + \ldots + \varphi_1\omega_T + \ldots$$
$$+ \varphi_p\omega_{T-p+1} - \theta_1\hat{\omega}_T - \ldots - \theta_q\hat{\varepsilon}_{T-q+1} + \delta \quad (1.20)$$

If 1>p and 1>q, this forecast will be

$$\hat{\omega}_T(h) = \varphi_1\hat{\omega}_T(h-1) + \ldots + \varphi_p\hat{\omega}_{(1-p)}$$

Once the differenced series ω_T has been forecasted, a forecast can be obtained for the original series y_t simply by summing ω_t, 'd' times.

2.4.3.1. Section I

- Stationarity and Integrability of Spot Rate and Forward Rate Series

Spot rate (S_t) and forward rate (tF_{t+1}) series have been subject to ADF unit root test for ascertaining the state of *stationarity* and *integrability* of the series concerned. Table 1 presents the results of the unit root tests on the series concerned.

Series	ADF Test Result	Inference
S_t (Level)	Non-Stationary	I(1)
tF_{t+1} (Level)	Non-Stationary	I(1)
$dS_t = S_t$ (First Difference)	Stationary	I(0)
$d_t F_{t+4} = tF_{t+4}$ (First Difference)	Stationary	I(0)

Table 1: Result of ADF Unit Root Tests on S_t and tF_{t+1} Series

The series S_t and tF_{t+1} are I (1) and offer a scope for examining their cointegrability over the period of study.

- ARIMA Forecast for one – period ahead Future Exchange Rate:

Univariate stochastic structure for S_t has been identified as ARIMA (1, 1, 0) such that

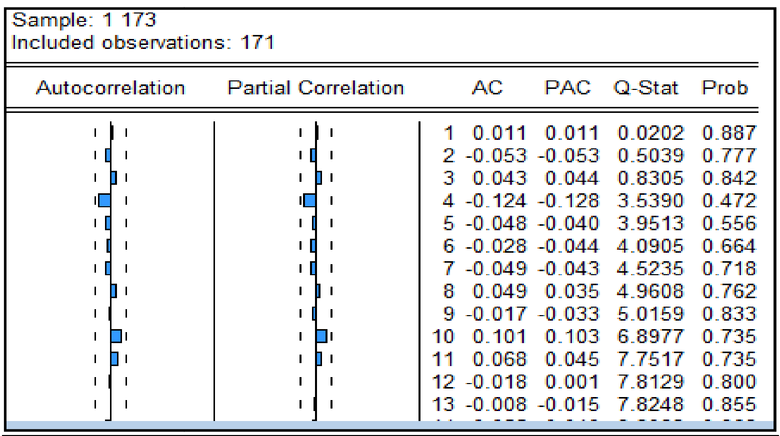
$S_t = [(1-L)(1-\varphi L)]^{-1}\epsilon_t;\ \epsilon_t \sim \text{iid } N(0, \sigma^2_\epsilon)$

The estimated equation is

$$(1-L)\, S_t = 0.000311 + 0.293937(1-L)\, L\, S_t \qquad (1)$$

SE	(0.000367)	(0.072957)
t	[0.845395]	[4.028890]
Prob	0.3991	0.0001

Residuals (ϵ_t) of the estimated equation are white noise as evidenced from its correlogram presented through Figure -1


Sample: 1 173
Included observations: 171

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.011	0.011	0.0202	0.887
		2	-0.053	-0.053	0.5039	0.777
		3	0.043	0.044	0.8305	0.842
		4	-0.124	-0.128	3.5390	0.472
		5	-0.048	-0.040	3.9513	0.556
		6	-0.028	-0.044	4.0905	0.664
		7	-0.049	-0.043	4.5235	0.718
		8	0.049	0.035	4.9608	0.762
		9	-0.017	-0.033	5.0159	0.833
		10	0.101	0.103	6.8977	0.735
		11	0.068	0.045	7.7517	0.735
		12	-0.018	0.001	7.8129	0.800
		13	-0.008	-0.015	7.8248	0.855

Figure 1: Correlogram of residuals from Equation (1)

The estimated ARIMA (1, 1, 0) structure for S_t as given by the equation (1) may be used for generating one – period ahead forecast for S_{t+1}. The forecast series is S^e_{t+1}. Time plots of S_{t+1} series along with S^e_{t+1} series are being given by Figure 2 below.

