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A Mathematics Model for Parking Space Match-Making Mechanism

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Abstract
Many cars have the advantage of transporting people or object from one location to another accurately. The objective of designing roads is to facilitate the movement of cars, while the purpose of constructing parking spaces is to allow cars to park in still. Even though roads and parking spaces serve different purposes, parking spaces often appear next to busy traffic road. Such design not only sacrifices the safety of roads for cars to travel but also worsens the traffic in terms of vehicle volume and traveling time, hence, increasing social costs. The car parking space match-making mechanism reported in this study is designed in the point of view of the government, in which owners of private parking can offer their spaces to others who are in need during non-use period. Alternatively, the mechanism can also induce people to purchase private parking spaces, which can be offered to those who are in need in exchange of rental fees. These incentives, therefore, support the formation and operation of this match-making mechanism. Through the operability of this mechanism, private parking space owners are more willing to transfer their parking space property right to the government for management, which in turn increases the number of parking spaces available for the government. Moreover, the parking spaces offered in this match-making mechanism are interchangeable.

The lifetime and stability of the mechanism from "nothing" to "something" depend on whether or not the interchangeability of such mechanism can catalyze subsequent operations spontaneously as well as the degree of catalytic power. If the mechanism possesses spontaneous catalytic power for sustainability, and the catalytic process has the "good" to benefit other people without compromising your own interest, the formation of this mechanism will be regarded as "good formation". On the other hand, if the mechanism cannot be catalyzed spontaneously, occasional operation is needed. Anything that is created from advertisement will eventually be gone. The mechanism reported in this study not only is regarded as "good formation", the target function can be designed depending on the beneficiary, giving the flexibility for selection. In other words, the rate of car space rental per unit time can be determined depending on the weighted average of interest from different groups as shown below.

- *Interest of the group who need to rent car parking space*
- *Interest of the group who transfer their private parking space to the government for management during non-use period*
- *Interest of the companies assigned by the government to handle car parking space match making (the company can be listed company at over-the-counter market or stock exchange market)*

Keywords: *Parking space, interchangeability, consumer surplus, trading cost*

1. Literature Background

Motorcycle and car has the advantage of transporting people or object from one location to another accurately. Such advantage has increased the number of motorcycles and cars rapidly along with urban development. In the perspective of the vehicle operator, riding motorcycle is far less safe and comfortable comparing to driving car. In terms of carrying objects or people, the performance of car also far exceeds that of motorcycle. Therefore, only the parking space supply and demand match-making of cars is considered in this study, the match-making of motorcycle parking spaces is out of the scope of this study. Insufficient parking space is a very serious problem in many urban areas of Taiwan. Taking Chiayi City as an example, due to the design of car parking spaces along the roads, at least 73 roads in Chiayi City have inadequate width of under 6 meters (reference [2]).

The amount of car parking spaces available not only will affect the quality of traffic, but also economic growth, particularly in the calculation of trading cost between buyer and seller ([3][4]). This is because in the perspective of a buyer, the trading cost is not based on simply how much the product is cost, but also other costs which are required for acquiring the product, such as the time in driving the car to the shopping mall as well as the time for finding a parking space. These two will directly or indirectly related to the convenience of car parking. As for the trading cost for the seller, besides the product manufacturing and material cost, the time for delivering the product to the store or the time for delivering the product to the customer's house and the time and cost for finding a parking space should also be considered. This suggests that the convenience of car parking will affect both the traffic and the economy of each individual. Therefore, it is believed in this study that how to build a car parking space match-making mechanism is an important topic worth for investigation.

To think about how to carry out car parking space match-making, we will automatically begin by considering those who have private car parking spaces and how to match or interexchange car parking spaces among those people during non-use period. Assuming Mr. A who has private car parking space and lives in the east side of the city is driving to work in the morning from east side of the city to west side, while Mr. B who has private car parking space and lives in the west side of the city is driving to work in the morning from west side of the city to east side. If Mr. A and Mr. B is willing to offer their car parking space during non-use period to each other in the morning, the car parking problem in the morning for both of them will be solved. Even though the mutual benefit of Mr. A and B originates from the philosophy of "unused space reutilization", the match-making process is performed only by interchanging the non-use period of car parking space among the group with private car parking spaces. It is not a "good formation" match-making mechanism because such interexchange operation of car parking space during non-use period will not activate the interexchange operation of other car parking spaces during the non-use period. In other words, the exchange operation of unused car parking space in such match-making mechanism cannot act as the driving activate subsequent operations for sustainability. Therefore, the aim of the parking space match-making mechanism reported in this study is to expand the target of match-making not only to those who drive cars (whether or not they own private car parking space), but also to those who do not drive cars and are willing to act as investors by purchasing car parking spaces in exchange of the rental fee. If a car driver C who owns private car parking space parks his/her car in the car parking space owned by driver D, he/she will automatically consider the possibility of transferring his/her parking space right to the government for management in exchange for car parking rental fee. This suggests that the interexchange operation of lending car parking space of driver D to driver C during non-use period in return for rental fee will activate driver C to offer his/her car parking space during non-use period to the government for management in exchange for rental fee and continue to active subsequent operations. The above-mentioned operation may also activate driver E who used to carpool with his friend who had the same parking experience. Due to such experience, driver E is willing to purchase car parking space and transfer the management of such property to a company (the company can be listed company at over-the-counter market or stock exchange market) assigned by the government in exchange for parking space rental fee. This implies that the car parking space match-making mechanism reported in this study not only is a "good formation" mechanism, but also a mechanism with two activation powers; first one is the activation for car parking space public ownership, and the second one is the activation for car parking space investment. These two activation powers are the key to the price-volume relationship of car parking space match-making mechanism, which will be described in the following section. The people involved in the car parking space match-making mechanism can be categorized into three different groups, namely the car parking space user group, car parking space owner group (including public owned and private owned parking space), and companies assigned by the government for carrying out the car parking space match-making operation.

