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Prediction of the Volatility of Ghana Stock Exchange Main Index by Using the Generalised Autoregressive Conditional Heteroskedasticity Model

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Abstract:
Since 2014, the Ghana Stock Exchange (GSE) main index has experienced turbulent stock market volatility and its overall performance continues to follow a downward trend. Consequently, this has awakened unprecedented interest in key stakeholders in order to halt the downward performance of the index. The call for further investigations and research into the stock market volatility put this study in the forefront. The first objective of this research was to analyse the statistical properties of GSE main index. The second objective was the determination of relation between stock price movement and volatility. Finally, the study also sought to find out the extent to which the prediction of volatility and returns of GSE main index could minimise the risk incurred by investors. The most empirically proven model called the Generalised Autoregressive Conditional Heteroskedasticity (GARCH), which takes care of all the statistical properties and stochastic dynamics of asset returns was used in capturing the stylish features of GSE main index: volatility clustering, excess kurtosis, leverage effects and unit roots. The distribution innovations of the residuals term considered in the application of the GARCH model were normal, Student't' and Generalised Error Distribution (GED). The result from the study indicated that EGARCH (1, 1) model with GED was the best model fitted for the GSE main index series during the in-sample period estimation. It is also observed surprisingly that higher GARCH orders could not out-perform lower GARCH orders. The prediction of volatility and returns on assets of GSE main index was good. The results of the prediction were validated by tools like Root Mean Square Error, Theil Inequality coefficient and simple regression line. On the whole, investors can possibly rely on the findings of this study to make further financial decisions regarding stock market volatility.

Keywords: Volatility clustering, prediction, GARCH, assets returns

1. Introduction

1.1. Statement of the Problem

One of the emerging markets in West Africa is the Ghana Stock Exchange (GSE). It was established in 1990. The purpose for its establishment was to deregulate the financial structure of the country. For more than a decade, the GSE main index, a market capitalization index which measures the performance of the entire market, performed creditably. This led to the listing of banks, insurance companies, and brokerage and asset management firms on the stock market. By the end of 2013, the index made a gain of 78.81 % with total market capitalization of GHS 11 694.93 (Anon., 2014a).

However, at the beginning of 2014, the microeconomic variables became very volatile in the country. The increase in prices of goods and services led to high inflation rate of 15.9 %. The high interest rate of 25% invariably induced high cost of borrowing money in the financial sector. The crude oil price moved up to US $ 112 by the middle of 2014. Consequently, domestic petroleum prices were adjusted upward between the ranges of 13.9 % and 29.2 %. On 14th September, 2014, the Bank of Ghana Monetary Policy Committee (B0G-MPC) Press Release indicated that the Ghana cedi depreciated by 26.7 % (Anon., 2014b). This also reflected the volatility in the foreign exchange market and affected the country's economic performance negatively (Anon., 2015a).

All these variables caused adverse market movement on the GSE. This confirmed the empirical statement that the stock market is uncertain or risky (Laopodis, 1997). At the end of 2014, the GSE main index performed abysmally with a record low gain of 6.5 %. The losses on investment completely derailed the future expectations of the investors. The current volatility of the GSE is so discouraging that most investors are losing confidence in the stock market. The issue is, if

care is not taken, investors remaining in the country would leave to different prominent markets. Of much concern to investors and the investing public is how to measure the future volatility (risks) of their investment on the market.

Investors want to know the amount of risk associated with their investment portfolio in order to avoid or minimise future losses. The prediction of market volatility and expected returns is, however, a major challenge even to experts in the field of finance. The Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model which takes care of all the statistical parameters and stochastic nature of volatility is considered to be a robust method for studying volatility.

Some researchers have used methods such as Historical Volatility models and Implied Volatility models to predict volatility. The main weaknesses of these models are the underlying assumptions that asset returns follow a normal distribution and volatility is always constant. Some of the models tend to significantly overestimate volatility and are not suitable for all financial products (Sergiy, 2009). Empirical evidence also suggests that financial asset returns are not an independently and identically distributed process and volatility is non-constant (Carol, 2007).

Apparently, there is not enough literature on GSE volatility forecasting. However, Okyere and Abakah (2000) used the GARCH model to study the exchange rate volatility. Also, Frimpong and Oteng-Abayie (2006) used the GARCH model to investigate Databank Stock Index (DSI) volatility. Unarguably, the GARCH models are able to analyse volatility (risks) of holding an asset and predict the long-run average volatility (Bollerslev, 1986; Engle, 1982 and Lars, 2002). But because researchers do not focus on the GSE main index, their works do not completely address the concern of investors that is how to measure the future volatility of their investment in GSE. This study focuses on predicting volatility of the GSE main index (which comprises the GSE Composite Index and Financial Stock Index) by using the GARCH models.

1.1. Objectives of the Research
The objectives of the study are to:
- Analyse the statistical properties of daily GSE Composite Index and Financial Stock Index returns;
- Investigate the relation between stock price movements and volatility; and
- Predict the market volatility and returns on assets of the GSE using the GARCH models.

1.3. Methods Used
The methods used in this study are:
- Literature Review;
- Descriptive Statistics for data analysis; and
- GARCH models for prediction of volatility and returns on assets.

1.4. Organisation of the Thesis
This thesis is organised in the following manner: Chapter 1 is on introduction, Chapter 2 introduces literature review and Chapter 3 provides method used. The Chapter 4 is on data analysis and discussion of results and finally Chapter 5 dwells on conclusions and recommendations.

2. Literature Review

2.1. The Concept of Volatility
An understanding of investment and business activities such as risk management, portfolio management and derivative pricing in today's financial world is a key to reducing transaction costs and making of profits. A financial activity like purchasing tradable assets such as stocks, bonds, mutual funds, real estate, etc. with the hope of generating expected returns (or income) in the future is referred to as investment (Anon., 2015b). The expected return on these assets is crucial because of the trade-off between returns and volatility since investors want to be compensated for the risks they bear. As the saying goes, 'the higher the rate of returns on an investment, the higher the volatility'.

What then is volatility? In everyday usage, volatility refers to the fluctuations in the stock prices of assets. These fluctuations often affect the future expected returns set by the investors. However, volatility has also been defined in various ways by several authors. According to Baillier et al (1996), "it is the measure of the intensity of unpredictable changes in asset returns and it is commonly time varying dependent". Also Sergiy (2009) explains volatility to mean "the spread of all positive and negative outcomes of an uncertain variable". The spread or distribution of asset returns (uncertain variable) about the expected returns is quantified by the investment volatility.

2.2. Measurement and Effect of Volatility
In statistics, the distribution of any random values around the mean is measured by the standard deviation. In similar vein, investment volatility is measured by standard deviation or the variance of returns. Hull (1997) confirms this by stating that "volatility of a stock price can be defined as the standard deviation of the returns provided by the stock in one year when the return is expressed using continuous compounding". Since stock price changes with time, volatility can be measured or predicted over different time periods such as daily, weekly, monthly, annually or for a long horizon. Any new information reaching the market is always considered in the fixing of stock price (Fama, 1965).

This information asymmetry causes investors to change the price of an asset depending on the magnitude of the news (good or bad). For example, news about recessions, bankruptcy and economic crisis often increase volatility on the stock market. An investor like the American Chamber of Commerce warned that the recent news of judicial corruption scandal in

Ghana could negatively affect investor confidence in thecountry (Ampah, 2015). Good news about a company launching new products, declaring high dividends and high growth rate decrease the rate of volatility.

Information about adverse market movements which is contingent on volatile macroeconomic indicators can affect investor perception of future price movement. Some of these macroeconomic performance indicators are inflation rate, interest rate, exchange rate, Consumer Price Index (CPI), Producer Price Index (PPI), and Gross Domestic Product (GDP). At the beginning of 2014, some of these variables became very volatile making the economy unstable for business and investment activities to function well.

Since 2014, the increase in prices of goods and services led to high inflation rate of 15.9 %. The cost of borrowing money from the bank was very high at the rate of 25 %. By June 2014, crude oil prices on the international market went up to US $ 112. The domestic petroleum prices were also adjusted upward between the ranges of 13.9 % to 29.2 %. In September 2014, the BOG-MPC Press Release indicated depreciation of the cedi by 26.7 % against its major trading partners like US dollar. The Gold Coast Fund Management Limited (GCFM) Cedi index, a measure of the holistic performance of the cedi on the interbank market observed the cedi has depreciated by 17.85 % within the period 14th - 18th September, 2015. The cedi has been slumping back and forth for sometimes now.

A global rating agency, Fitch, noted "high domestic yields and a 60 % depreciation in the currency since 2012 have pushed up borrowing costs, with interest payments now accounting for one-third of government revenue, the highest level among Fitch-rated Sub-Sahara African sovereigns" (Ampah, 2015). The current power outage in the country is forcing Small Scale Enterprises to fold up due to high cost of production. Some of the large corporations are also laying off workers in an attempt to cut down operational costs and recurrent expenditures (e.g. wages). This is leading to a reduction in the country's GDP and also an increase in CPI because of high demand for fewer consumer goods and services produced in the country. The volatility in the foreign exchange market is affecting the importation and exportation of goods and services since the cedi is losing its strength almost every day.

The Ministry of Finance in a Press Release published that the escalating public debt stock by June 2015 stood at GHS 94.5 billion which was mainly due to exogenous factors affecting government revenues and the depreciation of the cedi. The volatile cedi has significantly bloated the debt figures when expressed as a ratio of GDP. The debt to GDP ratio increased from 59.9 % in January to 71.32 % in June, 2015. The Ghanaian economy has reached a point in which the government has resorted to the International Monetary Fund (IMF) for a bailout programme. An amount of US$918 million was approved and would be injected into the economy in tranches for a period of three years. The first tranche of US$ 114.8 million has been received to purposely halt the declining gross international reserve of the Central Bank of Ghana. According to GCFM Research Economic Report (2015), this has already contributed largely to the depreciation of the cedi by 17.68 %   as at the end of the first quarter of 2015. This has affected most importers as a result of increasing cost of borrowing and high inflation rates due to high prices of goods and services. The government has also issued bonds (at least twice) in the international market in order to raise money to resuscitate the ailing economy.

The rippling effect of this economic instability is seen in the GSE main index performance. The GSE Composite Index made a low record gain of 6.5 % at the end of 2014 as compared to 78.81 % recorded by the same index in the previous year, 2013. At the end of 2015, the index recorded a negative 11.77 %. Most investors lost their investments and confidence in the stock market again. The concern of many investors is how to measure the future volatility of their returns on the stock market.

2.3. Some Theories and Model Classifications

Some theories have been propounded concerning stock price movement. The theories underpinning stock price volatility are often based on the analysis of certain variables or indicators. Some analysts (fundamental) value stocks by considering variables like economic forecasts, revenue projections, interest rates, firm's debts, management quality and political climates. Others (technical) examine stocks value by drawing graphs of historical data to observe trends or patterns in the stock returns (Scott, 2005). Notwithstanding, it has been stated that "financial indicators are complicated by complex interconnections which are often convoluted and cannot accurately predict the value of stock returns" (Sergiy, 2009). Also, Davis, *et al.* (2012) studied the U.S. stock returns since 1926 using dozens of metrics (indicators). It was found that "many commonly cited signals had very weak and erratic correlations in the subsequent returns even at the long investment periods". Again, Anon. (2013) agreed with Sergiy and Davis that some of these indicators contradict each other most of the time. Their findings show that "expected stock returns are best stated in probabilistic framework, not as a point forecast and should not be forecast over short periods". Empirical evidence suggests mathematical models (probabilistic framework) for analysing and predicting volatility of asset returns are varied and numerous. Most of these models have gone through series of modifications which aim at improving forecast accuracy. For purposes of this study, these models are classified as Historical Volatility (HV) models, Implied Volatility (IV) models, Autoregressive and Heteroskedasticity (AH) models. The HV models consist of Historical Average, Simple Moving Average, Exponential Smoothing and Exponential Weighted Moving Average (EWMA). Some of these models give poor forecast results, and are not suitable for all financial products. However, they can be used for quick forecast.