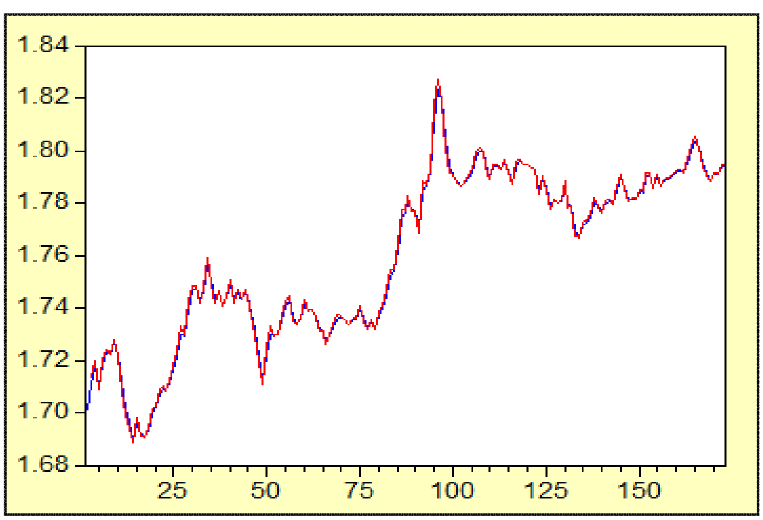


Figure 2: Time Plots of (S_{t+1}----) and (S^e_{t+1}----) series

It appears from the Figure (2) that one – period ahead forecast for S_{t+1} i.e, S^e_{t+1} almost coincide with the corresponding actual S_{t+1}. Basic statistics of the forecast error ($e_{t+1} = S_{t+1} - S^e_{t+1}$) are given in the Table 1.

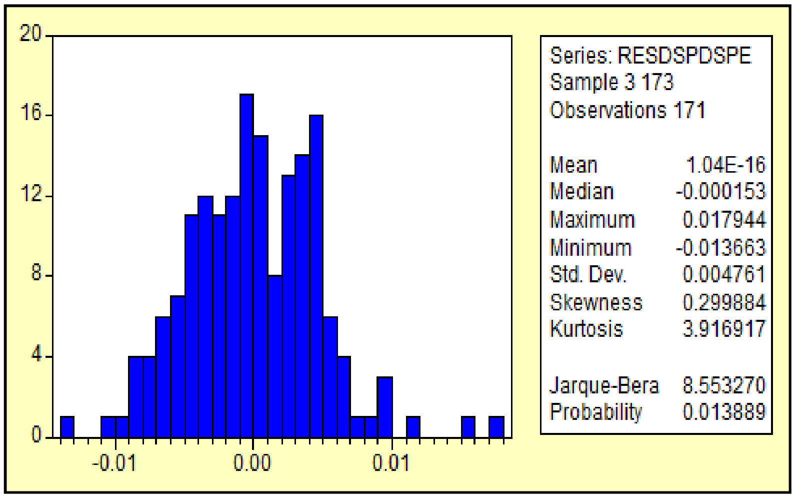


Figure 3: Statistics of the Forecast Errors (e_{t+1})

Mean value of e_{t+1} almost collapses on zero and the very small standard deviation of e_{t+1} indicates that S^e_{t+1} series represents the *Minimum Mean Square Error (MMSE)* forecast for S_{t+1} .

The estimated equation (1) may use to generate 4 – period ahead forecast for S_{t+4} . The forecast series represent the series for tE (S_{t+4}) = S^e_{t+4}. Time plots of S_{t+4} series along with the corresponding 4 – period ahead forecast S^e_{t+4} series are being presented through the Figure 3.