It is possible that someone might play the role of group one and two or group one, two and three simultaneously, meaning that someone might own private car parking space and at the same time a driver who rent a private car parking space owned by other people. Due to such mechanism, the feedback of the interexchange operation to the mechanism will be even more powerful.

The aim of this study is to transform the interests of car parking space match-making operation between the above-mentioned three groups into mathematical models which can be presented for detailed investigation. The objective is to find the optimal solution for the mathematical models and discuss the characteristics of the optimal solution.

2. Cars Parking Spaces Match-Making Mechanism

The mechanism includes two trading market, namely the front-end market and back-end market. The buyers of the front-end market are those people who need to rent car parking spaces, whereas the sellers are the car parking space match-making companies assigned by the government. The buyers of the back-end market are the car parking space match-making companies, whereas the sellers are those people who own the car parking spaces (including public and private car parking spaces); in other words, they are the car parking space suppliers. The price of front-end market is the parking fee per unit time, whereas the quantity is the number of parking space per unit time. The price of back-end market on the other hand is the parking fee paid successfully by the match-making company to the parking space supplier, whereas the quantity is the total parking hour per unit match-making company per unit time. The ratio between the quantity of car parking spaces supplied by the back-end market and the quantity of car parking spaces demanded by the front-end market will be used to evaluate the degree of convenience in car parking. With regard to the parking space match-making company, its source of operation consists of three parts, namely the front-end market price, back-end market price and car parking convenience index. Among them, car parking convenience index can be viewed as the average quality of product (car parking space) offered by the parking space match-making company. To make the outcome of this study closer to the assumptions, the average quality of product offered by the parking space match-making company will be kept at a certain level, only the price of front-end and back-end market will be manipulated. Under such assumption, the parking space demand of the front-end market and the parking space supply of the back-end market will eventually reach equilibrium with time followed by the operation of the activation power for parking space public ownership and the activation power for parking space investment. The benefits among the three groups, car parking space demanding group, car parking space supplying group and car parking space match-making companies, after equilibrium will be elaborated, which is also the main component of this mechanism.

2.1. Assumptions

The mechanism, the car parking demand rate at different time points of the same time frame varied insignificantly. The so-called car parking demand rate is the car parking demand per unit time. The car parking demand rate not only is related to parking fee per unit time, but also will be affected by degree of parking convenience. The degree of parking convenience can be regarded as the quality of parking space match-making company's product (parking space). In other words, it can also be regarded as the service quality of parking space match-making company. The so-called degree of parking convenience is the average length of time, which starts from the point when the driver begins to look for a parking space to the point when the driver finds a parking space without considering the influence of parking fee per unit time. In this study, the index of parking convenience is the quotient between parking space (vacant) supply per unit time q_2 and parking space demand q_1. It indicates the average number of vacant parking space available for each individual driver. This mechanism assumes that the fixed cost of parking space match-making company, i.e., the cost for establishing parking space information system (reference [5]), have already been settled. The information system is able to detect cars in the metropolis area at any location any time and the number of vacant parking space available nearby can also be detected.

The biggest difference between the parking space supply-demand function of this mechanism and that of other products in conventional economics is that: in this mechanism, the parking convenience index *l* must be determined first before the relationship between parking fee per unit time p_1 and parking space demand q_1 can be further described, which can be written as $p_1 = f_l(q_1)$. The rest of the mathematical symbols are described as follows.

2.2. Symbols Description

p_1 : The parking fee per unit time paid by front-end buyer to seller, where p_1 is the decision variable of parking space match-making company.

q_1 : Total parking hour of parking space demand per unit time for front-end market buyer

p_2 : The fee per unit time that must be paid by the back-end market buyer (parking space match-making company) to the seller (parking space supplier) once single match-making process is completed successfully, where $p_2 < p_1$, meaning that $(p_1 - p_2)$ is the commission per unit parking time received by the parking space match-making company for each successful match-making process.

This suggests that the type of trading performed between back-end buyer and seller is commission-type trading rather than outright rental-type trading.

q_2 : Total length of parking time that can be allocated per unit time by the back-end market buyer (parking space match-making company).

l : $l = \frac{q_2}{q_1}$ is the index of parking convenience, *l* can be viewed as the service or product quality of parking space match-making company; it is one of the trading cost of parking space demanding group; since *l* represents the average number of parking spaces available for the parking space demander, it is therefore regarded as the index for parking convenience. The degree of people willing to park not only is related to the parking fee per unit time, but also is affected by the parking convenience index *l*.

$\frac{1}{l}$ ⊙1 : a parking space waiting to be parked, ratio of average parked time.