The IV model like the Black-Scholes model is used to price derivative instruments (e.g. European and American put / call options). This model was the novel work of three prominent economists namely Fischer Black, Myron Scholes and Robert Merton (Black & Scholes, 1973). The weaknesses of these models are the underlying assumptions that asset returns follow a Gaussian normal distribution and volatility is always constant. Some assumptions of this model are still unrealistic and unattainable in this changing financial world.

The AH models comprise of Autoregressive Moving Average (ARMA), Autoregressive Conditional Heteroskedasticity (ARCH), and the Generalised Autoregressive Conditional Heteroskedasticity (GARCH). This class of

models seems to be the most robust of all for predicting future volatility of financial time series. The basic GARCH is symmetric while others are asymmetric e.g. Exponential GARCH, Integrated GARCH, GJR-GARCH, Quadratic GARCH, Component GARCH, GARCH-in Mean, etc. The GARCH-family model is able to capture all the financial time series characteristics such as excess kurtosis, volatility clustering and leverage effect.

Some researchers have used the GARCH family models to predict stock volatility in the daily returns, weekly returns, monthly returns, stock indices and also volatility of real estate index returns. For instance, French et al. (1987) studied the daily returns of S&P stock indices using the ARCH / GARCH models and found that expected market risk premium is positively related to the volatility of stock returns. Attanasio (1991) used EGARCH model   to investigate the impact of index futures trading on the price volatility of Denmark and France stock markets respectively. His findings indicate future trading has lowered price fluctuations in France while in Denmark index futures have changed stock returns distributions. There was evidence of high volatility persistence and asymmetry in these two European stock markets. Wong et al. (1988) examined the volatility spillovers between the spot and forward index returns of the Hong Kong real estate market through a bivariate GARCH model. Their results reveal that volatility of the forward market was more sensitive than the spot market and volatility was specifically transmitted from forward market to the spot market only.

Again, Caiado (2004) modeled the volatility of the Portuguese Stock Index PSI-20 by using GARCH family models: Simple GARCH, GARCH – Mean, Exponential GARCH, and Threshold GARCH. The data covered the period of January 2, 1995 to November 23, 2001 making a total of 1708 observations. This interval was mainly characterised by political instability in Europe and United States, Israel – Palestinian conflict and the terrorist attack on September 11, 2001. He found there were significant asymmetric shocks to volatility in the daily stock returns, except, in the weekly returns. The GARCH, GARCH in- Mean, Exponential GARCH and Threshold GARCH models gave a better forecast in the daily and weekly returns than in the sub-periods. The Exponential GARCH gave a better daily multistep forecast and GARCH model provided a superior weekly forecast when PSI-20 was included in the variance equation.

Of much importance is the finding that by reducing the sample period for estimation, forecast accuracy can be greatly improved. In the contrary, Figlewski and Green (1999) gave empirical evidence to the fact that having a long estimation period gives a better forecast over a long horizon.

Sergiy (2009) addressed the problems of forecasting volatility in the financial markets by testing accuracy of several models using the S & P 500 Stock Index. This index consists of 500 stocks from US companies with the largest capitalization. In all, 360 data points were sampled from December, 1978 to November, 2008. He concluded that HV models are the simplest but give poor forecast results. The IV models are difficult to implement while AH models are complex to implement and optimise. In short he indicates there is no single perfect approach to modeling and forecasting volatility of asset returns.

Figlewski (1997) finds that among all the volatility models tested, the GARCH model is more superior in forecasting stock market volatility. Akgiray (1989) also supports assertion of Figlewski that GARCH consistently outperforms EWMA and other HV models in all sub-periods and evaluation activities.

One area which has not been massively explored empirically is the relation between stock price and volatility. Bollerslev et al. (2008) indicate a negative correlation between return and volatility of assets. Also,Duffee (1995) finds there is a negative relation between returns and volatility when individual stocks return and volatility of individual firms were studied. Black (1976) referred to this market condition as leverage effect. This means an increase in the stock price of an asset leads to corresponding increase in the expected returns and a decrease in the price of an asset leads to a drop in the future expected returns. These fluctuations in prices raise volatility or risk of the assets (i.e. changes in stock prices tend to be negatively correlated with changes in volatility). A firm with a large capital base has more leverage because of its ability to absorb or minimise any future losses or volatility than a firm with a small capital. Such small firms cannot manage certain amount of risks or volatility beyond their capabilities. For this reason, the first Basel Accord in 1996 requires the setting aside of excess capital reserve to manage any future losses of the firm. One of the objectives of this research is to investigate the relation between the stock price and volatility. Thus the research would focus on finding out whether there exists a relation between the stock price and volatility of the GSE Main Index.

Financial asset return series have been noted for certain stylish features apart from leverage effect. One of these characteristics is fat-tail distribution of asset returns (excess kurtosis). It is usually thought that returns follow identically independent distributed patterns or simply put a normal distribution. However, some studies have shown empirically that returns are non-stationary and do not follow normal distributions (Wright, 1999). The third feature found to be associated with financial time series is often called volatility clustering. This means the time of high volatility is followed by the time of high volatility and the period of low volatility is followed by the period of low volatility (Peters, 2001).

It has also been reported that the long- run variance reverts to the average of the returns (i.e. mean reverting). Recent findings reveal the presence of unit roots in the variance equation, long memory property and co-movements of volatility across assets and financial markets (Poon and Granger, 2003).

2.4. Development of the GARCH Model

The systematic mathematical procedures involve in the GARCH model are shown here and most of the notations are drawn from Hull (2009). The individual stock prices $\left(s_i\right)$ are converted to logarithms return $\left(r_i\right)$ series which is given by:

$$r_i = \log s_i - \log s_{i-1} \qquad\qquad (2.1)$$

where $i = 1, 2, 3, 4 \ldots n$; $n+1$ is the number of daily observations. The standard deviation or volatility (δ_n) of the daily return series is found by:

$$\delta_n = \sqrt{\frac{1}{m}(r_{n-i} - u)^2} \tag{2.2}$$

where u is the average index returns and always assumed to be zero and m is the moving average recent observation. Since the variance is also used to calculate the volatility, thus the variance becomes:

$$\delta_n^2 = \frac{1}{m}\sum_{i=1}^{m} r_{n-i}^2 \tag{2.3}$$

In order to add more weights to recent data, a model similar to model (2.3) is written as:

$$\delta_n^2 = \frac{1}{m}\sum_{i=1}^{m} \alpha_i r_{n-i}^2 \tag{2 4}$$

where $\sum_{i=1}^{m} \alpha_i = 1$. This model is extended by assuming that there is a long-run average variance rate, v_l, which is given a weight of γ. Hence the model is written as:

$$\delta_n^2 = \gamma_{v_l} + \sum_{i=1}^{m} \alpha_i r_{n-i}^2 \tag{2.5}$$

where $\gamma + \sum_{i=1}^{m} \alpha_i$.

This Equation (2.5) is the ARCH (m) model proposed by Engle (1982). Bollerslev (1986) extended the ARCH model to include the residual or error factor denoted by δ_{n-i}^2 with the weight of β and order k. The resulting equation is called the GARCH model (Bollerslev, 1986) and written as:

$$\delta_n^2 = \gamma_{v_l} + \sum_{i=1}^{m} \alpha_i r_{n-i}^2 + \sum_{i=1}^{k} \beta_i \delta_{n-i}^2 \tag{2.6}$$

where $\gamma + \alpha + \beta = 1$. The model is stable when $\alpha + \beta < 1$. In order to make the predicted conditional variance always positive, it is required to add regressors denoted by θ_n^r in the variance equation and replacing γ_{v_l} by ϖ. The Equation becomes:

$$\delta_n^2 = \varpi + \sum_{i=1}^{m} \alpha_i r_{n-i}^2 + \sum_{i=1}^{k} \beta_i \delta_{n-i}^2 + \theta_n^r \pi \tag{2.7}$$

The EGARCH, TGARCH OR GJR models help to determine the relation between the stock price movement and volatility. This is most often referred to as leverage effect or asymmetry effect. This means that when volatility rises then the stock price falls and when it falls, the stock price rises. It is observed there is a negative relation between stock price movement and volatility on the stock market (Schwert, 1989).

The most common distribution assumptions about the residual (error) term often employ in ARCH-GARCH models are: normal (Gaussian) distribution, student's t- distribution and the Generalised Error Distribution (GED). Based on any chosen distributions, the parameters of the models are estimated by the method of Maximum Likelihood.

3. Methods Used

3.1. Descriptive Statistics of GSE Main Index Returns

For the preliminary analysis of data, the summary descriptive statistics is obtained to specify among other things the minimum and maximum values, mean, standard deviation, skewness and kurtosis of the data. The skewness (S) measures the symmetrical shape of a distribution. Mathematically, it is calculated as:

$$S = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{r_{i-\mu}}{\hat{\delta}}\right)^3 \tag{3.1}$$

where $\hat{\delta}$ is an estimator for standard deviation.

A normally skewed distribution has a symmetric bell shape whereas positively and negatively skewed distributions have a long right and left tail respectively.

The kurtosis (k) measures the peakedness or flatness of the shape of a distribution. It is calculated as:

$$k = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{i-\mu}}{\hat{\delta}}\right)^4 \quad (3.2)$$

It can either be leptokurtotic (where k exceeds 3) or platy kurtotic (where k is less than 3).

3.2. Testing for Normality in GSE Main Index Returns

The next step is to carry out the normality test of the GSE-CI and GSE-FSI returns. The Jacque-Bera (JB) test, the Quantile - Quantile plot (QQ -plot) and the histograms are often used in testing for normality of the returns. The JB statistic tests whether the returns follow normal distributions. It also indicates the differences between the skewness and kurtosis of a distribution. It is written as:

$$JB = \frac{N}{6}\left(S^2 + \frac{(k-3)^2}{4}\right) \quad (3.3)$$

where S is the skewness and k is the kurtosis.

The JB statistic is distributed as Chi-square with 2 degrees of freedom. Under the normality testing, the following hypothesis would be conducted:

H_0 : The GSE-CI and GSE-FSI returns are normally distributed;

H_a : The GSE-CI and GSE-FSI returns are not normally distributed.

When the p-value of JB statistic is greater than 5% significance level, one would fail to reject the null hypothesis which is desirable for a good fit model.

The QQ-plot is the plotting of quartiles against its theoretical quartiles. In other words, quartiles are points that show shift in location, scale, changes in symmetry and presence of outliers in the series. These shifts and changes would happen along a constructed 45 degrees reference line. When the points are linear with this reference line, then it indicates that the series comes from the same normal distribution. To double check the normality of the series, histograms of the distributions are drawn to prove whether the distributions have a bell shape.

3.3. Testing for Serial Correlation in GSE Main Index Returns

The GARCH model requires that the residuals (errors) of the returns are not serially correlated. The serial correlation test is performed on the GSE-CI and GSE- FSI returns to ascertain whether they are serially correlated or not. The following tests are mostly used for this purpose: Ljung Box Q - statistics tests, Breusch-Pagan-Godfrey (BPG) LM test and Durbin Watson (DW) test. The Ljung - Box Q-statistic normally tests high order serial correlations in the series and is written as:

$$Q^* = n(n+2)\sum_{i=1}^{k}\frac{1}{(n-1)^i}\rho_i^2 \quad (3.4)$$

where n is the sample size, h is the lag (may be equal to degree of freedom) and ρ is the autocorrelation, and i is the individual observations. It is Chi-squared distribution with degrees of freedom equal to the number of autocorrelations.

The Q-statistic comes with both the Autocorrelation (AC) and Partial Autocorrelation (PAC) functions. The AC and PAC are measures of the relation between current returns and past returns and show which past returns are most useful in predicting future returns. The graph of AC and PAC is called correlogram. The Correlogram lies within 95 % confidence interval of the normal curve. The Q-statistic is significant when the probability value is less than 5% level and vice versa. The correlogram is applied in two ways. First, it helps to identify the order of an Autoregressive (AR) and a Moving Average (MA) processes. When the AC is significant at first lag and all subsequent lags are not then it indicates first- order MA process. An AC which dies off more quickly geometrically with increasing number of lags is termed first- order AR returns series. The PAC coefficients normally display up to a certain specified order of lags. The PAC shows AR series if the coefficients cut off at a certain number of lags whilst a MA series will gradually approximate to zero. Secondly, the correlogram can be used to check the randomness of the returns series. When the AC coefficients are near zero for all lags then the returns series are said to be random.