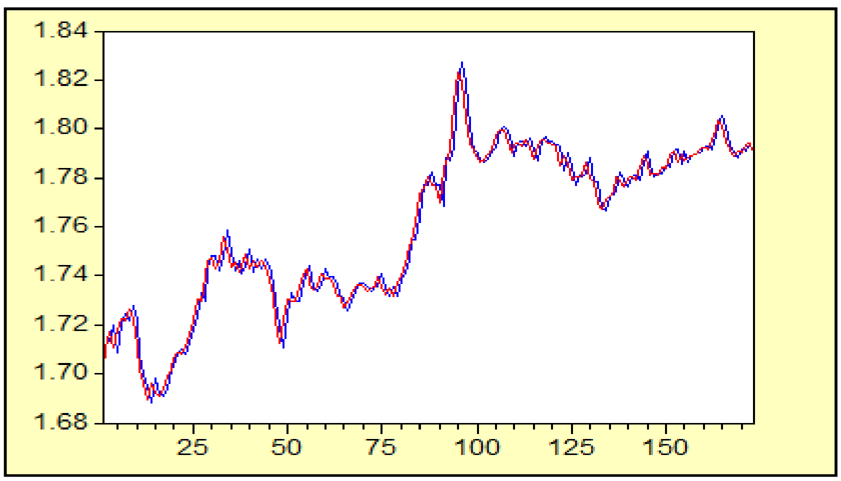


Figure 4: Time Plots of (S_{t+4}----) Series and 4 – Period Ahead forecast (S^e_{t+4}----) series.

Figure 4 shows that dynamic features of both the series are almost alike. Basic statistics of the corresponding forecast error ($e_{t+4} = S_{t+4} - S^e_{t+4}$) are being given by the Table 2.

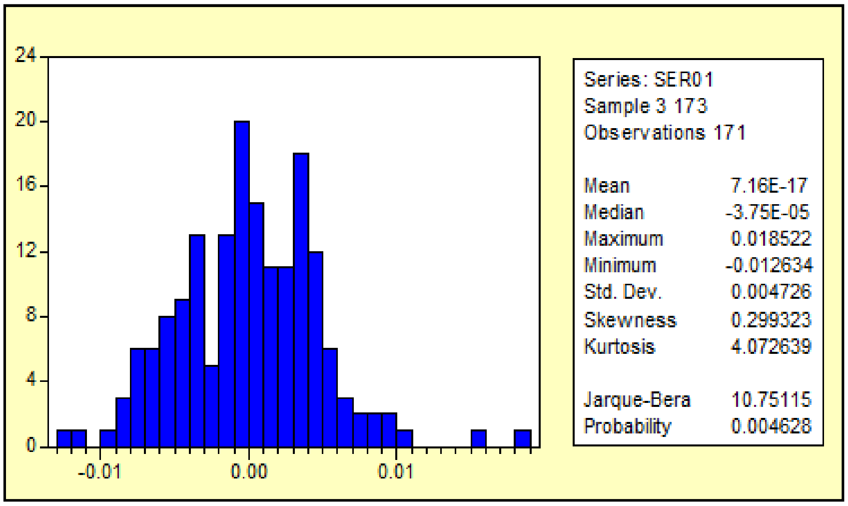


Figure 5: Basic Statistics of the Forecast Error (e_{t+4}) Series

Figure 5 shows that the mean and standard deviation of the e_{t+4} series almost coincide with zero. *Jarque – Bera Test* establishes normality of the series. Thus the e_{t+4} series is virtually normal with zero mean and standard deviation equal to 0.004726.

2.4.3.2. Section II

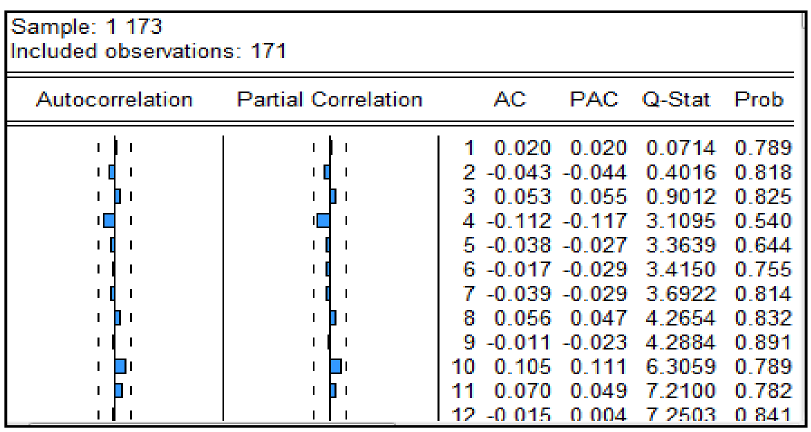
- Cointegration Between S_{t+4} and S^e_{t+4} Series

$S_{t+4} \sim I(1)$ and $S^e_{t+4} \sim I(1)$ series are found to be *cointegrated*. The estimated cointegration equation is

$$\hat{S}_{t+4} = 0.03116 + 0.982323\ S^e_{t+4} \quad (2)$$

SE	(0.019722)	(0.011203)
t	[1.577704]	[87.68693]
Prob	0.1165	0.0000

Correlogram of the residuals (u_{t+4}) of the equation (2) is given by the Figure 6.