The front-end market price-volume relationship can be specifically expressed only if the product quality, *l*, of the parking space match-making company is given; therefore, the mechanism treats *l* as the exogenous variable.

f_l : The demand function of the variable relationship between *l* value corresponded front-end market price p_1 and its quantity q_1 is described as $p_1 = f_l(q_1)$, where $f_l(q_1)$ is the strictly decreasing concave downward function of q_1; i.e.,

$$f_l^{'}(q_1) < 0 \text{ and } f_l^{''}(q_1) < 0 \text{ , } \forall q_1 \qquad (1)$$

When the parking convenience index *l* is increased to $\bar{l}$, the demand function between price variable y and quantity variable *x* described as: $y = f_l(x)$ will be entirely shifted to the right as the demand function: $y = f_{\bar{l}}(x)$ shown in Figure 1. Since $\bar{l} > l$, $\bar{l}$ corresponded parking convenience will be larger than l corresponded parking convenience; therefore, under the same parking fee per unit time, p, the parking demand $q_{\bar{l}}$ corresponded to $\bar{l}$ must be greater than the parking demand q_l corresponded to *l* as shown in Figure 1.

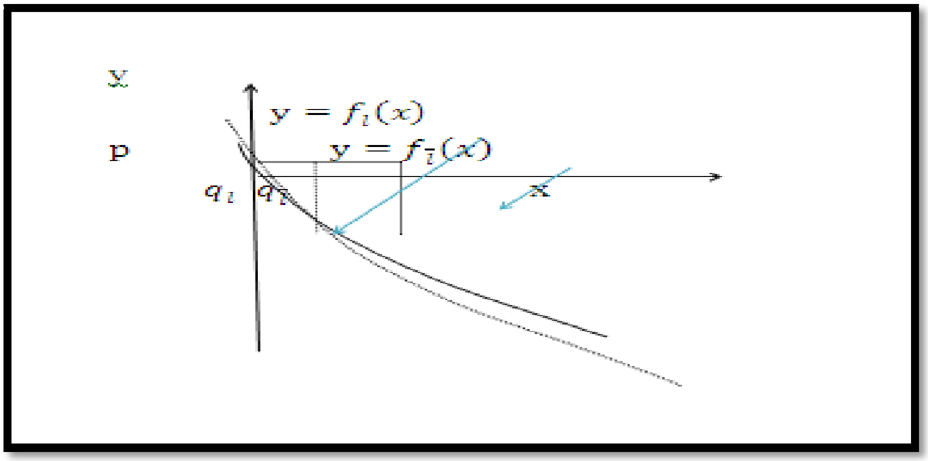

Figure 1: When l < $\bar{l}$, $y = f_l(x)$ Plot is on the Left Side of $y = f_{\bar{l}}(x)$ plot

g: Back-end market parking space supply function; g is the is the relationship between average income per unit time of a parking space waiting for rental $p_2\left(\frac{1}{l}\odot 1\right)$ and parking space supply quantity q_2: $p_2\left(\frac{1}{l}\odot 1\right) = g(q_2) = g(lq_1)$, where g is the strictly increasing concave upward function of q_2; the function y= $g(x)$ satisfies:

$g'(x) > 0$, $g''(x) > 0$ $\forall x \geq 0$ (2)

c_0 : The average personnel and process cost per unit time required for match-making one parking space by the parking space match-making company.

a⊙b : any given real number a, b, symbol a⊙b refers to the minimum number between a and b; in other word, a⊙b = min{a, b}

2.3. Mathematical Model Making

The expression of target function π of this mechanism is flexible. The choice of front-end market price p_1 can be either considering the maximization of parking space demander benefit π_d (parking consumer surplus) per unit time, or the maximization of parking space supplier profit π_s per unit time, or the maximization of parking space match-making company's benefit π_b per unit time. The choice of these three target functions can determined by using the regular weighted average and expressed simultaneously as:

$\pi = w_1\pi_d + w_2\pi_s + w_3\pi_g$, where the weighted value w_1 , w_2 , w_3 is a parameter between 0 and 1, In addition, $w_1 + w_2 + w_3 = 1$ this suggests that the values of w_1 , w_2 , w_3 are determined by the parking space match-making company once the policy has been confirmed by the government. The parking space match-making company may not be a complete profit company, it should take into consideration orders from the government in determining the weighted value for different groups. Under the circumstances when parking convenience index *l* is given and when the front-end market price p_1 is determined by the parking space match-making company, the back-end market price p_2 and front-end as well as back-end market quantity q_1 and q_2 will approach equilibrium with time. The symbols π_d , π_s , π_b and π are the benefit values of various groups under equilibrium.

According to the principle of economics, in any trading behavior with a deal price of p_1, both buyer and seller must benefit from such trading. The unit benefit of the buyer is the gap between the top-limit of the desired price and the actual purchased price. The total benefit of the buyer under the deal price p_1 is regarded as the consumer surplus of p_1, which is symbolized as $\pi_d(p_1)$.The value is represented by the area of the most top region in Figure 2 and Figure 3. Among them, the fee willing to pay by the parking space demander per unit time is $\int_0^{q_1} f_l(x)dx$ and the actual fee paid is p_1q_1. The difference between those two is shown by area of the most top region in Figure 2, π_d, where $\pi_d(p_1)$ has the following characteristics.

