THE INTERNATIONAL JOURNAL OF BUSINESS & MANAGEMENT

Determination of the Optimal Cost of Training Staff in Tertiary Institution Using Linear Programming and Integer Linear Programming Model

Akanbi, Olumuyiwa O.
Lecturer, Department of Mathematics & Statistics, Federal Polytechnic, Ilaro, Ogun State, Nigeria

Abstract:
Some Tertiary Institutions fail to give quality education to attract foreign exchange due to their inability to provide their staff quality training and re-training from functionality of service delivery to certification. As such, this paper employs an operational research constrained optimization models to minimize cost of training both academic and non-academic staff. Two mathematical programming techniques are exploited, namely Linear Programming (LP) by Dual simplex method and Integer Linear Programming (ILP) by Branch and Bound Method. The result shows that, some constraints in models I, II & III are violated when optimal results are rounded off to nearest integer. But in the case of ILP none of the constraints is violated, since the optimal results is already in integer form. Therefore, the study recommends that the best constrained optimization model for solving personnel management problem is the ILP model. Because it alleviates the burden of round off error.

Keywords: *Linear Programming (LP), Integer Linear Programming (ILP), Branch and Bound method (B&B), Dual Simplex Method and Operations Research (OR)*

1. Introduction

One of the areas which has attracted much attention in the literature about the public sector efficiency during the last few decades is the educational sector. As such, all hands must be on desk to protect this sector during this period of economy recession where prudency is the only way out.

Most tertiary institutions in Nigeria make use of intuition or trial and error method to select number and the category of their staff to be sent for training (academic/professional) at a minimum cost within a specified period of time.

As such, these institutions will find it difficult to allocate limited resources within their capacity to ensure minimum cost on training programs.

Personnel management is the ability to manage problems relating to recruitment, selection, training and development of man power to different areas. Its Optimization is an area of wide research. Highly complex problem can be modeled and solved to optimality or near optimality using LP or ILP. This paper hopes to give the reader an idea about real world scenario that is modeled to differentiate between LP and ILP. The two classes of constrained optimization model considered in this work share the same general structure of optimization with restriction. Linear programming is the simplest of all and is still the most widely used type of constrained optimization model. Integer linear programming requires one or more of the variables to take on positive integer values and are harder to solve.

Some researchers make it pertinent that the use of scientific methods, particularly LP and ILP in the allocation of scare resources is of vital importance for any enviable growth of the organization see Richard (1991), Taha (2008), Biniyam and Tizazu (2013), Johannes (2016), and the references therein.

Human resources of any organization are the largest factor of production. Any citadel of learning without creative mind and innovative personnel will bring zero growth to the nation. Therefore, capacity building should be given adequate attention to achieve the millennium goal.

Linear programming can be viewed as part of a great revolutionary development which has given mankind the ability to state general goals and to lay out a path of detailed decisions to make in order to "best" achieve its goals when faced with practical situations of great complexity (Dantzig 2002).

Linear programming and its many extension have come into wide use. In academic circles, industries, military, business and others.

For convenience we define a pure integer problem as linear programs in which all the variables are integer. Otherwise the problem is a mixed integer problem.

More so, if all the variables in the optimal solution are allowed to take 0 or 1, such is referred to as 0-1 or standard discrete programming problem (Kalavathy, 2002). Meanwhile, our approach in this study is a pure integer linear programming techniques.

The significances of ILP are numerous, several occurring situations in business and industry that extend to planning models involve integer valued variables. In manufacturing, production is frequently scheduled in terms of batches, lots or runs. In allocation of goods, shipment must involve discrete number of trucks and in particular personnel management where numbers of staff should strictly assume positive integers.

Application of LP began in 1947, (in connection with the planning activities of the military) by George B. Dantzig, shortly after world war II and has been keeping the pace ever since with the extraordinary growth of computing power.

Integer linear programming began in 1958 by Gomory, unlike the earlier work on the travelling salesman problem (TSP) by Fulkerson (1954). Land and Doig in 1960, introduced another method called Branch & Bound (B&B) which has turned out to be one of the most successful ways to solve practical ILP (Kurtz, 1992).