Sample: 1 173
Included observations: 171

	AC	PAC	Q-Stat	Prob
1	0.020	0.020	0.0714	0.789
2	-0.043	-0.044	0.4016	0.818
3	0.053	0.055	0.9012	0.825
4	-0.112	-0.117	3.1095	0.540
5	-0.038	-0.027	3.3639	0.644
6	-0.017	-0.029	3.4150	0.755
7	-0.039	-0.029	3.6922	0.814
8	0.056	0.047	4.2654	0.832
9	-0.011	-0.023	4.2884	0.891
10	0.105	0.111	6.3059	0.789
11	0.070	0.049	7.2100	0.782
12	-0.015	0.004	7.2503	0.841

Figure 6: Correlogram of Residuals(u_{t+4}) of the Equation (2)

Figure (6) testifies for the fact that residuals (u_{t+4}) of the cointegrating equation [equation (2)] are '*white noise*'. Thus S_{t+4} and S^e_{t+4} series are cointegrated.

The estimated equation (2) shows that

(i) the constant term is not statistically different from zero (even at 10%) level.
(ii) the regression coefficient, i.e, the coefficient of S^e_{t+4} is statistically significant even at 1% levels and
(iii) the absolute value of the coefficient is not statistically different from unity.

It, therefore, follows that

$$S_{t+4} = S^e_{t+4} + u_{t+4} \quad (3)$$

where u_{t+4} is white noise

Consequently, equation (3) indicates that 4 – period ahead forecast S^e_{t+4} emerges as the 'Rational Expectations' forecast for S_{t+4}.

2.4.3.3. Section III

- Relation between Forward Rate (tF_{t+4}) and the Expected Spot Rate (S_{t+4}^{e})

S_{t+4}^{e} = tE(S_{t+4}) series represents the 4 – period ahead forecasts at period t for the future exchange rates (S_{t+4}) by the market agents like speculators and hedgers. However, Reserve Bank of India also makes forecasts at period t for S_{t+4} by quoting 1 – month forward rate through weekly statements of forward premium (annualized percentage). These official forecasts i.e, forward exchange rates (t F_{t+4}) series may be different from S_{t+4}^{e} series. In such case scope of profit arbitrage emerges out of the forecast differentials. If S_{t+4}^{e} exceeds the corresponding (t F_{t+4}) rate, then market agents take a 'Long position' in the foreign exchange market at time t and hope for reaping profit by forward buying of dollar and selling dollar spot at the higher expected exchange rate at period (t+4). If, on the other hand, $S_{t}^{e} < tE(S_{t+4})$, market agent take a 'short position' in the foreign exchange market. Market agents hope for reaping profit by selling dollar forward and buying spot at (t+4) period. This calls for examining the relationship between S_{t+4}^{e} series and tF_{t+4} series in the Indian Foreign Exchange Market over the period of study.

Time plots of S_{t+4}^{e} series and tF_{t+4} series are being presented through the figure 7.