The BPG serial correlation LM test is carried out on the GSE-CI and GSE-FSI returns to check for serial correlation dependency in the mean Equation (2.5). The probability of the Obs.*R-squared statistic in the BPG - LM test output is the BPG - LM test statistic. It is computed as the number of observations times the R-squared from the Ordinary Least Square (OLS) regression. This is the regression of the residuals on the original regressors and lagged regressors up to a certain order. The LM test is chi-squared distributed. The Obs.* R- squared value is used to determine the presence of the serial correlation in the residuals at 5% significance level. The hypothesis to be tested under the serial correlation is:

- H_O: The returns of the GSE-CI and GSE- FSI are not serially correlated;
- H_a: The returns of the GSE-CI and GSE-FSI are serially correlated.

For our model to be well fitted, it is required that the null hypothesis is preferred to the alternative hypothesis. This is because OLS are no longer Best Linear Unbiased Estimator (BLUE), standard errors and test statistics (e.g. student- t and F- statistic) are no longer valid hence the need to adopt a different method to correct autocorrelation. The removal of serial correlation from the returns includes the following approaches: Heteroskedasticity-Autocorrelation Consistent

(HAC) standard errors method, Feasible Generalised Least Squares (FGLS) and the use of Cochrane - Orcutt method. In this study, HAC Newey–West standard errors method is used.

3.4. Testing For Heteroskedasticity in GSE Main Index Returns

A random distribution of residuals on a scatter graph is sometimes believed to be a constant variance and follows an identically independently distributed sequence (i.i.d). This assumption of constant variance (often called homoskedasticity) is no longer applicable to financial time series. This is proven by numerous researches confirming the non-constant variation in returns commonly referred to as heteroskedasticity. The Breusch-Pagan-Godfrey (BPG-LM) and White Heteroskedasticity tests (White, 1980) are used to check heteroskedasticity in the returns. The BPG-LM test is based on AR (q) model which is written as

$$u_n = \alpha_1 u_{n-1} + \alpha_2 u_{n-2} + \cdots \alpha_q u_{n-q} + v_n \quad (3.5)$$

Thus the hypothesis to be tested is as follows:

H_O: There is no heteroskedasticity in the returns of GSE-CI and GSE-FSI;

H_a: There is heteroskedasticity in the returns of GSE-CI and GSE-FSI.

It is desirable to reject the null hypothesis of no heteroskedasticity in favour of the alternative. A large chi-squared value indicates the presence of heteroskedasticity. When a series is heteroskedastic, then the following techniques can be used to correct it. First, the model is respecified by transforming the variables using Equation (2.1). Second, use robust standard errors method and third by employing weighted least squares. It is important to note that OLS is no longer BLUE of the variance. This is because its estimated variance is not small enough to be used for hypothesis testing and forecasting.

The White Heteroskedasticity test generally does not make any assumption about the form of heteroskedasticity and can identify specific errors. Thus it is used to check for the remaining heteroskedasticity by allowing for more regressors to be added. The White test is a special case of the BPG – LM test. These two tests are fitted by the residuals. The White test statistic is asymptotically distributed as a chi-square with degrees of freedom equal to the number of slope coefficients excluding the constant, in the test regressions.

3.5. Testing for Stationarity of GSE Main Index Returns

The Unit Root (and non-stationarity) test checks whether a series has transitory or permanent effect by using AR model. A non-stationary series can be made stationary by taking the first difference of the variable or by logarithm transformation. This supports the random walk model. To carry out the unit root test, the following steps are required:

- Transform data series by taking its logarithms;
- Choose the model to be tested e.g. ARCH and GARCH model;
- Determine appropriate number of lags by using Bayesian Information Criterion(BIC)
- Conduct unit root test at multiple lags.

ADF test recommends maximum number of lags in order to assess the significance of the coefficients of the lag terms.

3.6. Basic GARCH Model Specifications

A unique characteristic of a stock volatility is that it is indirectly observable. A stock price as an input in IV models like the Black- Scholes formula is often used in derivative markets (e.g. options markets). The IV model among other things assumes volatility or variance to be constant always. This assumption has been heavily criticized by practitioners and experts in finance. Some also predict volatility using sample standard deviations (like the HV models) over a short period of time with the assumption that volatility is constant. This model too is plagued with inconsistency since volatility is never constant as time elapses. The GARCH model uses exact function to predict the conditional variance. The variance of the dependent variable is predicted as a function of passed values of the dependent variable and independent or exogenous variables. The ARCH and GARCH models are restated as follows:

$$\delta_n^2 = \varpi + \sum_{i=1}^{m} \alpha_i r_{n-i}^2$$

$$\delta_n^2 = \varpi + \sum_{i=1}^{m} \alpha_i r_{n-i}^2 + \sum \beta_i \delta_{n-i}^2$$

where $\varpi, \alpha \ and \ \beta$ are the model parameters, r_n is GSE–CI or GSE–FSI returns, $r_{(n-i)}^2$ is the ARCH term (lag of squared returns), δ_{n-i}^2 is the GARCH term (previous period forecast variance).

The basic GARCH (m, k) model is symmetric and cannot determine the leverage effects observed in financial time series. There are different types of the GARCH model depending on the order of the parameters chosen. For example, GARCH (0, 1), GARCH (0, 2), GARCH (0, 3), GARCH (1, 1), GARCH (1, 2), GARCH (2, 1), and GARCH (2, 2). These basic GARCH types can capture features like volatility clustering in returns and volatility shocks to persistence. The GARCH model has also been modified in various forms purposely for capturing other characteristics of financial time series such as the leverage effects, asymmetric information (news impact) and distribution properties of returns, which the basic GARCH model is unable to determine. Some of these extensions include Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), GJR GARCH, GARCH-in Mean, Component GARCH, Integrated GARCH (IGARCH), Quadratic GARCH (QGARCH), Fractionally Integrated GARCH (FIGARCH), and Fractionally Integrated Exponential GARCH (FIEGARCH). For instance, the

EGARCH is designed to capture the leverage effects often observe in financial series. The mean model of EGARCH   is same as Equation (2.5) and the variance model is given as:

$$\ln\left(\delta_n^2\right) = \varpi + \alpha\left(\frac{r_{n-1}}{\delta_{n-1}} - \sqrt{\frac{2}{\pi}}\right) + \gamma\frac{r_{n-1}}{\delta_{n-1}} + \beta\left(\delta_{n-1}^2\right) \qquad (3.9)$$

where ϖ is the long run variance (average / mean) of the model. When $\alpha + \beta \approx 1,$ then volatility shock is persistent. The symbol γ captures the leverage effect in the index returns. If the $\gamma \neq 0$ means asymmetry information is having an impact on stock returns and can simply be described as leverage effect.

The information (asymmetry) flow in the stock market is very crucial because to a large extent, it determines the price of a stock (Fama, 1965). This information asymmetry can be good or bad economic news. Bad economic news increases stock volatility while good economic news induces high returns and dividends to investors. The TGARCH or GJR model is named after Glosten et al. (1993) accounts for the news impact effect on the conditional variance. The GJR mean model is same as Equation (2.5) and the variance model   is stated as:

$$\delta_n^2 = \varpi + \alpha_1 r_{n-1}^2 + \gamma - r_{n-1}^2 l_{n-1} + \beta\,\delta_{n-1}^2 \qquad (3.10)$$

where $l_{n-1} = 1$ if $r_{n-1} < 0$ or $l_{n-1} = 0$ if $r_{n-1} > 0$; $\varpi \geq 0,\ \alpha \geq 0,\ \beta \geq 0, \alpha + \beta \geq 0$ (for the variance to be positive). The $r_n > 0$ represents good economic news, and $r_n < 0$ stands for bad economic news may have different influence on the conditional variance $\left(\delta_n^2\right)$. Good news affects the conditional variance through α whilst bad economic news affects through $\alpha + \beta$. Therefore the leverage effect exists when $\gamma > 0$, an indication that news impact is asymmetry.

3.7. Distribution Innovations of the Residuals
        The estimation of GARCH models by maximum likelihood function is based on certain distributions aimed at obtaining approximately good accurate forecasts. The most often used distributions are the Gaussian normal distribution, student-t distribution, Generalised Error Distribution (GED) (Nelson, 1991), skewed student-t distribution and the Normal Inverse Distribution (Jensen and Lunde, 2001). This thesis would focus on the Gaussian normal distribution, student-t distribution and GED innovations since they are most often applied to financial time series models. The Gaussian normal distribution has the density function defined by:

$$G(r) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(r-u)^2/2\sigma^2} \qquad (3.11)$$

 where the mean $u = 0$ and the variance $\left(\sigma^2 = 1\right)$. The normal distribution is symmetric in form. The density function of student 't' distribution with n degrees of freedom greater than two (2) is given by:

$$T(r) = \frac{\tau\left(\dfrac{n+1}{2}\right)}{\sqrt{n\pi}\,\phi\left(\dfrac{n}{2}\right)}\left(1 + \frac{r^2}{n}\right)^{-(n+1)/2} \qquad (3.12)$$

     Thus Equations (3.11) and (3.12) are applied during the application of the GARCH models in order to select the best model that fits the data for prediction of GSE main index returns and volatility.

3.8. Residual Diagnostic Analysis and Prediction of Volatility of GSE Main Index
        One of the objectives of this research is to predict the returns and volatility of GSE main index by choosing the best GARCH model that fit the data. The ARCH/GARCH model has two main equations: the conditional mean Equation (2.5) and the conditional variance Equation (2.6). These equations and EGARCH model are estimated simultaneously for best model selection. The selection of the model depends on the model that gives minimum values of information criterion such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan- Quinnin Criterion (HQC). Residuals diagnostic analysis would be conducted since it helps to reveal whether the following assumptions are met:
- There should not be correlation in the residuals.
- The mean of the residuals should be approximately zero.
- Residuals should have constant variance; and
- Residuals must be normally distributed.

        When these assumptions are satisfied by the model then it makes it easier for the prediction or forecasting to be done. Forecast can be done with AR terms, MA terms (or both) and also with lagged dependent variables. The researcher would predict with lagged dependent variables of each stock index. The statistical package used is E views software which incorporates static and dynamic forecasting methods. A dynamic forecast involves the future values of lagged residuals formed from using the forecasted values of the dependent variables. The static forecast is a one-step ahead forecast and uses the actual lagged residuals and actual values for the dependent variables to produce the forecasts. A static forecast is

employed here to do out- of- sample fore cast within the period of January 2nd, 2014 to December 31st, 2015. The following mathematical tools and their equations are often used to evaluate predictions: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and the Theil Inequality coefficient. These equations are as follows:

$$RMSE = \sqrt{\sum_{n=N+1}^{N+h} \left[\frac{\left(\hat{r}_n - R_n \right)^2}{h} \right]} \qquad (3.13)$$

$$RMSE = \sum_{n=N+1}^{N+h} \left(\frac{\left| \hat{r}_n - R_n \right|}{h} \right) \qquad (3.14)$$

$$MAPE = 100 * \sum_{n=N+1}^{N+h} \frac{1}{h} \left| \frac{\hat{r}_n - R_n}{R_n} \right| \qquad (3.15)$$

Theil inequality coefficient $= \dfrac{RMSE}{\sqrt{\sum_{n=N+1}^{N+h} \left(\dfrac{\hat{r}_n}{h} \right)} + \sqrt{\sum_{n=N+h}^{N+h} \left(\dfrac{\hat{R}_n^2}{h} \right)}}$ $\qquad (3.16)$

The Theil inequality can be categorized into three proportions. The first one is biased proportion which determines the systemic error in the forecast model. It is expected to be close to zero. The variance proportion measures the distance between the forecast variance and the actual variance of the series. A good and acceptable forecast should have the bias and variance proportions to be small. The last one is the covariance proportion which indicates the unsystematic error of the forecast and expected to be high. The sum of the three proportions should be one. A simple regression line would also be constructed to see how the forecasted values and the actual values fit the line of best fit.