Christodoulos and Xiaoxia, (2005), reviewed the advances of mixed-integer linear programming (MILP) for the scheduling of chemical processing systems.

Akinyele (2007), applies LP model based on integer programming to the determination of effective size of manpower to be engaged. His study also incorporates global constraints such as production capacity/demand rate and allowable time of operation into the model to reflect the reality activities in production organization in developing countries.

Linear programming technique is a very resourceful method in various fields. In the study by Snezana and Milorad (2009), they present a method for modeling and optimizing an industrial steam condensing system by linear programming techniques. LP is used to minimizing the total cost for energy net costs in steaming condensing systems.

John, Ganesh, and Narayanan (2010), in their paper proposed a vendor selection model using ILP model for multi-product, multi-vendor environment. As such, their model is validated with a case study by implementing the model for Agricultural equipment whole sale company.

Stephen, (2012) considers the application of integer programming to an investment firm in Ghana to maximize the investment of the company. He also performed a sensitivity analysis to assess the model stability to slight variations of some selected parameter.

Waheed (2012), demonstrates the use of linear programming methods as applicable in the manufacturing industry where KASMO industry limited, Osogbo, Nigeria was taken as a case study.

Kourosh, et al (2013), solve transportation problems using linear programming in Services Company. The paper reveals that an evaluation of 500 largest companies in the world showed that 85% of them have used linear programming.

In 2013, Mina, et al exploit LP to establish the optimal combination of production and the optimal allocation of human resources in a beverage company.

Biniyam and Tizazu (2013), worked on personnel scheduling using ILP model in which Avantis Blue-Nile Hotels, in Ethiopia serves as a case study. They used ILP to determine an optimal weekly shift schedule for the Hotel's engineering department personnel.

In the study by Agarana, Anake and Adeleke (2014), LP was applied to the management of loan portfolio of banks, where an answer is provided to the question of how to avoid possible occurrence of non-performing loans, bad and doubtful debts in banks.

Jean et al (2016) proposed the optimization of the beam layouts of the multibeam satellites where the strength of the methodology proposed is in mixed-integer linear programming to incorporate explicitly technological feasibility constraints of the subsystem involved.

For the solution approaches used in this work, the study is to choose which of the approaches is the best constrained optimization model.

From the available literature little or no attention is paid to application of ILP to personnel management and as such this paper is dedicated to the full utilization of ILP to personnel management.

2. Methodology

A realistic model in minimizing cost of training the staff in tertiary institution is formulated in term of LP and ILP.

A linear programming is the problem of maximizing (or minimizing) a linear function subject to a finite number of linear constraints.

Min $\sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} c_j x_j \geq b_i (i = 1,2, \dots m)$

with $x_j \geq 0$ (j = 1,2, … n).

if $x_j \geq 0$ *and* x_j *are integers then LP model becomes an ILP model.*

2.1. Mathematical Formulation of the Model

(i) Decision variables:

Let x_1 represent an individual in the senior category

Let x_2 represent an individual in the junior category

(ii) Objective function:

Minimize (z) = $c_1 x_1 + c_2 x_2$

Where c_1 *and* c_2 are the cost coefficients of training senior and junior staff respectively. c_1 *and* c_2 are taken as unity for simplicity and flexibility of the model.

(iii) Constraints:

The constraints for this work are basically the least time available for both academic and professional training. The available time for academic staff is 36 months (equivalent to minimum time for Ph.D. degree) while non-academic is 18 months (equivalent to minimum time for professional & academics masters)

Therefore, the general model governing the work is given as:

$$min \sum_{j=1}^{2} c_j x_j$$

Subject to:

$\sum_{j=1}^{2} a_j x_j \geq bi$ (i = 1,2, … n)

With $x_j \geq 0$, *in case of ILP* $x_j \in \mathbb{Z}^+$

(iv) Non negativity condition:

Case I

$x_1, x_2 \geq 0$

Case II

$x_1, x_2 \in \mathbb{Z}^+$

Case II renders the system as pure ILP models

2.2. The Models

The models are developed according to Academic and Non-academic units.