$$\pi_d(p_1) = \int_0^{q_1} f_l(x)dx - p_1q_1$$

$\frac{d}{dp_1}\pi_d(p_1) = [f_l(q_1) - p_1]\frac{dq_1}{dp_1} - q_1 = -q_1 < 0$ (3)

y

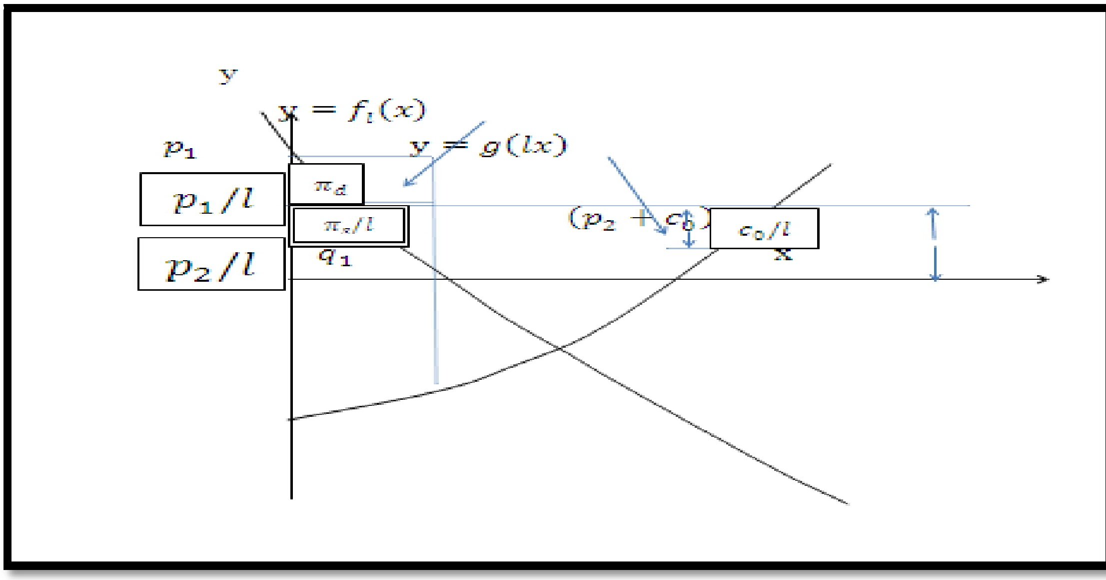

Figure 2: the Benefit of Various Groups Corresponded by the Price-Volume (p_1, q_1) *when l* > 1

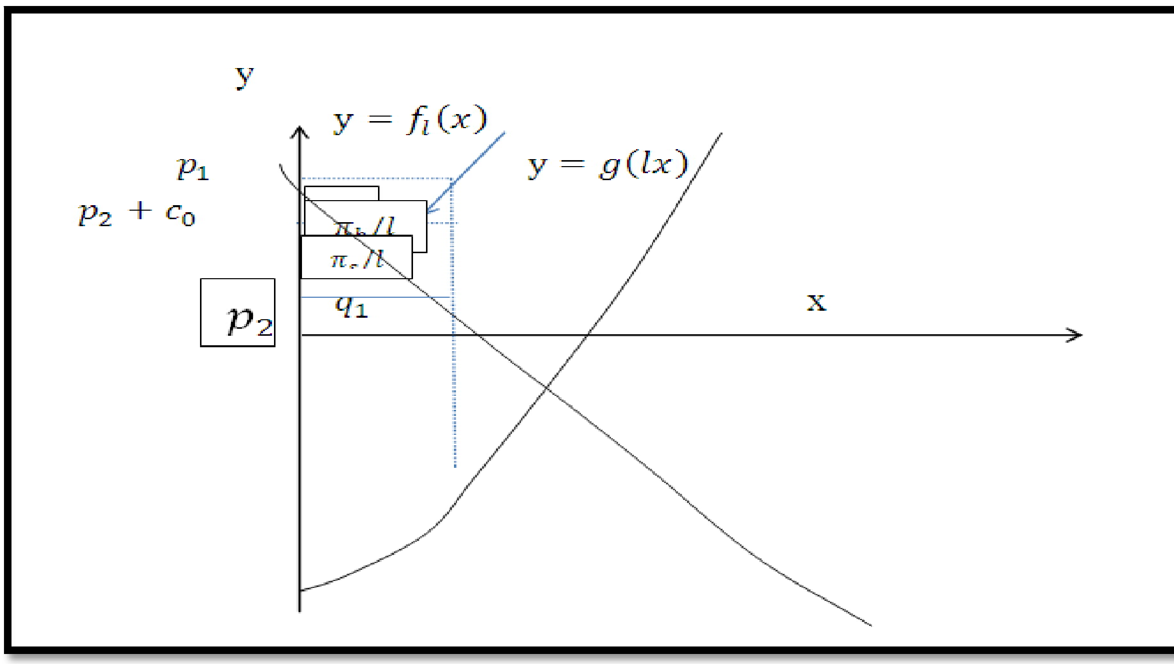

Figure 3: The Benefit of Various groups Corresponded by the Price-Volume (p_1, q_1) *when l*≤ 1