4. Data Analysis and Discussion

4.1. Sources of Data

The data for the research is obtained from the GSE and consists of 1237 daily observations of Composite Index (GSE-CI) and Financial Stock Index (GSE-FSI). It covers the period of 5 years starting from January 4th, 2011 to December31st, 2015. The whole data set is divided into two sections. The in-sample estimation period consists of 779 observations beginning from 4th January, 2011 to 31st December, 2013. A total of 458 observations constitute out-of sample estimation period which covers the interval of 2nd January, 2014 to 31st December, 2015. The samples of GSE main index prices are displayed in Table 1.

DATE	GSE -CI	GSE-FSI	DATE	GSE -CI	GSE-FSI
31-Dec-10	1,000	1,000	21-Jan-11	1,016.50	1,009.85
4-Jan-11	992.25	992.36	24-Jan-11	1,019.52	1,013.46
5-Jan-11	991.08	985.81	25-Jan-11	1,022.35	1,013.46
6-Jan-11	993.89	984.69	26-Jan-11	1,024.19	1,016.04
7-Jan-11	995.5	984.69	27-Jan-11	1,044.90	1,044.72
10-Jan-11	1,001.63	991.16	28-Jan-11	1,052.98	1,049.95
11-Jan-11	1,004.15	994.09	31-Jan-11	1,057.14	1,056.30
12-Jan-11	998.37	990.08	1-Feb-11	1,049.66	1,042.96
13-Jan-11	998.05	992.41	2-Feb-11	1,051.25	1,044.75
14-Jan-11	1,000.64	992.08	3-Feb-11	1,059.28	1,054.44
17-Jan-11	997.75	990.86	4-Feb-11	1,059.38	1,054.60
18-Jan-11	1,001.24	992.88	7-Feb-11	1,042.01	1,050.35
19-Jan-11	1,004.80	996.62	8-Feb-11	1,049.11	1,041.54
20-Jan-11	1,007.72	999.67	9-Feb-11	1,052.14	1,048.06

Table 1: Samples of GSE Main Index Prices

4.2. Data Transformation and Graphs

The daily data in Table 1 is first plotted to see the behaviour pattern exhibited by the two indexes. Figure 4.1(a) and (b) show the graphs of the actual prices of GSE-CI and GSE-FSI. The GSE-CI prices exhibit more upward trend across the greater part of the range than the GSE-FSI prices. The stock indices follow a Random Walk (RW) model. The daily GSE-CI and GSE-FSI prices are transformed by means of logarithms returns Equation (2.1) to obtain returns series which are unit free. One of the conditions required for execution of the GARCH models is that the returns must exhibit volatility clustering. This means volatility is high for certain time interval and low for other time interval. Figure 4.1(c) and (d) also show GSE-CI returns being highly volatile than GSE-FSI returns. The GSE-CI returns show relatively high frequencies, an implication for small daily changes in stock prices. This characteristic of volatility clustering has been observed in most empirical financial time series. This volatility clustering is more pronounced in the GSE-CI returns than the GSE-FSI returns for the period considered. This implies investment in assets by non-financial institutions is highly risky as compared to financial institutions like the banks and asset management firm.
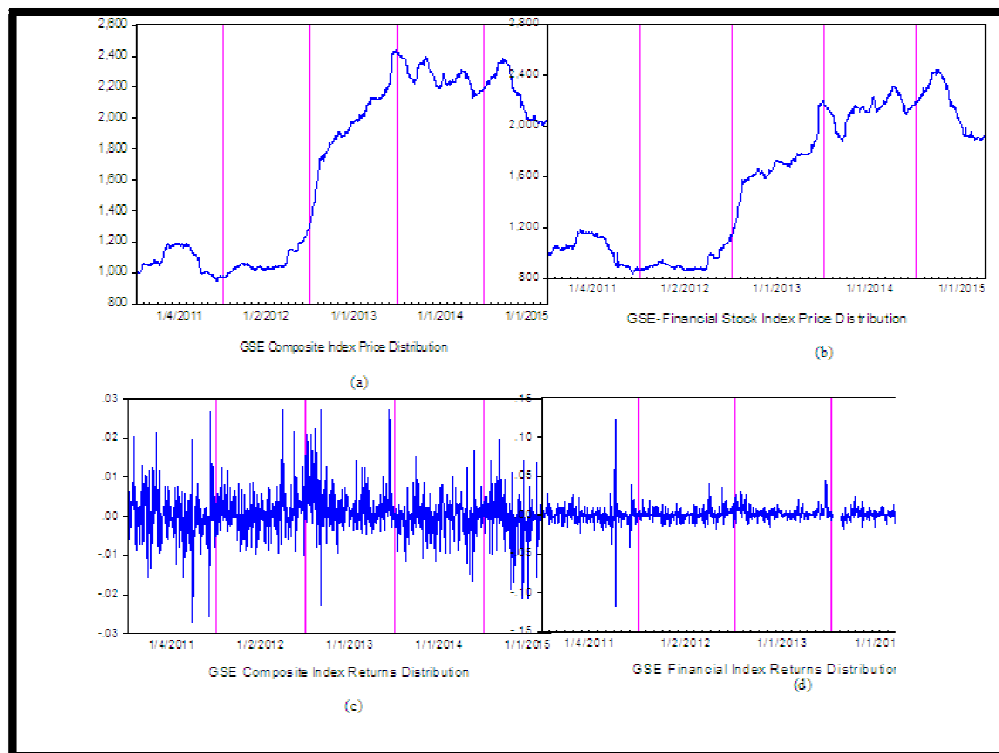

Figure 1

	Prices		Returns	
STATISTIC	**GSE-CI**	**GSE-FSI**	**GSE-CI**	**GSE-FSI**
Mean	1684.027	1553.066	0.0005627	0.0005382
Median	1898.983	1638.230	0.0002207	0.0000000
Maximum	2439.203	2446.190	0.0272100	0.1226000
Minimum	940.0422	823.5700	-0.0270500	-0.1177000
Std. Dev.	547.2357	541.0858	0.005460566	0.008934277
Skewness	-0.124180	0.017823	0.361498	0.511643
Kurtosis	1.240775	1.387765	7.850262	57.38733
Jarque-Bera	162.6942	134.0379	1238.5	152390
Probability	0.000000	0.000000	2.2e-16	2.2e-16
Sum	2083142.	1921143.		
Sum Sq. Dev.	3.70E+08	3.62E+08		
Observations	1237	1237	1237	1237

Table 2: Descriptive Statistics for GSE Main Index

Table 2shows the descriptive statistics for GSE main index prices. From the table there is a moderate difference between the minimum and maximum values of both the GSE-CI and the GSE- FSI raw data series respectively. The standard deviation of GSE-CI is a bit higher than GSE-FSI confirming higher fluctuations in the stock prices of listed equities. The GSE-CI returns are positively skewed while the GSE-FSI returns are negatively skewed. The kurtosis of both indices is less than the normal value of 3 showing that the index prices are platykurtic.

Table 2also shows the summary statistics of GSE-CI and FSI returns series. There is no much difference between the means of both the GSE-CI and FSI returns as well as their minimum and maximum values. The standard deviation of GSE-FSI (i.e. 0.008934) is higher than the standard deviation of GSE-CI (i.e. 0.00546). This shows that with the actual returns, it is highly risky for investors to invest in banking institutions than the assets of non-banking institutions. The GSE-CI returns distribution is less positively skewed (0.361) than the GSE-FSI returns series (i.e. 0.512). This means that the fatter part of the distribution is more pronounced on the left for GSE-FSI returns than the GSE-CI returns since both have positive skewed values. The implication is that both stock indices return is asymmetric (or having non-normality distributions). Again the kurtosis of GSE-CI and GSE-FSI returns exceed the normal standard value of 3 (i.e. 7.850 and 57.387 respectively) which is leptokurtic. The high value of kurtosis for GSE-FSI returns show that its distribution is highly peaked than the GSE-CI return series. This result also is consistent with the non-normality distribution often observed in financial time series.

4.3. Testing For Normality of GSE Main Index Returns

4.3.1. Jarque Bera (JB) Statistic Test
Under the normality testing, the following hypothesis is conducted:
- H_0 : The GSE-CI and GSE-FSI returns are normally distributed;
- H_a : The GSE-CI and GSE-FSI returns are not normally distributed.

According to Table 4.2, the JB statistic test reported probability value for GSE-CI is 2.2e-16 which is less than 5 % significance level. This induces the rejection of the null hypothesis and acceptance of the alternative hypothesis that the GSE-CI returns is not normally distributed. Similarly, the JB statistic associated p-value (2.2e-16) for GSE-FSI returns also indicates the rejection of the null hypothesis in favour of the alternative hypothesis.

4.3.2. Histogram Distributions of GSE Main Index Returns
The non-normal behaviour shown by the GSE-CI and GSE-FSI returns series is also confirmed by the histograms distributions for both indexes in Figure 4.2(a) and (b). The indices have too long tails which suggest that the GSE-CI returns are thick tail and positively skewed. The GSE-FSI returns also reveal thickness at the tails and positive skewedness.

4.3.3. Quantile - Quantile Plot (QQ - Plot) of Returns
The QQ - plot in Figure 4.2(c) shows that the GSE-CI returns are not normally distributed since the points are deviated from the straight reference line. These deviations are heavily pronounced at the tails. This nonlinear pattern is often described as leptokurtic and hence the GSE-CI returns are a poor fit. On the other hand, the QQ - plot for GSE-FSI returns on Figure 2(d) also shows evidence of non-normal distributions with few outliers. The returns of GSE-FSI do not therefore satisfy "goodness of fit". The QQ-plot, J.B statistic and the histograms all confirmed that the GSE main index is not normally distributed.
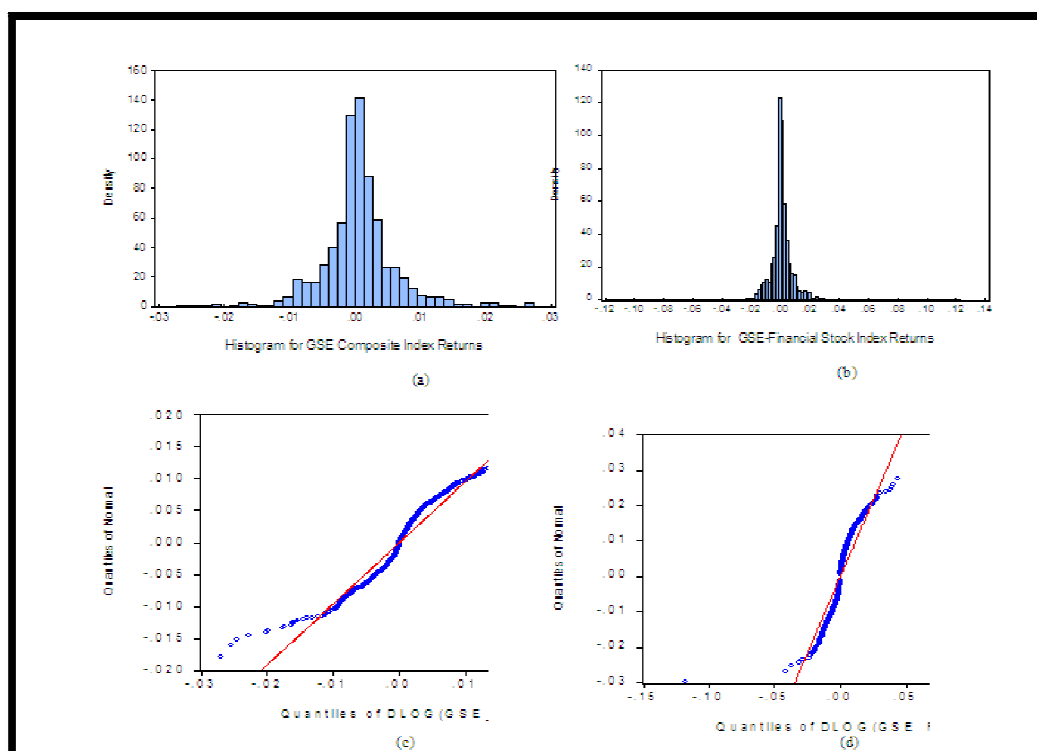