Academic units are divided into four schools namely: School of Applied Science, School of Management Science, School of Engineering and School of Environmental Studies. Therefore, the work focuses on formulation, analysis and interpretation of five models as LP and ILP.

Let: Non-academic model = model I
School of Applied Science = model II
School of Management Studies = model III
School of Engineering = model IV
School of Environmental Studies = model V

2.3. Model Assumptions

i. The coefficient of the objective function is assumed to be one million naira.
ii. The time constraints for non-academic staff is assumed to be at least 18 months.
iii. The time constraints for academic model is assumed to be at least 36 months.
iv. The constraints & objective function is linear.

2.4. Valuation of the Model

The model was validated with a polytechnic academic structure in Nigeria, where the non-academic staff are sub-divided into Rectory, Bursary, Library, Registry, Works & Services and Medical. We collected the data from Personnel Establishment department of a federal polytechnic to validate the models. Due to the agreement of confidentiality the name of the Polytechnic is withheld. Academic staff are divided into various schools and departments.

2.4.1. Non Academic Units Model:

LP		ILP
Min (z) = $x_1 + x_2$		Min (z) = $x_1 + x_2$
Subject to:		
$27x_1 + 37x_2 \geq 18$	(Rectory)	$27x_1 + 37x_2 \geq 18$
$36x_1 + 10x_2 \geq 18$	(Bursary)	$36x_1 + 10x_2 \geq 18$
$20x_1 + 22x_2 \geq 18$	(Library)	$20x_1 + 22x_2 \geq 18$
$80x_1 + 23x_2 \geq 18$	(Registry)	$80x_1 + 23x_2 \geq 18$
$55x_1 + 50x_2 \geq 18$	(Works & Services)	$55x_1 + 50x_2 \geq 18$
$13x_1 + 15x_2 \geq 18$	(Medical)	$13x_1 + 15x_2 \geq 18$
with $x_1, x_2 \geq 0$		with $x_1, x_2 \geq 0$&$x_1, x_2 \in \mathbb{Z}^+$

2.4.2. Academics Units Model:

- School of Applied Science Model

LP		ILP
Min (z) = $x_1 + x_2$		Min (z) = $x_1 + x_2$
Subject to:		
$12x_1 + 14x_2 \geq 36$	(Food Technology Dept.)	$12x_1 + 14x_2 \geq 36$
$20x_1 + 23x_2 \geq 36$	(SLT Dept.)	$20x_1 + 23x_2 \geq 36$
$13x_1 + 15x_2 \geq 36$	(Maths & Stats Dept.)	$13x_1 + 15x_2 \geq 36$

$9x_1 + 8x_2 \geq 36$	(Comp. Science Dept.)	$9x_1 + 8x_2 \geq 36$
$10x_1 + 9x_2 \geq 36$	(OTM Dept.)	$10x_1 + 9x_2 \geq 36$
$9x_1 + 6x_2 \geq 36$	(HMT Dept.)	$9x_1 + 6x_2 \geq 36$
$7x_1 + 7x_2 \geq 36$	(Nutrition & Dietetics Dept.)	$7x_1 + 7x_2 \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_{1,}x_2 \geq 0\&x_{1,}x_2 \in \mathbb{Z}^+$

- School of Management Studies Model

LP | **ILP**

Min (z) = $x_1 + x_2$ | Min (z) = $x_1 + x_2$

Subject to:

$7x_1 + \geq 36$	(Bus. Admin. Dept.)	$7x_1 + \geq 36$
$8x_1 + 4x_2 \geq 36$	(Public Admin. Dept.)	$8x_1 + 4x_2 \geq 36$
$3x_1 + 4x_2 \geq 36$	(Insurance Dept.)	$3x_1 + 4x_2 \geq 36$
$14x_1 + 4x_2 \geq 36$	(Accountancy Dept.)	$14x_1 + 4x_2 \geq 36$
$10x_1 + 4x_2 \geq 36$	(Banking & Finance Dept.)	$10x_1 + 4x_2 \geq 36$
$9x_1 + 6x_2 \geq 36$	(GNS Dept.)	$9x_1 + 6x_2 \geq 36$
$7x_1 + 4x_2 \geq 36$	(Marketing Dept.)	$7x_1 + 4x_2 \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_{1,}x_2 \geq 0\&x_{1,}x_2 \in \mathbb{Z}^+$

- School of Engineering Model

LP | ILP

Min (z) = $x_1 + x_2$ | Min (z) = $x_1 + x_2$

Subject to:

$21x_1 + x_2 \geq 36$	(Electrical/Electronic Dept.)	$21x_1 + x_2 \geq 36$
$22x_1 + 4x_2 \geq 36$	(Mechanical Engineering Dept.)	$22x_1 + 4x_2 \geq 36$
$13x_1 + x_2 \geq 36$	(Civil Engineering Dept.)	$13x_1 + x_2 \geq 36$
$9x_1 \geq 36$	(Computer Engineering Dept.)	$9x_1 \geq 36$
with $x_1, x_2 \geq 0$		with$x_1, x_2 \geq 0\&x_{1,}x_2 \in \mathbb{Z}^+$

- School of Environmental Studies Model

LP | ILP

Min (z) = $x_1 + x_2$ | Min (z) = $x_1 + x_2$

Subject to:

$9x_1 + 10x_2 \geq 36$	(Surveying & Geoinformatics. Dept.)	$9x_1 + 10x_2 \geq 36$
$8x_1 + 7x_2 \geq 36$	(Architecture Dept.)	$8x_1 + 7x_2 \geq 36$
$6x_1 + 8x_2 \geq 36$	(Quantity Surveying Dept.)	$6x_1 + 8x_2 \geq 36$
$11x_1 + 9x_2 \geq 36$	(Urban & Regional Planning Dept.)	$11x_1 + 9x_2 \geq 36$
$7x_1 + 7x_2 \geq 36$	(Estate Management Dept.)	$7x_1 + 7x_2 \geq 36$
$10x_1 + \geq 36$	(Building Technology Dept.)	$10x_1 + \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_1, x_2 \geq 0\&x_{1,}x_2 \in \mathbb{Z}^+$

3. Data Analysis

The work exploits two techniques Dual simplex and Branch & Bound techniques to obtain solutions to our LP and ILP models respectively via TORA Mathematical package.

Model	LP Optimum Solution	ILP Optimum Solution
1	Min (z) = 1.23 $x_1 = 0.22$ $x_2 = 1.01$	Min (z) = 2 $x_1 = 1$ $x_2 = 1$
2	Min (z) = 5.14 $x_1 = 1.71$ $x_2 = 3.43$	Min (z) = 6 $x_1 = 2$ $x_2 = 4$
3	Min (z) = 10.29 $x_1 = 5.14$ $x_2 = 5.14$	Min (z) = 11 $x_1 = 6$ $x_2 = 5$
4	Min (z) = 4 $x_1 = 4$ $x_2 = 0$	Min (z) = 4 $x_1 = 4$ $x_2 = 0$
5	Min (z) = 5.40 $x_1 = 3.60$ $x_2 = 1.80$	Min (z) = 6 $x_1 = 4$ $x_2 = 2$

Table 1: Optimal results of different models

MODELS	LP					ILP		
Model I	Z	x_1		x_2		x_1	x_2	Z
		Actual Value	Rounded Value	Actual Value	Rounded Value	Actual Value	Actual Value	
	1.23	0.22	0	1.01	1	1	1	2
Model II	5.14	1.71	2	3.43	3	2	4	6
Model III	10.29	5.14	5	5.14	5	6	5	11
Model IV	4	4	4	0	0	4	0	4
Model V	5.40	3.60	4	1.80	2	4	2	6