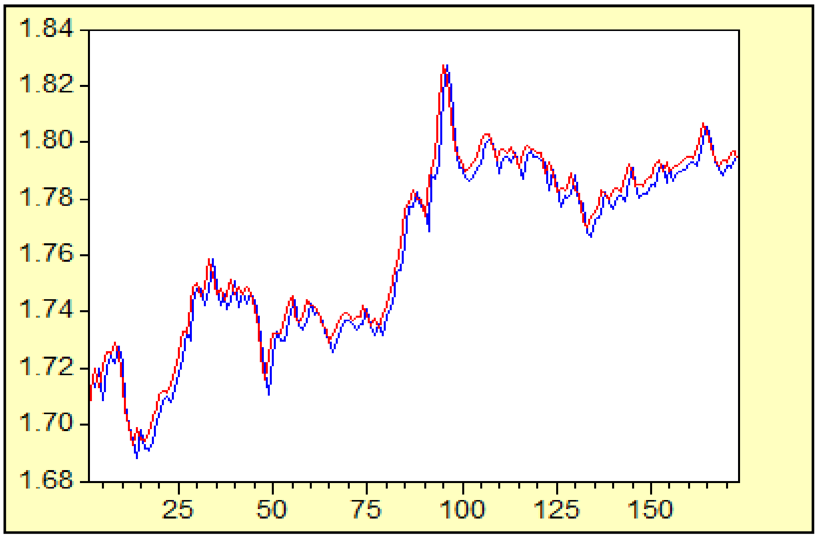


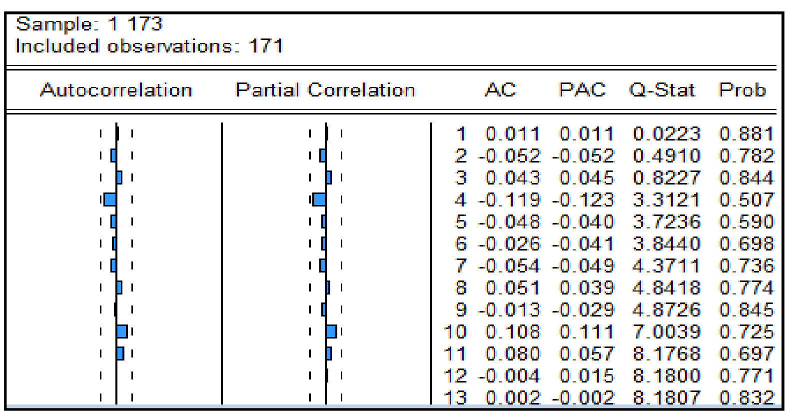
Figure 7: Time plots of (S_{t+4}^{e}----) series and (t F_{t+4} -----) series

Figure 7 shows that both the series possess almost the same dynamic texture and these series appear to share '*common trends*' S_{t+4}^{e} ~I(1) series and tF_{t+4} ~ I(1) series, therefore, may be '*cointegrated*'. The estimated '*cointegrating equation*' is

$$tF_{t+4} = 0.010170 + 0.992541\ S_{t+4}^{e} \qquad (4)$$

SE	(0.019901)	(0.11285)
t	[0.511028]	[87.94951]
Prob	0.6100	0.0000

The residuals of the cointegration equation (eqn. 4) constitute ω_{t+4}series. Correlogram of ω_{t+4}series is presented in Figure 8.



Sample: 1 173
Included observations: 171

	AC	PAC	Q-Stat	Prob
1	0.011	0.011	0.0223	0.881
2	-0.052	-0.052	0.4910	0.782
3	0.043	0.045	0.8227	0.844
4	-0.119	-0.123	3.3121	0.507
5	-0.048	-0.040	3.7236	0.590
6	-0.026	-0.041	3.8440	0.698
7	-0.054	-0.049	4.3711	0.736
8	0.051	0.039	4.8418	0.774
9	-0.013	-0.029	4.8726	0.845
10	0.108	0.111	7.0039	0.725
11	0.080	0.057	8.1768	0.697
12	-0.004	0.015	8.1800	0.771
13	0.002	-0.002	8.1807	0.832

Figure 8: Correlogram of Residuals ω_{t+4} from the Equation (4)

The residuals appear to be '*white – noise*' such that ω_{t+4} ~ iid N (0, σ^2 ω_{t+4}). Consequently, tF_{t+4}andS_{t+4}^{e} series are cointegrated.
It is further observed from the estimated equation (4) that

(i) the constant term in the regression equation is not statistically different from zero (even at 10% level).
(ii) the regression coefficient is significant event at 1% level.
(iii) the regression coefficient is not statistically different from unity.

All these observations indicate that

(i) $tF_{t+4} = S^e_{t+4} + \omega_{t+4}$ (5)

where $\omega_{t+4} \sim$ iid N $(0, \sigma^2 \omega_{t+4})$

and $tF_{t+4} \cong S^e_{t+4}$

or, $tF_{t+4} \cong tE(S_{t+4})$

Therefore, there exists no scope for profit arbitrage in the Indian rupee / dollar foreign exchange market over the period (*11th November, 2011 - 27th February, 2015*).

(ii) the absence of making profit through arbitrage testifies for the fact that foreign exchange market in India over the period of study was '*efficient*'.