Figure 2: Histograms and QQ-Plot for GSE Composite Index and GSE Financial Stock Index Returns

4.4. Testing for Serial Correlations in GSE Main Index Returns

4.4.1. Correlogram and Q- Statistic Test

The autocorrelation shown in Table 4.3 (a) gives a strong indication of first- order serial correlation in the GSE-CI returns since all the coefficients are nonzero and decreases towards zero. The spikes of the Correlogram decrease geometrically as the number of lag increases. This is an evidence of first-order autoregressive AR (1) series. The probabilities of the Q–statistics are all zero, emphasizing that the GSE-CI returns are serially correlated. The GSE-CI returns exhibit a clear pattern of randomness. From Table 4.3(b), the GSE-FSI returns show a strong evidence of autocorrelation of first-order in the residuals since all the coefficients are nonzero and decrease towards zero. The bars of the Correlogram decrease geometrically as the number of lag increases. This is an evidence of first- order autoregressive series. The probabilities of the Q- statistics are all zero except the first probability value (0.279) which is statistically not significant and hence the GSE-FSI returns follow first – order autoregressive model.

```
Date: 04/08/16   Time: 16:11
Sample: 1/04/2011 9/30/2015
Included observations: 1236
```

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.159	0.159	31.277	0.000
		2	0.145	0.123	57.459	0.000
		3	0.134	0.098	79.807	0.000
		4	0.136	0.092	102.85	0.000
		5	0.086	0.031	112.05	0.000
		6	0.159	0.113	143.34	0.000
		7	0.157	0.098	174.14	0.000
		8	0.100	0.026	186.65	0.000
		9	0.092	0.022	197.10	0.000
		10	0.145	0.080	223.28	0.000
		11	0.087	0.014	232.67	0.000
		12	0.129	0.063	253.53	0.000
		13	0.110	0.030	268.69	0.000
		14	0.030	-0.057	269.79	0.000
		15	0.066	0.010	275.24	0.000
		16	0.033	-0.035	276.61	0.000

Table 3: Correlogram and Q-Statistics Results of GSE-CI Returns

```
Date: 04/08/16   Time: 16:13
Sample: 1/04/2011 9/30/2015
Included observations: 1236
```

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.031	0.031	1.1706	0.279
		2	0.155	0.154	31.002	0.000
		3	0.065	0.057	36.214	0.000
		4	0.108	0.084	50.797	0.000
		5	0.079	0.060	58.600	0.000
		6	0.129	0.099	79.265	0.000
		7	0.078	0.048	86.787	0.000
		8	0.094	0.050	97.846	0.000
		9	0.056	0.018	101.76	0.000
		10	0.088	0.043	111.33	0.000
		11	0.083	0.047	119.99	0.000
		12	0.120	0.076	138.06	0.000
		13	0.048	0.003	140.94	0.000
		14	0.041	-0.018	143.01	0.000
		15	0.045	0.002	145.52	0.000
		16	-0.027	-0.075	146.45	0.000

Table 4: Correlogram and Q-Statistics Results of GSE-FSI Returns

4.4.2. Breusch -Pagan - Godfrey (BPG-LM) Serial Correlation Test

The following hypothesis has been tested under the serial correlation:
- H_O: The returns of the GSE-CI and GSE-FSI are not serially correlated;
- H_a: The returns of the GSE-CI and GSE-FSI are serially correlated.

From Table 5 the BPG-LM test statistic values (Obs.* R-squared) for GSE-CI and GSE-FSI returns are 40.01578 and 45.86819 respectively. Their corresponding p-values of 0 % each is less than 5 % significance level. This indicates that the null hypothesis should be strongly rejected for both series and the alternative hypothesis of GSE-CI and GSE-FSI returns being serially correlated (at 5 lags) accepted. This therefore means that the model must be modified or corrected in order to remove any serial correlations in the stock indices returns. If left unchecked it can invalidate standard hypothesis tests and interval estimate of predictions. Therefore, the Table 6 uses the White HAC standard errors and covariance method to correct for serial correlation in the GSE-CI and GSE -FSI returns respectively. By this method the coefficients of both series remain the same while the standard errors have changed and make the model free from serial correlation.

| BPG-LM Serial Correlation Test | | | | HAC standard Error and Covariance Method | | | |
| GSE-CI Returns | | | | Correction for GSE-CI Returns | | | |
Variable	Coefficient	Standard Error	t – statistic	Probability	Coefficient	Std Error	t – Statistics	Probability
C	1.696072	0.830439	2.042339	0.0423	1.696072	0.687541	2.466865	0.138
GCE – CI(-1)	1.157527	0.028133	41.14467	0.000	1.157527	0.48763	23.7379	0.0000
GSE – CI(-2)	-0.158127	-0.158127	-5.62356	0.000	-0.158127	0.048724	-3.245344	0.0012
GSE – FSI Returns				Correction for GSE – FSI Returns			
C	1.535544	1.069774	1.435591	0.1571	1.535544	0.999501	1.536312	0.1247
GSE – FS(-1)	1.030049	1.039949	0.028476	0.000	1.030049	0.084531	12.18787	0.0000
GSE – FS(-2)	-0.30261	-0.030261	0.028468	0.2801	-0.030261	0.084627	-0.36348	0.7163

Table 5: Results of Serial Correlation Test for GSE-CI and GSE-FSI Returns
Results of BPG-LM Serial Correlation Test for GSE-CI and
GSE -FSI Returns

	GSE – CI	GSE – FSI
F – Statistic	8.217516	9.45775
Probability	0.000	0.000
Obs.* R Squared	40.01578	45.86819
Probability	0.0000	0.000

Table 6: Results of Serial Correlation and HAC Standard Error Covariance Method

4.5. Testing for ARCH Effect in GSE Main Index Returns
The hypothesis for the ARCH test is
- H_O: There is no ARCH effect in the returns of GSE-CI and GSE-FSI.
- H_a: There is ARCH effect in the returns of GSE-CI and GSE-FSI.

The results from Table 7 suggest strongly the rejection of the null hypothesis of no ARCH effect in the GSE-CI returns since the p-value of 0 % is less than 5% significance level. Thus there is ARCH effect in the GSE-CI returns series. The ARCH test results for GSE-FSI returns (from Table 7) also suggest strongly the rejection of the null hypothesis of no ARCH effect in the GSE-FSI returns since the p-value of 0 % is less than 5% significance level. Thus there is ARCH effect in the GSE-FSI returns series. This is also confirmed by the large values of F-statistic (19.45980 and 45.6723 respectively which are more than the critical value) for both stock indices suggesting the rejection of the null hypothesis and the acceptance of the alternave hypothesis of ARCH effect in the stock indices.

4.6. Testing for Heteroskedasticity in GSE Main Index Returns

4.6.1. Breusch- Pagan- Godfrey (BPG) Heteroskedasticity Test
The hypothesis to be test is as follows:
- H_O: There is homoskedasticity in the returns of GSE-CI and GSE-FSI;
- H_a: There is heteroskedasticity in the returns of GSE-CI and GSE-FSI.

According to Table 7, the BPG Heteroskedasticity test statistic (Obs.* R-squared) value of 12.34627 is for the GSE-CI returns. Its corresponding p-value of 0 % is less than 5% significance level. This is a strong indication that the null hypothesis of homoskedasticity in the GSE-CI returns is rejected and the alternative hypothesis of heteroskedasticity presence is accepted. Therefore, there is heteroskedasticity in the GSE-CI series and should be corrected. The same Table 7 reports the BPG heteroskedasticity test for GSE-FSI returns and its test statistic value (Obs.* R-squared is 69.09861) with associated p-value of 0.00 % is less than 5% significance level. It suggests the rejection of null hypothesis of homoskedasticity in the GSE-FSI returns. The reported F-statistic (12.34623 and 36.50802 for each index is greater than the critical value) also ascertains the rejection of the homoskedasticity in favour of heteroskedasticity. It means there is heteroskedasticity in GSE-FSI returns and must be addressed for further analysis of the data.

	Arch Test		Heteroskedasticity Test	
	GSE-CI	GSE-FSI	GSE-CI	GSE-FSI
Obs.*Squared	90.5760	80.4865	24.2685	69.0986
Probability	0.0000	0.0000	0.0000	0.0000
F-Statistic	19.4598	36945.6723	12.3462	36.5080

Table 7: Results of BPG Heteroskedasticity and ARCH tests for GSE Main Index Returns

4.6.2 Correction for Heteroskedasticity Problem in the Model

The heteroskedasticity issues can affect the variance of the regression line, and consequently affect the variance of the estimated coefficients and also the 95% confidence interval for prediction of the dependent variable. There are various methods for addressing heteroskedasticity problem. These include Feasible Generalised Least Square method (FGLS), Cochrane-Orcutt method and the White Heteroskedasticity test. One robust standard errors method used in this research is the Heteroskedasticity-Autocorrelation Consistent (HAC) by Newey-West to correct the heteroskedasticity and autocorrelation in returns series. This method re-estimate the equation by adjusting the standard errors for heteroskedasticity (and serial correlation) of the unknown form. In the model output (not reported here) the coefficients of the parameters remain the same. However, the HAC estimated standard errors reduced and its corresponding t-statistics changed from the original regression. The Table 6 confirms the correction of the heteroskedasticity by the HAC standard errors and covariance method. This therefore means that the model is now free from heteroskedasticity and serial correlation and hence prediction can be done.

4.6.3. Randomness and Graphs of Residuals

The randomness of the residual series can be observed in Figure 4.3 (a) as already illustrated by the correlogram. This shows that the GSE-CI and the GSE-FSI returns are heteroskedastic and have been addressed for any further analysis of the data.
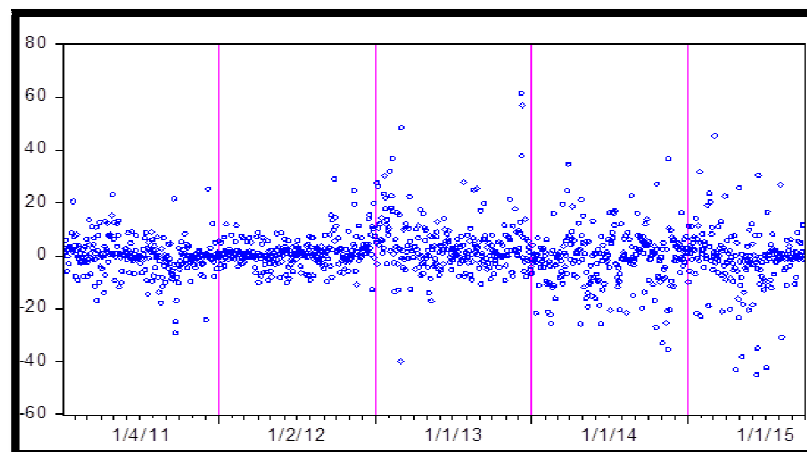

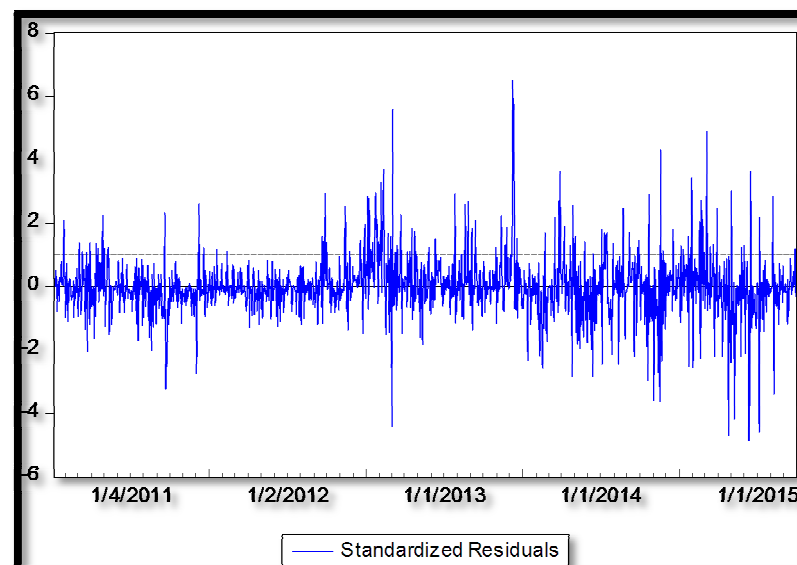*Figure 3: Graph of Randomness*

Figure 4: Graph of Standardized Residuals

4.7. Testing for Stationarity in the GSE Main Index Returns

The hypothesis to be tested is
- H_0 = The returns of GSE-CI and GSE-FSI are non-stationary (unit root);
- H_a = The returns of GSE-CI and GSE-FSI are stationary (no unit root).