When the parking convenience index *l* is given, the average rental time per unit parking space is $\frac{1}{l}\odot 1$, symbol $\frac{1}{l}\odot 1$ refers to the minimum one among $\frac{1}{l}$ and 1. Therefore, the average net profit per unit time of p_1 corresponded parking space match-making company $\pi_b(p_1)$ is: the net profit per unit time per unit parking space waiting to be parked $\left[(p_1 - p_2 - c_0)\left(\frac{1}{l}\odot 1\right)\right]$ times the number of parking space waiting to be parked q_2 ,
where $q_2 = lq_1$, therefore, by using (1) and (6) , the profit per unit time of the parking space match-making company $\pi_b(p_1)$ can be obtained (see Figure 2 and Figure 3), which has the following characteristics

$\pi_b(p_1) = (p_1 - p_2 - c_0)\left(\frac{1}{l}\odot 1\right)lq_1$ and

$\frac{d}{dp_1}\pi_b(p_1) = (1\odot l)\left[\left(1 - \frac{dp_2}{dp_1}\right)q_1 + (p_1 - p_2 - c_0)\frac{dq_1}{dp_1}\right]$ (4)

Where the first term in the bracket is a positive value, while the last term in the bracket is a negative value (see (1) and (8)) .

Since the function $g(x)$ of the price-volume relationship of parking space supply $p_2\left(\frac{1}{l}\odot 1\right) = g(q_2)$ is the marginal cost of parking space supply quantity x, by using integration variable, $\int_0^{q_2} g(x)dx = \int_0^{q_1} g(lx)l dx$ is obtained, which is the total cost of parking space supply quantity lq_1. Therefore (p_1q_1) , $q_1 = f_l(p_1)$, the corresponding profit of the parking space supplying group $\pi_s(p_1)$ is equal to: the total income of the parking space supplying group $p_2\left(\frac{1}{l}\odot 1\right)lq_1$ minus its total cost (see Figure 2 and Figure 3), which means

$\pi_s(p_1) = p_2\left(\frac{1}{l}\odot 1\right)lq_1 - \int_0^{q_1} g(lx)ldx$

$$\frac{d}{dp_1}\pi_s(p_1) = (1\odot l)\left[q_1\frac{dp_2}{dp_1} + p_2\frac{dq_1}{dp_1}\right] - g(lq_1)l\frac{dq_1}{dp_1}$$

$$= [(1\odot l) - l]p_2\frac{dq_1}{dp_1} + (1\odot l)q_1\frac{dp_2}{dp_1} \quad (5)$$

Where the front term of (5） is greater or equal to 0 (When $l > 1$, the value is greater than 0; when $l \leq 1$, the value is equal to 0); the back term of (5) is less than or equal to 0 (see (8））.

If the parking space match-making company is requested by the government, the weighted average of (3）, (4）, and (5）is used:

$$\pi(p_1) = w_1\pi_d(p_1) + w_2\pi_s(p_1) + w_3\pi_b(p_1)$$

used as its business operation target function; then the mathematical model of the parking space match-making company is described as below

$$\begin{cases} Max_{p_1}\ \pi(p_1) = w_1\left[\int_0^{q_1} f_l(x)dx - p_1q_1\right] + w_2\left[p_2\left(\frac{1}{l}\odot 1\right) - \int_0^{q_1} g(lx)ldx\right] + w_3(p_1 - p_2 - c_0)\left(\frac{1}{l}\odot 1\right)lq_1 & (6) \\ \text{限於：} p_1 = f_l(q_1);\ p_2\left(\frac{1}{l}\odot 1\right) = g(lq_1)\ ;\ p_1 \geq p_2 + c_0 & (7) \end{cases}$$

Where l , w_1 , w_2 , w_3 are all non-negative model exogenous variables (parameters). $w_1 + w_2 + w_3 = 1$ and f_l and g are all model exogenous functions. Even though they are model exogenous variables, the given l value must contain p_1 to allow the inequality of (7）to hold：$p_1 = p_2 + c_0$

3. Optimization Solution

Assuming the best solution for the above-mentioned model exist, and it is represented by the symbol p_1^*. Given p_1 and let $q_1(p_1)$, $p_2(p_1)$, $q_2(p_1)$ satisfies the following equation, respectively： $p_1 = f_l(q_1)$, $l = \frac{q_2}{q_1}$, $p_2\left(\frac{1}{l}\odot 1\right) = g(lq_1)$ and use symbol q_1^* , p_2^* , q_2^* to represent：$q_1 = f(p_1)$ ‧ $p_2 = \left(\frac{1}{l}\odot 1\right)^{-1} g\left(lf_l^{-1}(p_1)\right)$ and $\frac{d}{dp_1}p_2\left(\frac{1}{l}\odot 1\right)^{-1} g'\left(lf_l^{-1}(p_1)\right)lf_l^{-1}(p_1) < 0$ (8)

Consider (6）and integrate with respect to p_1 and use (3） (4） (5）to obtain

$$\frac{d}{dp_1}\pi(p_1) = w_1\frac{d}{dp_1}\pi_d(p_1) + w_2\frac{d}{dp_1}\pi_s(p_1) + w_3\frac{d}{dp_1}\pi_b(p_1)$$

$$= -w_1q_1 + w_2\left[\left((l\odot 1) - l\right)p_2\frac{dq_1}{dp_1} + \left(\frac{1}{l}\odot 1\right)q_1\frac{dp_2}{dp_1}\right]$$

$$+w_3(1\odot l)\left[\left(1 - \frac{dp_2}{dp_1}\right)q_1 + (p_1 - p_2 - c_0)\frac{dq_1}{dp_1}\right] \quad (9)$$