(iii) $tF_{t+4} \cong S^e_{t+4}$ Indicates that *'forward rate emerged as the unbiased predictor of future exchange rate'* in the Indian foreign exchange market over the period of study.

2.4.3.4. Section IV

- Relation between Forward Rate (tF_{t+4}) Series and Spot Rate (S_{t+4}) Series.

Figure 9 presents the time plots of Forward Rate tF_{t+4} series and spot rate (S_{t+4}) series. Both the series exhibit the same type of dynamic texture over the period of study.

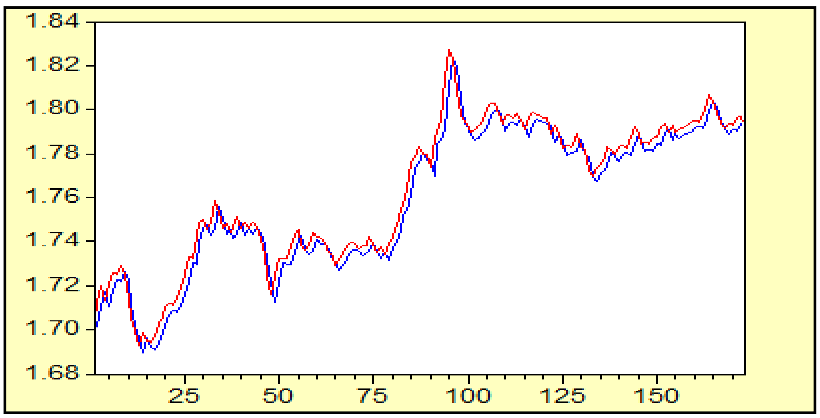


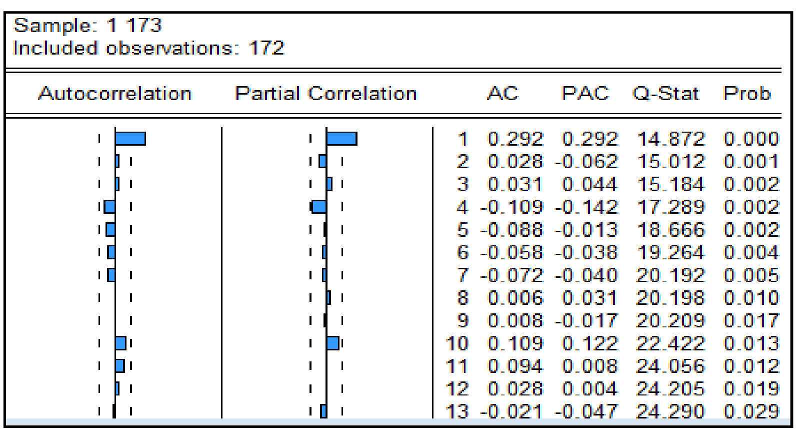
Figure 9: Time plots of (tF_{t+4}----) Series and (S_{t+4}----) Series

These series appear also to share the '*common trend*' and, therefore cointegrated. The estimated cointegrated equation is

$$t\hat{F}_{t+4} = 0.012111 + 0.991146\ S_{t+4} \quad (6)$$

SE	(0.020714)	(0.011748)
t	[0.584675]	[84.36468]
prob	0.5595	0.0000

Correlogram of the residuals u_{t+4} of the equation (6) is given by the figure (10)



Sample: 1 173
Included observations: 172

	AC	PAC	Q-Stat	Prob
1	0.292	0.292	14.872	0.000
2	0.028	-0.062	15.012	0.001
3	0.031	0.044	15.184	0.002
4	-0.109	-0.142	17.289	0.002
5	-0.088	-0.013	18.666	0.002
6	-0.058	-0.038	19.264	0.004
7	-0.072	-0.040	20.192	0.005
8	0.006	0.031	20.198	0.010
9	0.008	-0.017	20.209	0.017
10	0.109	0.122	22.422	0.013
11	0.094	0.008	24.056	0.012
12	0.028	0.004	24.205	0.019
13	-0.021	-0.047	24.290	0.029

Figure 10: Correlogram of Residuals u_{t+4} of the Equation (6)

u_{t+4} Series has been subject to ADF Unit Root Test with exogenous constant. The ADF test statistics = - 9.689490 exceeds the corresponding 1% critical value (- 3.468740). Therefore, there exists no 'unit root' in the series u_{t+4} series and the series is *'stationary'* indicating $u_{t+4} \sim$ I (0). Consequently, $t\hat{F}_{t+4}$andS^e_{t+4} series are cointegrated.