4.7.1. ADF Test of Unit Root (or non-stationarity) in GSE-CI Returns

Equations (3.6), (3.7) and (3.8) have been used to calculate ADF unit root statistic for GSE-CI returns and the results displayed in Table 4.7. From the table, the following estimates: 1.119, 0.063 and 1.462 from each model respectively are less than the critical value at 5% significance level. This indicates the null hypothesis of non-stationarity in the GSE-CI returns cannot be rejected. The corresponding p-values of the ADF statistics in each model (0.710, 0.995 and 0.965 respectively) are more than 5% significance level. It also emphasizes that the null hypotheses cannot be rejected. Therefore, the GSE-CI returns have unit root or being non-stationary. In order to make the GSE-CI returns stationary as required by the GARCH model, the index is then converted to first difference and the results are shown in Table 4.7. From the second part of the same table, these values: 10.03599, 10.69621, and 9.89002 of each model respectively, are more than the critical value at 5% significance level which indicates that the null hypothesis of unit root must be rejected and the alternative hypothesis be accepted. The corresponding p-value of 0.00 % for each case is less than 5% significance level. This reveals that the null hypothesis is rejected in favour of the alternative. Therefore, at first difference the GSE-CI returns become stationary (meaning it has no unit root).

4.7.2. ADF Test of Unit Root (or Non-stationarity) in GSE-FSI Returns

The Table 9 shows results from Equations (3.6), (3.7) and (3.8) calculations for GSE-FSI returns using the ADF test. The test statistics of GSE-FSI actual returns are 0.808, 0.520 and 1.436 (for each model respectively) are less than the critical value at 5% significance level. The researcher fails to reject the null hypothesis of non-stationarity in the GSE-FSI returns. The corresponding p-values of the ADF statistics in each model (0.816, 0.983 and 0.963 respectively) are more than 5% significance level. This means that the null hypotheses cannot be rejected. Therefore, the GSE-FSI returns have unit root or are non-stationary. To make the GSE-FSI returns stationary as required by the GARCH model, the index returns are converted to first difference and the results from the models are shown in the same Table 9. The test statistic values: 10.442, 10.444, and 10.354 are more than the critical value at 5% significance level. The implication of this is that the null hypothesis of unit root must be rejected and the alternative hypothesis accepted. The corresponding p-values of the ADF statistic (0.000 % in each case) are less than 5% significance level revealing that the null hypothesis is rejected in favour of the alternative. Therefore, at first difference the GSE-FSI returns are stationary (have no unit root).

	GSE-CI Actual returns ADF Test statistic				First Difference of GSE-CI returns ADF Test statistic		
	T – statistic	Critical value (5%)	Probability		T – statistic	Critical value (5%)	Probability
Model (3.6)	1.119	2.863	0.710	Model (3.6)	10.036	2.864	0.000
Model (3.7)	0.063	3.413	0.995	Model (3.7)	10.696	3.413	0.000
Model (3.8)	1.462	1.941	0.965	Model (3.8)	9.890	1.941	0.000

Table 8: Results of Stationarity and Unit Root Test for GSE-CI Returns

	GSE-FSI Actual Returns			First Difference of GSE- FSI Returns		
	ADF Test Statistic			ADF Test Statistic		
	T – statistic	Critical Value (5%)	Probability	T – statistic	Critical value (5%)	Probability
Model (3.6)	0.808	2.864	0.816	10.442	2.864	0.000
Model (3.7)	0.520	3.413	0.983	10.444	3.414	0.000
Model (3.8)	1.436	1.941	0.963	10.354	1.941	0.000

Table 9: Results of Stationarity and Unit Root Test for GSE-FSI Returns

4.8. Application of GARCH Model Types with Residuals Distribution Innovations

The estimation of parameters in the GARCH variance Equation (2.6) is done by maximizing their log- likelihood functions. The following GARCH model types are estimated: the GARCH (0, 1), GARCH (0, 2), GARCH (0, 3), GARCH (1.1), GARCH (1, 2), GARCH(2,1),GARCH(2,2) and EGARCH (1,1) using the daily GSE-CI and GSE-FSI returns with three different distributions that is normal, student -t, and GED. The results are put in the Tables 4.9 to 4.14 with the various residuals distributions.

GARCH Parameters							Information Criterion		
GARCH Type	ϖ	α_1	α_2	β_1	β_2	β_3	AIC	BIC	HQC
(0, 1)	53.1705 (0.7255)	-	-	0.8182	-	-	6.5804	6.6187	6.5952
(0, 2)	187.9488 (0.8164)	-	-	-0.1930 (0.3855)	0.8051 (0.0004)	-	6.5808	6.6255	6.5981
(0, 3)	162.1073 (0.7900)	-	-	-0.7411 (0.0117)	0.8096 (0.0000)	0.5535 (0.14R97)	6.5821	6.6332	6.6019
(1, 1)	5.0170 (0.0684)	0.2108 (0.0359)	-	0.8038 (0.000)	-	-	6.5081	6.5528	6.5254
(1, 2)	47.9029 (0.0000)	0.1506 (0.0008)	-	-0.0795 (0.1058)	-0.0748 (0.1849)	-	6.6118	6.6629	6.6315
(2, 1)	6.4429 (0.0795)	0.1312 (0.1377)	0.1470 (0.2349)	0.7525 (0.000)	-	-	6.5088	6.5599	6.5286
(2, 2)	16.7853 (0.0477)	0.2450 (0.0403)	0.3487 (0.0197)	-0.2367 (0.0002)	0.6681 (0.0000)	-	6.4973	6.5548	6.5195

Table 10: Results from GARCH Variance Model (2.6) for GSE-CI Returns with Student - tDistribution

GARCH Parameters								Information Criterion		
GARCH Type	ϖ	α_1	α_2	β_1	β_2	β_3	γ	AIC	BIC	HQC
(0, 1)	1.8010 (0.005)	-	-	0.9686 (0.0000)	-	-	-	6.8546	6.8865	6.8669
(0, 2)	3.2082 (0.7707)	-	-	0.12602 (0.9834)	0.81814 (0.8889)	-	-	6.8574	6.8957	6.8722
(0, 3)	5.0017 (0.0003)	-	-	-0.5720 (0.0000)	0.4924 (0.0000)	0.9751 (0.0000)	-	6.8548	6.8996	6.8721
(1, 1)	3.6113 (0.000)	0.14602 (0.0000)	-	0.7936 (0.0000)	-	-	-	6.6676	6.7059	6.6824
(1, 2)	2.6527 (0.0000)	0.1137 (0.0000)	-	1.1483 (0.000)	-0.3040 (0.0165)	-	-	6.66806	6.7128	6.6853
(2, 1)	4.0386 (0.0000)	0.1147 (0.0000)	0.0573 (0.1027)	0.7633 (0.000)	-	-	-	6.6686	6.7133	6.6858
(2, 2)	11.2687 (0.000)	0.1690 (0.0000)	0.2575 (0.0000)	-0.2247 (0.000)	0.6339 (0.000)	-	-	6.6534	6.7047	6.6734
EGARCH(1,1)	0.1110 (0.0177)	0.2819 (0.0000)	-	0.9203 (0.000)	-	-	0.0278 (0.1570)	6.6709	6.7156	6.6882

Table 11: Results from GARCH Variance Model (2.6) For GSE-CI Returns with Normal Distribution

GARCH Type	Garch Parameters							Information Criterion		
	ϖ	α_1	α_2	β_1	β_2	β_3	γ	AIC	BIC	HQC
(0, 1)	106.9538 (0.0000)	-	-	-0.9215 (0.0000)	-	-	-	6.5410	6.5794	6.5558
(0, 2)	55.1839 (0.8991)	-	-	-0.3793 (0.9224)	0.3921 (0.9242)	-	-	6.5492	6.5939	6.5664
(0, 3)	71.6631 (0.0009)	-	-	-1.1756 (0.0000)	0.2894 (0.1358)	0.6267 (0.0000)	-	6.5430	6.5940	6.88706.5626
(1, 1)	3.57119 (0.0170)	0.1488 (0.0032)	-	0.7977 (0.0000)	-	-	-	6.4675	6.5122	6.4647
(1, 2)	2.7961 (0.0840)	0.1192 (0.0503)	-	1.0642 (0.0120)	-0.2238 (0.5301)	-	-	6.4688	6.5199	6.4885
(2, 1)	3.8033 (0.0225)	0.1147 (0.0753)	0.0499 (0.5174)	0.7793 (0.0000)	-	-	-	6.4701	6.5212	6.4898
(2, 2)	11.6435 (0.0055)	0.1703 (0.0039)	0.2437 (0.0000)	-0.2380 (0.0011)	0.6592 (0.0000)	-	-	6.4628	6.5202	6.4850
EGARCH (1,1)	0.0913 (0.3494)	0.2475 (0.0002)	-	0.93158 (0.0000)	-	-	0.0531 (0.1730)	6.4643	6.5154	6.4841

Table 12: Results from GARCH Variance Model (2.6) for GSE-CI Returns with GED Distribution

GARCH Type	GARCH Parameters							Information Criterion		
	ϖ	α_1	α_2	β_1	β_2	β_3	γ	AIC	BIC	HQC
(0, 1)	3.2467 (0.0000)	-	-	0.9730 (0.0000)	-	-	-	7.5587	7.5907	7.5710
(0, 2)	1.0765 (0.6402)	-	-	1.6725 (0.0409)	-0.6815 (0.3936)	-	-	7.5610	7.5993	7.5758
(0, 3)	11.5306 (0.0000)	-	-	-0.9394 (0.0000)	0.8652 (0.000)	0.9770 (0.0000)	-	7.5597	7.6037	7.5762
(1, 1)	4.63010 (0.0000)	0.1337 (0.000)	-	0.8386 (0.0000)	-	-	-	7.3022	7.3405	7.3170
(1, 2)	5.5744 (0.0000)	0.1580 (0.0000)	-	0.5567 (0.0356)	0.2501 (0.2787)	-	-	7.3048	7.3495	7.3220
(2, 1)	4.5397 (0.0000)	0.1398 (0.0001)	-0.0094 (0.8055)	0.8420 (0.000)	-	-	-	7.3049	7.3497	7.3222
(2, 2)	8.9153 (0.0000)	0.1506 (0.0000)	0.1209 (0.0000)	-0.16312 (0.0000)	0.8228 (0.0000)	-	-	7.2885	7.3396	7.3082
EGARCH(1,1)	0.0488 (0.0388)	0.24866 (0.0000)	-	0.9536 (0.0000)	-	-	0.0339 (0.0454)	7.2741	7.3188	7.2913

Table 13: Results from GARCH Variance Model (2.6) for GSE-FSI Returns with Normal Distribution

GARCH Type	GARCH Parameters							Information Criterion		
	ϖ	α_1	α_2	β_1	β_2	β_3	γ	AIC	BIC	HQC
(0, 1)	8502.129 (0.9983)	-	-	0.9016 (0.0000)	-	-	-	6.9950	7.0334	7.0099
(0, 2)	7533.506 (0.9967)	-	-	-0.0803 (0.3325)	0.9070 (0.0000)	-	-	6.9975	7.0423	7.0148
(0, 3)	2136 956	-	-	0.6490 (0.3380)	0.9596 (0.000)	-0.6798 (0.2888)	-	7.0003	7.0514	7.0200
(1, 1)	10259.12 (0.9980)	225.7073 (0.9980)	-	0.6164 (0.0000)	-	-	-	6.9449	6.9896	6.9621
(1, 2)	10064.97 (0.9981)	214.2674 (0.9981)	-	0.7354 (0.0127)	-0.0939 (0.6562)	-	-	6.9474	6.9985	6.9671
(2, 1)	11904.88 (0.9979)	188.4180 (0.9979)	63.1659 (0.9979)	0.5524 (0.0000)	-	-	-	6.9473	6.9984	6.6970
(2, 2)	5513.854 (0.9985)	292.1606 (0.9985)	-176.0130 (0.9985)	1.1943 (0.0242)	-0.3545 (0.2794)	-	-	6.9499	7.0075	6.9722
EGARCH (1,1)	0.69366 (0.0574)	0.7870 (0.1905)		0.8592 (0.0000)	-	-	0.2574 (0.2521)	6.9294	6.9805	6.9491

Table 14: Results from GARCH Variance Model (2.6) for GSE-FSI Returns with Student - Tdistribution

4.8.1. Discussion and Selection of Fitted model for GES Stock Indices

Table 4.15 (a & b) gives the summary of the model selection for both stock indices returns based on the information criteria: AIC, BIC and HQC. The model that gives the minimum values of this criterion becomes the fitted model. Thus the most acceptable model is EGARCH (1, 1) with GED for both GSE-CI and the GSE-FSI returns.