Therefore, the criteria for the best solution p_1^* is as follows

Best solution p_1^* primary condition：

$$0 = \frac{d}{dp_1}\pi(p_1^*)$$

$$= -w_1q_1^* + w_2\left[\left((l\odot 1) - l\right)p_2^*\frac{dq_1}{dp_1} + (l\odot 1)q_1^*\frac{dp_2}{dp_1}\right]_{p_1 = p_1^*}$$

$$+w_3(1\odot l)\left[\left(1 - \frac{dp_2}{dp_1}\right)q_1^* + (p_1^* - p_2^* - c_0)\frac{dq_1}{dp_1}\right]_{p_1 = p_1^*} \quad (10)$$

Best solution p_1^* secondary condition：

$\frac{d}{dp_1}\pi p_1$ in the neighborhood of p_1^* is the reduced function of p_1 (11)

4. Best Solution Characteristics

Hypothesis 1 the criteria for best solution p_1^* of model (6） are verified in equation (10） and equation (11） mentioned above： from the proof process of (1）, (10）, and (11）

Hypothesis 2 (The best parking space rental fee p_1^* of the complete non-profit parking space match-making company) Assuming $w_3 = 0$, meaning parking space match-making company accepting government's order：As long as its income is adequate to support the cost in inequality (7） and hold. Regardless of its profit to pursue maximum weighted average in the benefit of parking space demander, consumer surplus and parking space supplier.

(i) If $l \le 1$, then $p_1^* = p_2^* + c_0$

And p_2^* satisfies: $p_2^* = g\left(lf_l^{-1}(p_2^* + c_0)\right)$ (12)

(ii) If $l > 1$, then p_1^* satisfies

$0 = -w_1 q_1^* + w_2\left[(1-l)p_2^* \frac{dq_1}{dp_1} + q_1^* \frac{dp_2}{dp_1}\right]_{p_1=p_1^*}$ (13)

Proof: from the assumption of $w_3 = 0$, the third term of (9) is equal to 0

(i) When $l \le 1$, since the first two terms of equation (9) are negative values (see (8)), equation (9) will always be negative value. Under the condition that inequality (7) must hold, the value of p_1 must be reduced as much as possible in order to increase $\pi(p_1)$ to pursue p_1^*. It is also known from (8) that when p_1 increases, p_2 decreases, therefore, the best solution p_1^* must allow its corresponding inequality (7) to occur when the equal sign holds, which means the equation $p_1^* = p_2^* + c_0$ must hold. Furthermore, by using (2) and (8), equation (12) can be verified.

(ii) If $l > 1$, equation (15) can be verified by using (10).

Hypothesis 3 (The best parking space rental fee p_1^* of the complete profit parking space match-making company), if $w_3 = 1$ (parking space match-making company pursues maximum profit as its business operation target), its best solution satisfies

$\frac{1}{p_1^* - p_2^* - c_0}\left[\frac{d}{dp_1}(p_1 - p_2 - c_0)\right]_{p_1=p_1^*} = \frac{1}{q_1^*}\left[\frac{-dq_1}{dp_1}\right]_{p_1=p_1^*}$ (14)

Where the term on the left side of equation (14) means:

When parking space match-making company's unit parking space per unit time is rented, the percentage increase in net income $(p_1^* - p_2^* - c_0)$ with the unit rental fee.

On the other hand, the term on the right side of equation (14) means:

The percentage decrease of front-end market parking space demand $q_1^* = q_1(p_1^*)$ resulted from the increase of p_1^* for unit parking space per unit time.

Proof: Since $w_3 = 1$ and $w_1 + w_2 + w_3 = 0$, therefore $w_1 = w_2 = 0$, substitute into (10) to obtain (14).

Hypothesis 4 (The best solution p_1^* that pursues maximum parking space demander consumer surplus), If $w_1 = 1$, meaning that under the condition when the income from parking space match-making company itself and parking space supplier is adequate to support the cost (inequality (7) holds) to pursue maximum parking space demander consumer surplus; then its best solution p_1^* is equal to $p_2^* + c_0$, where p_2^* satisfies: $p_2^* = g\left(lf_l^{-1}(p_2^* + c_0)\right)$ (15)

Proof: From $w_1 = 1$ obtained $w_3 = w_2 = 0$ (since w_1, w_2, $w_3 \ge 0$ and $w_1 + w_2 + w_3 = 1$), obviously (the corresponding consumer surplus of the best solution of $w_1 = 1$ is symbolized as $p_1^*(w_1 = 1)$) >(The corresponding consumer surplus of the best solution of $w_3 = 0$ is symbolized as $p_1^*(w_3 = 0)$). From Figure 1 or Figure 2, it is known that: $p_1^*(w_1 = 1) < p_1^*(w_3 = 0)$ (16)

This means that from Figure 1 or Figure 2, we can obtain: To allow maximization in parking space demander consumer surplus, p_1^* should be reduced as much as possible under the condition that inequality (7) holds; in addition, it is known from (8) that reducing p_1^* also reduces $p_1^* - p_2^* - c_0$, therefore

$p_1^*(w_1 = 1)$ must satisfies: $p_1^*(w_1 = 1) - p_2^*(w_1 = 1) - c_0 = 0$, resulting in the verification of equation (15).