The estimated equation (6) shows that

i. the regression constant is not statistically different from zero (even at 10% level)
ii. the estimated coefficient of S^e_{t+4}significant at1% level.
iii. The absolute value of coefficient of S^e_{t+4} is not statistically different from one.

These findings indicate that the equation (6) can be modified as

$$tF_{t+4} \cong S^e_{t+4} \qquad (7)$$

Equation (7) shows that forward rate quoted at t for the period (t+4) virtually becomes equal to the actual spot rate at period (t+4) i.e., S_{t+4} . Since the regression constant is not different from zero, there exists no risk premium in the forward exchange market. This further supports the findings that the forward exchange market in India was '*efficient*' over the period (11th November, 2011 – 27th February, 2015).

2.4.3.5. Section V

- Further Test of Efficient Market Hypothesis for Indian Foreign Exchange Market:

It may however be noted that residuals (u_{t+4}) from the regression equation (6) are not '*white noise*' and there are exhibit first order auto – correlation: This indicates that there exists the scope of predicting future errors on the basis of past errors. This would be a sign of foreign exchange market '*inefficiency*' with the implication that there exist 'unexploited profit' opportunities'.

Cumby and *Obstfeld* argue that the equation (6) needs to be estimated as follows:

$$S_{t+4} - S_{t+3} = a_1 + a_2(tF_{t+4} - S_{t+3}) + u_{t+4} \qquad (8)$$

Given REH as in equation (7)

$$S_{t+3} = tF_{t+3} \qquad (9)$$

Then equation (8) can be written as

$$S_{t+4} - S_{t+3} = a_1 + a_2(tF_{t+4} - tF_{t+3}) + u_{t+4}$$
$$dS_{t+4} = a_1 + a_2 dtF_{t+4} + u_{t+4} \qquad (10)$$

Since, S_{t+4} ~ I (1) and tF_{t+4} ~ I (1), we have dS_{t+4} ~ I (0) and $dtF_{t+4} \sim I(0)$ consequently, dS_{t+4} and dtF_{t+4} series are 'Cointegrated' and the residuals from the '*cointegrating equation*' are '*white noise*'.

Cumby and Obstfeld hold that in the estimated equation $\widehat{a_1} = 0$ would imply absence of '*risk premium*' and '*efficiency*' of the foreign exchange market.

Again $\widehat{a_1} = 1$ would imply '*rational expectation hypotheses*'. Thus *Cumby* and *Obstfeld* hold that the estimation of equation (10) allows joint tests of '*Market Efficiency*' and '*rational expectation hypotheses*'.

The estimated equation is

$$d\hat{S}_{t+4} = -7.62E\text{-}06 + 1.003520\, d\,t\,F_{t+4} \qquad (11)$$

SE	(1.65E-05)	(0.003305)
T	[-0.460854]	[303.6811]
Prob	0.6455	0.0000

In the estimated equation (11)

i. $\hat{a}_1$ is not statistically different from zero even at 1% level. This testifies for the absence of '*risk premium*' in the market and, therefore the '*efficiency*' of Indian foreign exchange market.
ii. $\hat{a}_2$ is significant at 1% level and absolute value of $\hat{a}_2$ is not statistically different from unity. This testifies that official forward forecasts are '*Rational Expectations Forecast*' by nature.

The time plots of $dS_{t+4} = \dot{S}_{t+4}$and dtF_{t+4} series are being presented through the figure 11.