GARCH Type	Distribution	AIC	BIC	HQC
GARCH (2,2)	GED	6.4628	6.5202	6.4850
EGARCH(1,1)		6.4133	6.5154	6.4841
GARCH (2, 2)	Student-t	6.4973	6.5548	6.5195
EGARCH(1,1)		6.5833	6.5526	6.5655
GARCH (2, 2)	Normal	6.6534	6.7047	6.6734

Table 15: Summary of Model Selection for GSE-CI Returns

GARCH Type	Distribution	AIC	BIC	HQC
EGARCH (1,1)	Normal	7.2741	7.3188	7.2913
EGARCH (1,1)	Student-t	6.9294	6.9805	6.9491
EGARCH(1,1)	GED	6.8147	6.8658	6.8345

Table 16: Summary of Model Selection for GSE-FSI Returns

The selected EGARCH (1, 1) model output indicates our dependable variable (GSE-CI) has been influenced by its own lagged variable since the R-squared value is high (i.e. 0.999670). This means that at least one independent variable (GSE-CI (-1)), is significant in influencing or explaining the volatility of the GSE-CI returns. According to Table 12, the ARCH term $(\alpha = 0.2475)$ and GARCH term $(\beta = 0.9316)$ from EGARCH (1, 1) model sum up to 1.1791. This value is very close to one which suggests that the GSE-CI returns series have followed an Integrated GARCH (IGARCH) order proposed by Engle and Bollerslev (1986). This means volatility shock to persistence exists in the conditional variance (volatility) of GSE-CI returns series. The presence of leverage effect (inherent) in the GSE-CI returns is small and has been captured by $(\gamma = 0.0531)$. In Table 4.15(b), the EGARCH (1,1) model output shows our dependable variable GSE-FSI has been well explained by its own lagged variables since the R-squared value is high (i.e. 0.99003). Thus the two independent variables (GSE-FSI (-1) and GSE-FSI (-2)), are significant in influencing the volatility of the GSE-FSI returns. The ARCH term $(\alpha = 0.2987)$ and the GARCH term

$(\beta = 0.8709)$ for the GSE-FSI index sum up to 1.1696 which is very close to one suggesting that the GSE-FSI series has followed Integrated GARCH (IGARCH) order proposed by Engle and Bollerslev (1986). The implication for this is that shock to persistence exists in the conditional volatility of GSE-FSI returns series. The leverage effect $(\gamma = 0.0797)$ exists and is minimal which indicates its presence in the GSE FSI returns. It implies investors on the GSE are influenced by any economic news flowing into the stock market. It can be concluded that the GSE stock indices have shown features of asymmetric effect mostly exhibited by advance stock markets all over the world, however, the effect is on a low level.

4.8.2. Residual Diagnostic Analysis of EGARCH (1, 1) model

It is important to do residual analysis to see whether the residuals from our selected EGARCH (1, 1) model satisfy the following conditions: no serial correlations, no ARCH effects, no heteroskedasticity issues and whether the model is

normally distributed. From Table 4.16(a) and (b), the coefficients of all the Q-statistics are not significant revealing there is no ARCH effect in the squared residuals of GSE-CI and GSE-FSI returns series. The spikes of the bars also suggest obviously no autocorrelation in the squared residuals from the EGARCH (1, 1) model.

Table 17: Correlogram of GSE-CI Squared Residual

Table 18: Correlogram of GSE-FSI Squared Residuals

These results are also proven by the ARCH tests in Table 19 for both GSE-CI and GSE-FSI standardized residuals series. It shows the null hypothesis of no ARCH effect in the GSE-CI residuals cannot be rejected since the ARCH test statistic (Obs.* R-Squared = 8.19344) has a p-value of 0.1459 which is more than 5 %. Hence the standardized residuals from the EGARCH (1, 1) model are free from autocorrelations and serial dependency. The ARCH test for GSE-FSI standardized residuals from the second part of Table 19 also indicates that the null hypothesis of no ARCH effect cannot be rejected since the ARCH test statistic (Obs.* R-Squared = 0.539073) has p-value of 0.9906 which is more than 5 %. Hence the standardized residuals from the EGARCH (1, 1) model are free from autocorrelations and heteroskedasticity.

	ARCH TEST	
	GSE-CI	GSE-FSI
F-Statistic	1.64338	0.10705
Probability(5,779)	0.1461	0.9907
Obs.* R-squared	8.19344	0.53907
Probability(5,779)	0.1459	0.9906

Table 19: Results of ARCH Test for GSE-CI and GSE-FSI Standardized Residuals

On normality of the residuals, Figure 4.4 shows that the histogram distributions of both stock indices deviate from normal distributions even though the means and medians are approximately zero with the standard deviation close to one. The kurtosis of each index (8.576083 and 23.54920) exceeds the normal value of 3 and high JB test statistic values (1053.40 and 14346.33) for each index respectively reject normality of the standardized residuals. This is however expected since financial returns are not normally distributed.
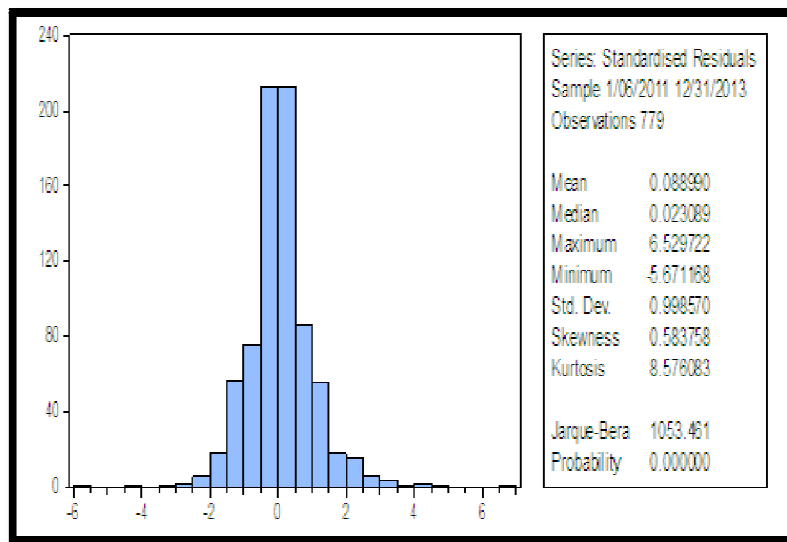
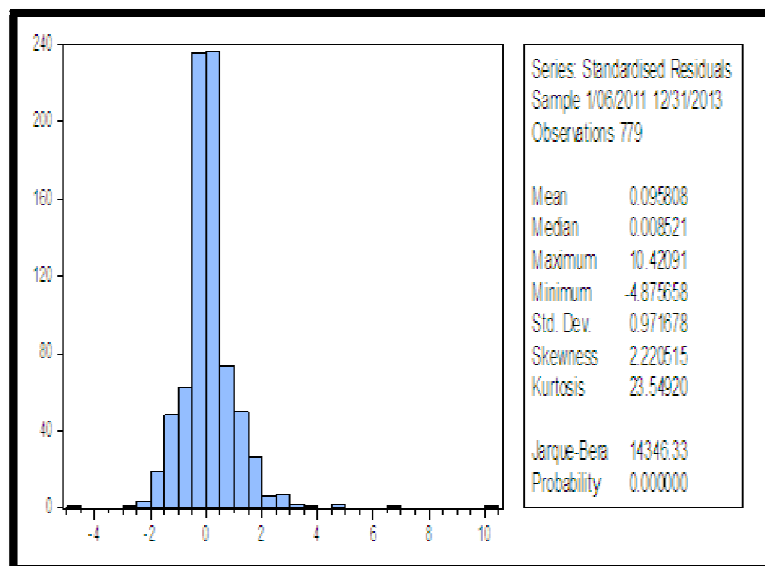*Figure 5: Histogram of GSE-FSI Standardized Residuals*

Figure 6: Histogram of GSE-CI Standardized Residuals

4.8.3. Prediction of Volatility and Returns of GSE Main Index

The final objective of this study is to predict the market volatility and returns of GSE main index. The prediction was carried out by first plotting the graphs of the actual and the predicted or fitted values of the GSE main index as shown in Figure 7and (b). From the graphs it can be observed that the actual values and predicted or fitted values are indistinguishable and shows the model selected has accurately fitted the GSE main index returns.
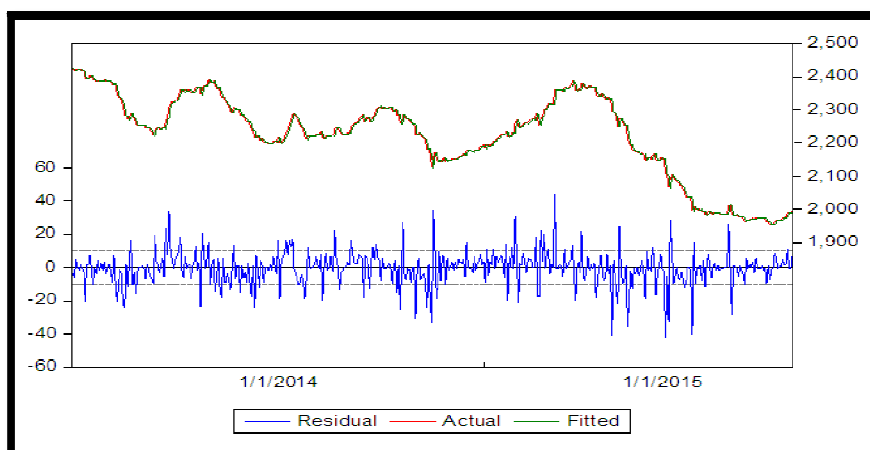
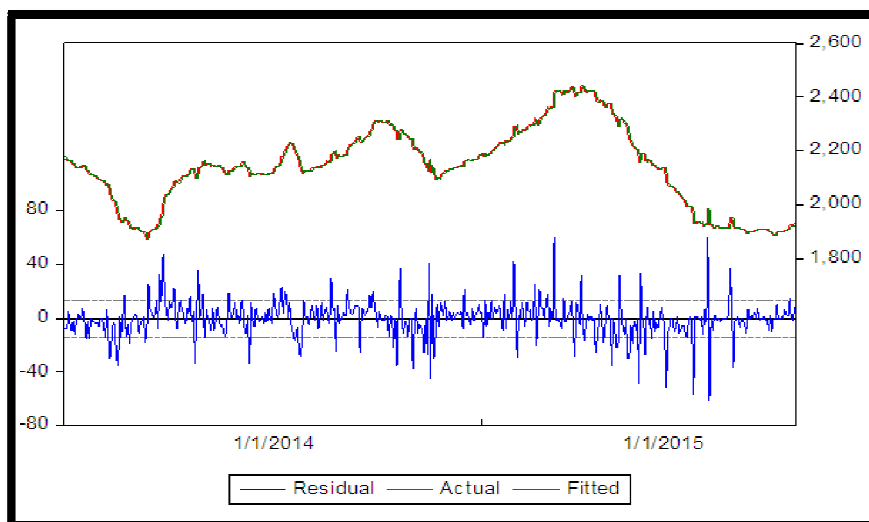*Figure 7: Graph of Residual, Fitted and Actual Values of GSE-CI Returns*