Hypothesis 5 (the best solution p_1^* to pursue maximum profit for parking space supplier) If $w_2 = 1$, meaning that under the condition when the income from parking space match-making company itself p_1 is adequate to support the unit cost $p_2^* + c_0$, to pursue maximum profit for parking space supplier, then its best solution p_1^* equals to $p_2^* + c_0$, where p_2^* satisfies (15) (17)

Proof: From $w_2 = 1$ obtained $w_3 = w_1 = 0$, further, from Figure 1 and Figure 2, we get

$p_2^*(w_2 = 1) > p_2^*(w_3 = 0)$ (18)

From the above equation and (8), we get

$p_1^*(w_2 = 1) < p_1^*(w_3 = 0)$ (19)

From Figure 1 or Figure 2, we get: Under the assumption of $w_2 = 1$, To allow maximum profit for parking space supplier, p_2 should be increased as much as possible under the condition that inequality (7) holds; holds; in addition, it is known from (8) that increasing p_2 will reduce $p_1 - p_2 - c_0$, therefore

$p_1^*(w_2 = 1) - p_2^*(w_2 = 1) - c_0 = 0$, resulting in the verification of (17)

Hypothesis 6. (i) When $l \le 1$, the parking space match-making company will face the following 3 problems: the best solution problem to allow maximization in weighted benefit for parking space demander and parking space supplier, the best solution problem to allow maximization in profit for parking space demander, and the best solution problem to allow maximization in benefit for parking space supplier, which have the same best solution $p_1^* = p_2^* + c_0$. When $l \le 1$, the above mentioned 3 problems will become the same problem.

(ii). When $l > 1$

$p_1^*(w_1 = 1) < p_1^*(w_3 = 0)$, and $p_1^*(w_2 = 1) < p_1^*(w_3 = 0)$

Proof：(i）can be verified from (12）(15） and (17）
(ii）can be verified from (16）and (19）
Hypothesis 7. (i）If ≤ 1 , then w_1，w_2，w_3 corresponded best solution p_1^* is symbolized as $p_1^*(w_1，w_2，w_3)$, which is always less than $w_3 = 1$ corresponded best solution p_1^*, which is p_1^*(with any given w_1，w_2，w_3） $< p_1^*(w_3 = 1)$
(ii）If > 1 , then at least one of the following 2 inequality holds,
p_1^*(with any given w_1，w_2，w_3） $< p_1^*(w_3 = 1)$ or
p_1^*(with any given w_1，w_2，w_3） $< p_1^*(w_2 = 1)$ (20）
Proof：(i）Since $l \leq 1$，$(l\odot 1) - l = 0$，
Use (8） to get：two terms on the right side of (10）are all negative values
Therefore, the plot of (10) when $w_3 = 1$, and the plot of $\pi(p_1)$ with any given w_1，w_2，w_3 are shown in Figure 4 (see (10）and (11））