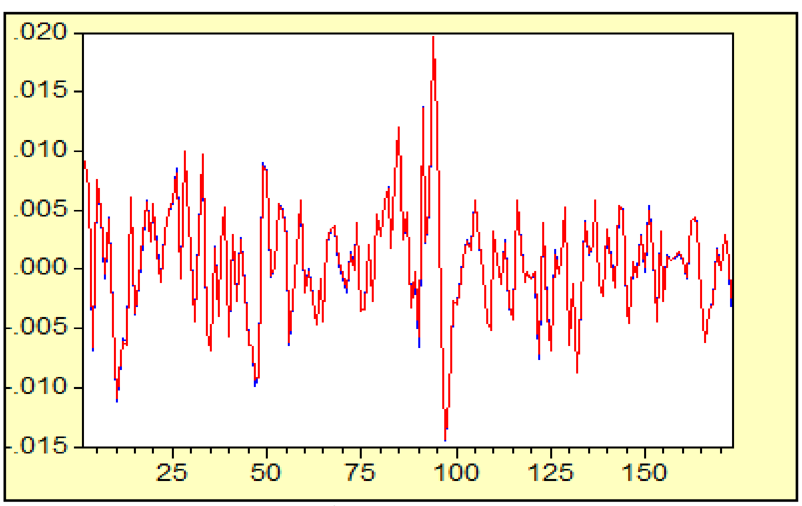


Figure 11: Time plots of ($\dot{S}_{t+4}$ -----) series and dtF_{t+4}-----) series

Figure (11) shows that one series appears to be almost superimposed on another which implying $dtF_{t+4} = \dot{S}_{t+4}$ such that differences between those two series over time are not statistically significant. These further testify for the '*absence of risk premium*' and for the '*efficiency'* of Indian Foreign exchange market over the period of study.

2.4.3.6. Section VI

3. Summary and Conclusion

Univariate stochastic structure for the spot exchange rate (S_t) has been identified as ARIMA (I, I, D). Estimated ARIMA (1, 1, 0) equation for S_t has been used to generate one – period ahead forecast (S^e_{t+1}) series and four – period ahead forecast (S^e_{t+4}) series.

(i) S^e_{t+1}~ I (1) and S_{t+1}~ I (1) series are cointegrated. Similarly, S^e_{t+1}~ I (1) and S_{t+4}~ I(1) series are cointegrated. Forecast error (S_{t+1}- S^e_{t+1}) series and (S_{t+4}- S^e_{t+4}) series are *'white – noise'* which imply that these forecasts are Minimum Mean – Squared Error (MMSE) forecasts by nature.

(ii) Again forecast errors, being *'white – noise'*, further imply that the forecasts are 'Rational' such as

$$S_{t+1} = S^e_{t+1} + \epsilon_{t+1}$$

and $$S_{t+4} = S^e_{t+4} + \epsilon_{t+4}$$

where ϵ_{t+1} and ϵ_{t+4} are *'white noise'*.

(iii) S^e_{t+4} ~ I(1) and tF_{t+4} ~ I (1) are '*cointegrated*' while residuals of the corresponding equation are *'white – noise'*. This implies that forward rates (tF_{t+4}) are the unbiased predictor of the corresponding spot rate (S_{t+4}) such that

$$tF_{t+4} = S^e_{t+4}$$

Thus the '*Covered Interest Rate Arbitrage Parity (CIRAP) Doctrine*' appears to hold good in the Indian foreign exchange market over the period of study (*11th November, 2011 - 27th February, 2015*)

(iv) $d\,t\,F_{t+4}$ series and $dS_{t+4} = \dot{S}_{t+4}$ series are found to be almost identical. Thus rate of change of exchange rate ($\dot{S}_{t+4}$) equals the forward premium ($t\,d\,F_{t+4}$) in the Indian foreign exchange market over the period of study.

(v) (a) In the equation of regression of $\dot{S}_{t+4}$ on dtF_{t+4} the regression constant is not different from zero. This implies the absence of risk premium in the foreign exchange market and, therefore the 'efficiency' of the market concerned over the period of study.
(b) Again the coefficient of dtF_{t+4} is not different from unity implying the holding of '*Rational Expectation Hypothesis*' in case of official forward rate forecast for spot rate at period (t+4).

The study, therefore, shows that in the Indian foreign exchange market over the period (11th November, 2011 - 27th February, 2015)

i. ARIMA forecasts like S^e_{t+1} and S^e_{t+4} are MMSE forecasts and these forecasts are '*Rational*' by nature.
ii. CIRAP holds such that forward exchange rate (tF_{t+4}) served as the unbiased predictor of the spot rate (S^e_{t+4}).
iii. there did exist no '*risk premium'* in the rupee – dollar foreign exchange market. There was no scope for spending arbitrage profit arising out of the differential forward rate and corresponding spot rate. This testifies for the 'efficiency' of Indian foreign exchange market over the period of study.

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