Figure 8: Graph of Residual, Fitted and Actual Values of GSE-FSI Returns

4.8.4. Evaluation of Results from Prediction of GSE Main Index Returns

The graphs in Figure 4.6 (a) and (b) on prediction of volatility of GSE-CI and GSE-FSI returns appear to be well fitted (this confirmed earlier results in Figure 7and (b)). The Theil Inequality and RMSE and Regression lines are used to validate the results of the prediction of GSE main index. From Figure 9 and (b), the Theil Inequality value for each index is 0.002314 and 0.003182 which indicates a perfect prediction for the series because they fall within the range of accurate predictions. The bias proportion values (i.e. 0.002075 and 0.001415 for the series respectively) and variance proportion (i.e. 0.000585 and 0.000317) are very small indicating that the forecasting errors are largely due to unsystematic forecasting errors (i.e. the covariance proportions have the values of 0.997608 and 0.997999 accordingly). The standard deviation (i.e. RMSE) of the forecast errors are 10.30918 and 13.653 for each index returns. Moreover the tables also show the forecast of the variances of both series for the out-of sample period.
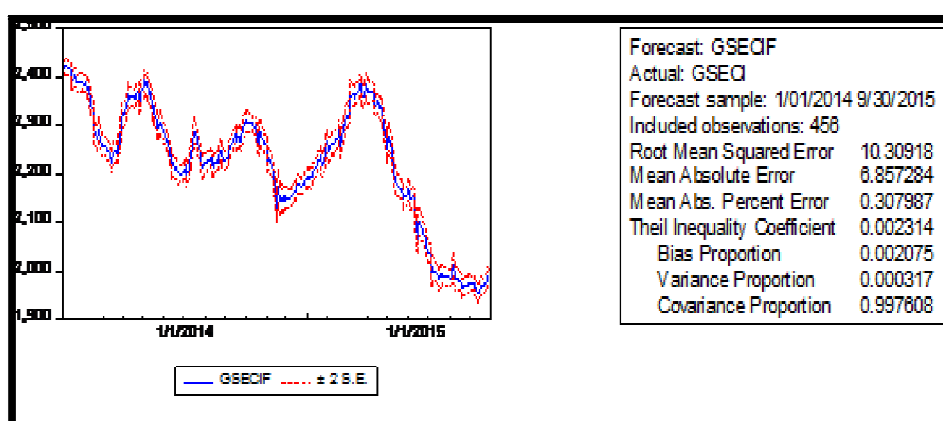

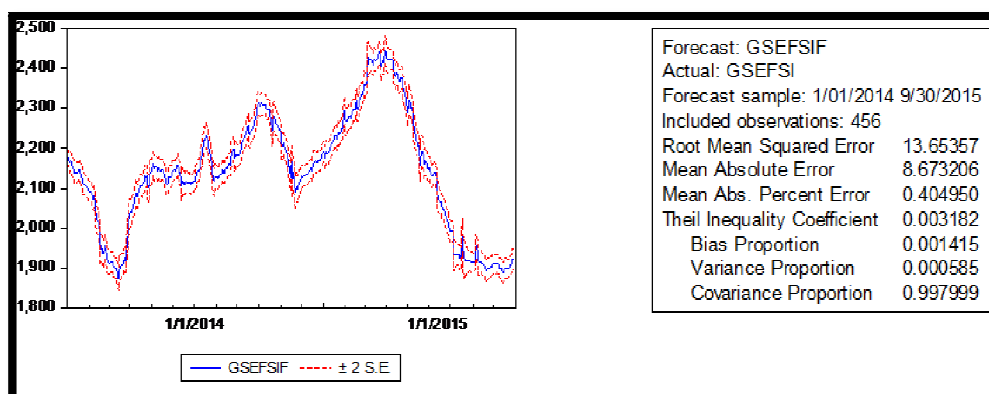*Figure 9: Prediction or Forecasting of GSE-CI Returns*

Figure 10: Prediction or Forecasting of GSE-FSI Returns

Another tool used to evaluate the accuracy of the prediction is the construction of regression lines between the GSE main index actual series and the predicted series. Figure 4.7 (a) and (b) show that the regression lines for both indices are perfectly fitting the series.
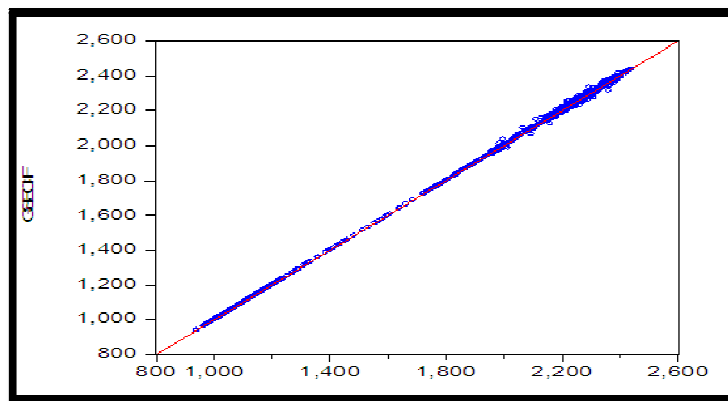
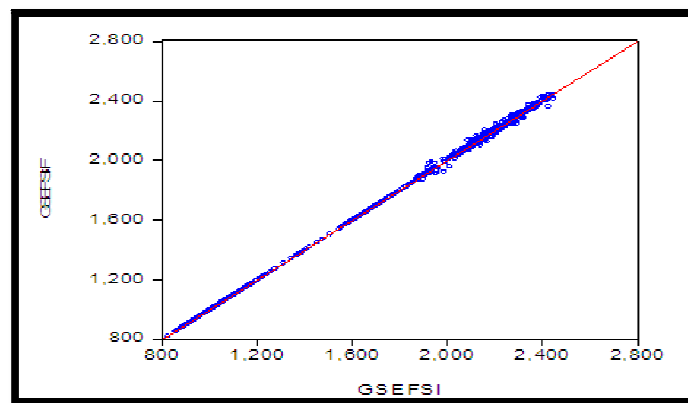*Figure 11: Regression of Actual GSE-CI Values against Predicted GSE-*

Figure 12: Regression of Actual GSE-FSI Values
Against Predicted GSE-FSIF Values

5. Conclusions and Recommendations

5.1. Conclusions

In this research the objectives were to address the following questions. One, does the GSE main index exhibits any statistical properties? Two, are there any relation between stock price movement and volatility on the Ghana stock exchange? Third, to what extent would the prediction of market volatility and returns on assets of GSE minimise the risks incurred by investors? The daily GSE stock indices prices were sampled for the study. The results of the descriptive statistics indicate that GSE main index prices follow a random walk model while the returns exhibit volatility clustering. This is more pronounced in the GSE-CI than the GSE-FSI returns. The means and the standard deviations of the stock indices show small differences between them. Moreover, the two indices do not follow a normal distribution due to high kurtosis and positive skewedness. The JB test value and the QQ-plot also reject normality of the GSE main index returns. Some tests findings from the analysis reveal the presence of ARCH effects, serial correlation and heteroskedasticity in the GSE main index.

With three residuals distributions normal, student-t and GED), the GSE main index returns were fitted to the following GARCH types: GARCH (0, 1), GARCH (0, 2), GARCH (0, 3), GARCH (1, 1), GARCH (1, 2), GARCH (2, 1), GARCH (2, 2) and EGARCH (1, 1). The EGARCH (1, 1) model with GED was the most robust of all during the in-sample estimation period. One significant and interesting result found is that GARCH models with higher ordersdo not give best AIC, BIC and HQC minimum values and hence fail to compete favorably with those with low orders. It is also found that when the whole data was estimated, GARCH (1, 1) model performed creditably than any other GARCH models but could not outperform EGARCH (1, 1) model during the in-sample estimation period.

The conditional variance or volatility of the main index estimated by the EGARCH (1, 1) model reveals very significant results. The findings suggest the existence of volatility shocks to persistence since the ARCH term and the GARCH term sum up to approximately one. The stock indices return therefore follow Integrated GARCH model, an evidence of covariance stationary series and the decay of the shock would be gradual and slow. The presence of leverage effect inherent in the stock indices as captured by the EGARCH (1, 1) model means that most firms listed on the exchange are financing and expanding their capital structure in order to be solvent. This is giving these institutions leverage over others which are not. It has been noted that the volume of assets traded by various listed equities on the GSE has risen tremendously showing that the more leverage firms assumed, the higher the rate of volatility on the stock market.

The prediction of GSE main index has been good as depicted by the graphs and the measures of forecast evaluations (i.e. RMSE, MAE, MAPE, Theil inequality coefficient and the line of best fit). The Theil Inequality coefficient suggests a perfect fit with bias proportion and variance proportion being very small for both stock indices. The errors in the predictions can

only be due to unsystematic errors during the prediction processes. The lines of best fit in each index clearly fit the actual values of the GSE main index against the predicted values as shown in the graphs.

5.2. Recommendations

The following recommendations are suggested for further studies. Since this research finds the GSE main index to follow integrated GARCH model which is covariance stationary in structure, it is highly recommended that to better capture the dynamics of "true volatility," long run persistence of volatility shocks can be modeled by using Fractionally Integrated GARCH (FIGARCH) model, Fractionally Integrated Exponential GARCH (FIEGARCH) model and Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) models and Multi-GARCH models. Moreover, optimization of trading operations of listed equities on the GSE should be explored for maximum profitability.

One important area this thesis could not focus on and it is therefore suggested for further research is the causative factors causing the volatility of the GSE main index. Some of these economic variables like the interest rate risk, inflation rate risk, exchange rate risk, credit rate risk was skyrocketing and making the country to be economically unstable for prospective investors both locally and internationally. Although these factors were mentioned in the background of the study, time could not allow the researcher to study specifically how each would influence the volatility of the GSE main index. The influence of these factors on volatility of GSE main index can be modeled by using the multivariate GARCH model or Artificial Neural Network model which can help to estimate the volatility relation between various markets and assets and determine the extent to which these factors are influencing volatility of the GSE main index. It is also recommended that further research could look at the mathematical modeling and optimization of trading operations of all listed equities on the Ghana stock exchange.

6. References

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Appendix

GARCH Type	GARCH Parameters							Information Criterion		
	ϖ	α_1	α_2	β_1	β_2	β_3	γ	AIC	BIC	HQC
(0, 1)	180.5745 (0.2592)	-	-	-0.5978 (0.6693)	-	-	-	6.8685	6.9070	6.8834
(0, 2)	232.1817 (0.0000)	-	-	-1.0527 (0.0000)	-0.1525 (0.3440)	-	-	6.8670	6.9117	6.8843
(0, 3)	53.0833	-	-	-0.9258 (0.0000)	0.5980 (0.1551)	0.8677 (0.0007)	-	6.8718	6.9229	6.8916
(1, 1)	12.3158 (0.0217)	0.2523 (0.0142)	-	0.6996 (0.0000)	-	-	-	6.8333	6.8787	6.8513
(1, 2)	13.13304 (0.0377)	0.2526 (0.0275)	-	0.4617 (0.2722)	0.2059 (0.5386)	-	-	6.8498	6.9000	6.8696
(2, 1)	11.9467 (0.0461)	0.2664 (0.0531)	-0.0390 (0.7842)	0.7215 (0.0000)	-	-	-	6.8261	6.8773	6.8459
(2, 2)	5.3758 (0.5912)	0.3314 (0.0286)	-0.2196 (0.1693)	1.1915 (0.1238)	-0.3253 (0.5409)	-	-	6.8729	6.9082	6.8729
EGARCH(1,1)	0.4042 (0.0765)	0.2988 (0.0011)	-	0.07968 (0.1693)	-	-	0.8709 (0.000)	6.8147	6.8668	6.8345

Table 20: Results from GARCH Variance Model (2.6) for GSE- FSI Returns with GED