Table 2: Comparison table between approximated optimal result of LP and ILP

4. Interpretation of Results

The various models output is presented in table 1. The approximated optimal value of IL is presented in table 2 to conform to reality (human beings cannot assume a fractional value)

i. Considering the LP models

From model 1, No senior staff is expected to be sent for training to minimize the cost of training at ₦1.23m. From model II, 2 senior staff and 3 junior staff can be sent for training to minimize the cost at ₦5.14m. From model III, equal number of senior and junior staff can be sent for training ($x_1 = x_2 = 5$) at ₦10.29m. From model IV, 4 senior staff can be sent for training and no junior staff should be sent for training to optimize cost of training at ₦4m. From model V, 4 senior staff can be sent for training and 2 from the junior cadre at a cost of ₦5m.

ii. Considering the ILP models:

The same number of senior and junior staff ($x_1 = x_2 = 1$) can be sent for training at a minimum cost of ₦2m. From model II, 2 senior staff and 4 junior staff can be sent for training at a cost of N6m. From model III, 6 senior staff and 5 junior staff can be sent for training at an optimal cost of N11m. From model III, 4 senior and no junior staff should be sent for training in order to minimize the cost at N4m. From model IV, 4 senior and 2 junior staff should be sent for training at a minimum cost of N6m

5. Conclusion

The work successfully established the models of optimizing the cost of building human capacity in the citadel of learning. Furthermore, the study obtained the optimal solution to the five models with the help of Dual Simplex Method (LP) and Branch & Bound Technique (ILP).

With critical observation from our findings, in the case of **model I;** when this model is taken as (LP) we obtained the optimum solution to be min (z) = 1.23, x_1= 0.22, x_2 =1.01. Since we are dealing with human beings, the results of our decision variables x_1 = 0.22 &$x_1 = 1.01$ are meaningless (fractional part of staff cannot be obtained as a living being). Hence we are forced to round them off to nearest integer i.e. $x_1 = 0, x_2 = 1$. With these rounded off values, we observed that the rounded values of x_1 *and* x_2 violate all the constraints except the first constraint. This implies that our solution is not a good optimal solution. Taken the model as ILP, we obtained solution given as min (z) = 2, $x_1 = 1, x_2 = 1$, in all ramifications values x_1 and x_2 satisfy all the constraints. Hence one junior and one senior staff should be sent for training within the cost implication.

Similar observation was made in **Model II,**if taken as LP; we have min (z) = 5.14, $x_1 = 1.71$ *and* x_2= 3.43, if these values are rounded to nearest integer i.e. x_1= 2 and x_2 = 3. The seventh constraint is going to be violated (hence not a good solution). But if the model is treated as ILP we shall obtain an optimum solution that satisfies all the constraints i.e. min (z) = 6,x_1 = 2, x_2 = 4.

Also another typical observation was made in **Model III**; If examined as LP, we have min (z) = 10.29, x_1 = 5.14 and x_2 = 5.14, rounding off gives x_1 = 5 and x_2 = 5. These value violate the third constraint. But if the model is viewed as ILP we have optimum solution as min (z) = 10, x_1 = 4, x_2 = 6, which automatically satisfies all the constraints.

Though models IV and V after rounding off to nearest integer do not violate any of the constraints. This is not always the case. In conclusion, it worth noting that the rounded values in **model I**, **Model II** and **Model III** do not give the exact objective when substituted into the objective function. In this research work, the best approach to personnel management model is the ILP model because it alleviates this computational burden dramatically, since B & B method embraces an intelligent search procedure designed to reach optimum integer solution without rounding off the result. Therefore, we recommend that a viable approach to solving personnel management of the type investigated in this work is ILP.