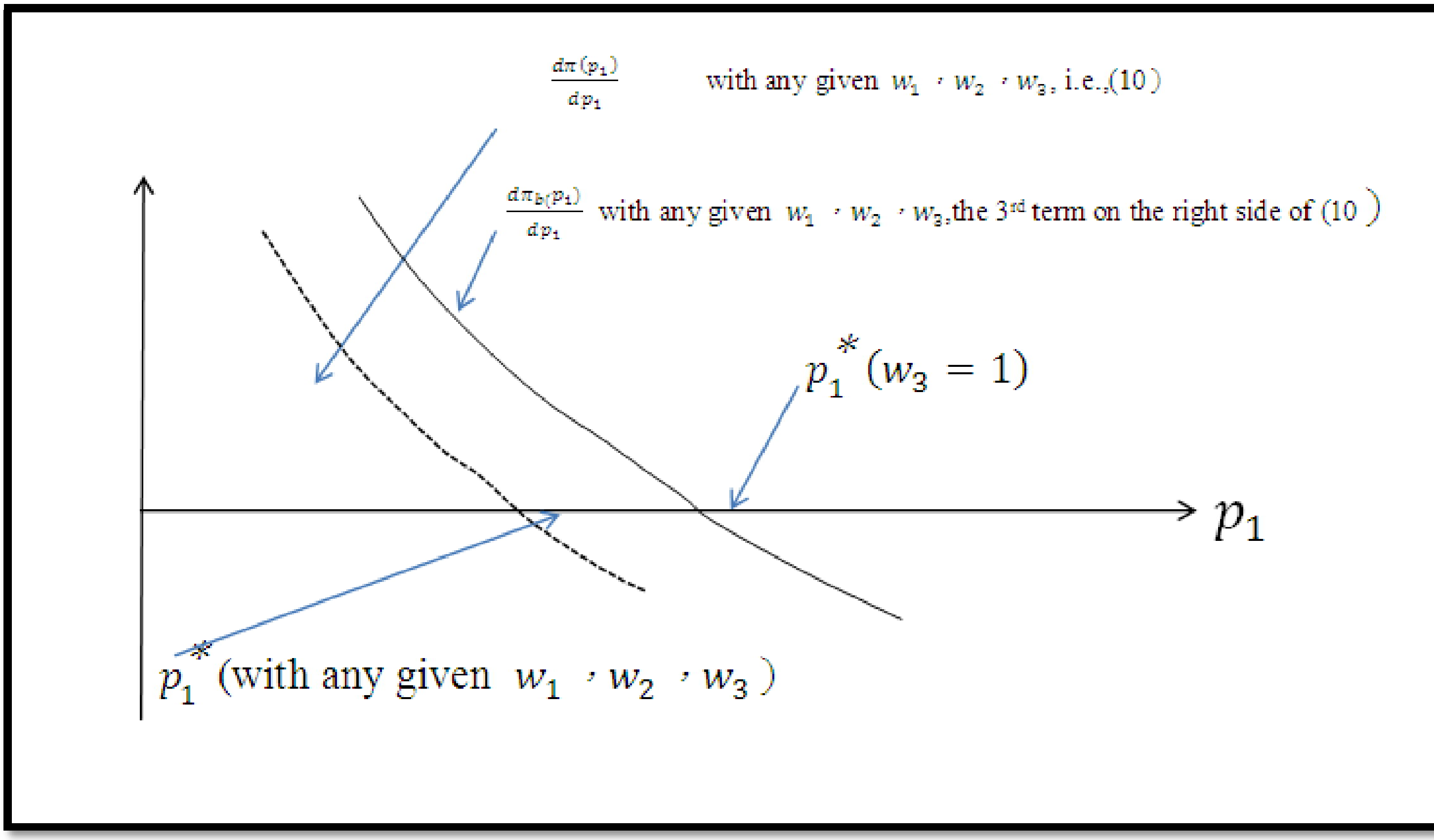

Figure 4: The Plot of p_1^* (with Any Given w_1，w_2，w_3） $< p_1^*(w_3 = 1)$ When $l \leq 1$

(ii）The proof by contradiction is as follows：If (20）is false, then the 3rd term and 2nd term on the right side of (10) can be obtained from (10） and (11）. Since p_1^*(with any given w_1，w_2，w_3）points are all negative values and the 1st term on the right side of (10) is always negative, this makes the value on the right side of the equal sign of (10) not equal to zero, which is the value on the left side of the equal sign of (10), resulting in contradiction.

The above hypotheses are the characteristics of the best solution after providing parking convenience index l, which means p_1^* will vary with l and can be represented by $p_1^* = p_1^*(l)$. With regards to the parking space demander, the price paid for a parking space per unit time not only is the parking fee $p_1^*(l)$, it also includes the average parking time cost due to parking convenience index, which starts from the point when the driver decided to find a parking space to the point when the driver finds a parking space. If $t(l)$ is used to represent the average length of lead time spent for parking; $C(t(l))$ is used to represent time cost spent corresponding to t; T represents the average length of time for subsequent consecutive parking after each parking, then the average parking cost per unit time is
$p_1 + C(t(l))/T$, where $C(t(l))/T$ is the indirect cost per unit time (21）

The reason why when we are describing parking space demand function f_l previously, a subscript l on the function symbol f is required. In fact, after providing l, the above mentioned $t(l)$, T value and parking time cost $C(t(l))$ can be obtained during practical parking space rental. Moreover, by utilizing market survey, the demand function f_l and supply function g can be determined, which can then be substituted in to equation (10) to obtain the best solution $p_1^*(l)$.

Hypothesis 8 (How the parking convenience index l is determined by the parking space match-matching company）. After providing the average price per unit time k, make s_k as： $s_k = \{(p_1, l)|p_1 + C(t(l))/T = k\}$, meaning that s_k are all pairings (p_1, l) that makes (21）always equal to constant k. Obviously, any element (p_1, l) of s_k corresponded parking space demander will have the same average parking price (all equal to constant k). The parking space match-making company will then choose from s_k a pairing (p_1, l) that gives maximum target value, which is represented by $(p_1, l)_k$. Then the target value of $(p_1, l)_k$ for different k values are compared to determine the best k value, which is represented by k^*, and k^* corresponding $(p_1, l)_{k^*}$ value.

5. Conclusions

Any production system that is invested must have a driving force that keeps the system to operate. Such driving force not only is limited to energy, but also includes targeted mechanism that consumes energy. For example, a vendor operates its production robots and personnel to produce merchandise and sell them in the market in exchange of income. This suggests that "operate" and "interexchange" are important attributes that make the production system to work effectively. Whether or not the interexchange behavior resulted after the system operates will "catalyze automatically" the next operation to continue production will be the key for the sustainable development of the system, because this will affect the lifetime and scale of the system, which is developed from "nothing" to "something". If the system developed can be "catalyzed automatically", then the system is referred to as a "good formation" system or mechanism as mentioned in this study. An example of the "good formation" mechanism is planting of trees "by people" in the beginning to reduce carbon dioxide. As the trees grow big and the number of trees increases over time, the reduction of carbon dioxide can be carried out automatically (by the increased number of trees and the increased CO2 consumption of big trees) "without any man-made action". The parking space match-making mechanism reported in this study not only is a "good formation" mechanism, but also a mechanism that exhibits: preference towards the benefit of parking space demander, preference towards the benefit of parking space supplier, or preference towards the benefit of parking space match-making company. The mechanism has generic attributes which gives flexibility in selecting target function.

The main content of this study is to convert the price-volume relationship of the two-parking space demand-supply markets formed by parking space demander, parking space supplier and parking space match-making company assigned by the government into specific mathematical model.

This study also demonstrated the criteria for best solution as well as the characteristics of the best solution. It is hoped that the results of this study will serve as useful reference to benefit the design of many underground parking lots that will soon be constructed in many new buildings of metropolis areas in Taiwan.

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