6. References

i. Agarana, M.C., Anake, T.A. and Adekele, O.J. (2014). Application of linear programming model to unsecured loans and bad debt risk control in banks. International journal of management, information technology and engineering (2) issue 7, 93-102.
ii. Akinyele Taiwo (2007). Determination of the optimal manpower size using linear programming model. Research journal of Business Management (1) 30-36.
iii. Biniyam, K. and Tizazu, A. (2013). Personnel Scheduling using an integer programming model an application at Avanti Blue-Nile Hotels. (Springer plus 2013, 2:333)
iv. Christodoulos, A. and Xiaoxia, L. (2005). Mixed integer linear programming in process scheduling modeling, algorithms and applications. Annals of operations research 139, 131-162
v. Dantzig, G, (2002). Linear Programming (INFORMS No 1, 42-47)
vi. Dantzig, G. Fulkerson, D. and Johnson, S. (1954). Solution of a Large-Scale Traveling-Salesman Problem (Operations Research 2, 393-410).
vii. Johannes, B. (2016). AIMMS Optimization Modeling (Bellenue, U.S.A.)
viii. John Rajan, Ganesh, K. and Narayanan, K. (2010). Supply chain Application of ILP model for vendor selection in a two stage. Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management Dhaka, Bangladesh, January 9-10.
ix. Jean T.c., Christian. A Laurent .H and Stephen M. (2016). Mixed integer linear programing for multibeam satellite systems design: Application to the beam layout optimization. Systems Conference (Syscon) Annual IEEE. DOI: 10. 1109/SYSCON.2016.7490613.
x. Kalavaty, S. (2002). Operation Research 2nd edition Vikas Publishing House PVT Limited.
xi. Kourosh Rajeoyan, Farhang Khalaghdoost Nejati, Reza Hajah (2013). Using Linear Programming in Solving the Problem of Services Company's cost. Singaporean journal of Business Economics and Management Studies.
xii. Kurtz S. (1992). Solving Integer Programming with Branch and Bound Technique. Retrieved from http:// www.columbia.edu.co
xiii. Martin, G. and George, L. (2008). George Dantzig's Contributions to integer programming. Discrete optimization 5, 168-173.S
xiv. Mina, A., Yeyhameh, B. Maryam, I. and Zeynab, N. (2013). Optimal allocation of Human Resources by Using Linear Programming in the Beverage Company. Universal Journal of Management and Social Sciences 3, No 5 5-12
xv. Richard B. (1991), Introduction to Linear Programming: application and extension. Marcel Dekker, Inc. New York.
xvi. Snezana Dragicevic and Milorad Bojic (2009). Application of linear programming in energy management. Serbian Journal of Management 4(2) 227-238.
xvii. Stephen Twum (2012). Application of integer programing to an investment problem of a firm in Ghana. Doi: 10.4028/www.scientific.net
xviii. Taha H.A. 2008. Operations Research: An Introduction 8th ed. New Delhi; Pearson Education.
xix. Waheed Babatunde (2012). Profit maximization in a product mix company using linear programming. European Journal of Business and Management 4(17).

APPENDIX

- List of academic and non-academic staff:

Units	Department	Senior Staff	Junior Staff
Non Academic unit	Rectory	27	37
	Bursary	36	10
	Library	20	22
	Registry	80	23
	Works & Services	55	50
	Medical	13	15
Academic unit: School of Applied Science	Food Technology	12	14
	SLT	20	23
	Maths & Statistics	13	15
	Computer Science	9	8
	Office Tech & Mgt.	10	9
	Hospitality Mgt.	9	6
	Nutrition & Dietetic	7	7
Academic unit: School of Mgt. Studies	Business Adm.	7	-
	Public Adm.	8	4
	Insurance	3	4
	Accountancy	14	4
	Banking & Finance	10	4
	General Studies	9	6
	Marketing	7	4
Academic unit School of Engineering	Electrical/Elect.Engineering	21	1
	Mechanical Engineering	22	4
	Civil Engineering	13	1
	Computer Engineering	9	-
Academic unit School of Environmental Studies	Surveying&Geo-Info.	9	10
	Architectural Design	8	7
	Quantity Survey	6	8
	URP	11	9
	Estate Magt.	7	7
	Building Tech.	10	-