THE INTERNATIONAL JOURNAL OF BUSINESS & MANAGEMENT

Raw Materials Optimization in a Bread Making Factory: An R Implementation

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Abstract:
This study tries to demonstrate the application of linear programming methods in a bread manufacturing industry. The concept of Simplex Algorithm; an aspect of linear programming was deployed to allocate raw materials to some competing variables (Big Loaf, Medium Loaf and Small Loaf) in Crystal Bakery, Keffi, for the purpose of profit maximization. The simplex algorithm was implemented in R, where codes were written to carry out iterations. From the result of analysis, it was gathered that approximately 40 units of Big loaf, 180 units of medium loaf and 0 units of small loaf be produced to achieve a profit of around N23,000. Also, it was seen that most of the profit came from the sales of the medium loaf.

Keywords: *Linear programming, simplex, r, profit, maximisation*

1. Background

It is Impossible to properly define Linear Programming without first exploring the field of Operations research. This is because linear programming is one of the prominent techniques adopted in this field of study. Operation Research, abbreviated 'OR' for short is a scientific approach to decision making, which seeks to determine how best to design and operate a system, under conditions requiring the allocation of scarce resources (Oyekan, 2019).

Major management decisions involve trying to make the most effective use of Organization resources (Adams, 1969). These resources include Machinery, Labour, Money, Time, Warehouse space or Raw materials to produce goods (machinery, furniture, food or cooking) or service (schedules for machinery and production, advertising, policies or investment decision). Linear Programming (LP) is a widely used mathematical technique designed to help managers in planning and decision making relative to resource allocations (Bierman, 1973). And also provide relatively simple and realistic solutions to these problems. A wide variety of production, finance, marketing, and distribution problems have been formulated in the linear programming framework. Linear Programming (LP) also called Linear Optimization is a technique which is used to solve mathematical problems in which the relationships are linear in nature (Stroud, 2003).

The usefulness of this technique is enhanced by the availability of several user-friendly computer software such as STORM, TORA, QSB, LINDO, R, etc. however, there is no computer software for building an LP models. Model building is an art that improves with practice.

1.1. Statement of Problem

The aim of every business organisation is purposely to make profits solving a problem. Bread has been a very common food and factories producing bread are established to meet a need and also make profit. Manufacturers are faced with the problem of scarce resources viz-a-viz the demand for bread daily. Studies have been conducted on combining raw materials for bread production of different sizes and in various markets as seen in (Oyekan, 2019). This study hopes to address the problem of producing different sizes of bread in Keffi and its environs in view of the scarce resources and competing market demand.

1.2. Aim

The aim of the study is to utilize the concept of simplex algorithm in the allocation of scarce raw materials in a bakery for profit maximization implemented in R.

1.3. Objectives
- To formulate the linear programming problem.
- To set up the simplex tableau
- To obtain the optimal product mix that would maximize contribution to profit

1.4. Justification of Study

An optimal as well as a feasible solution to an LP problem is obtained by choosing one set of values from several possible values of decision variables $x_1, x_2, \ldots \ldots \ldots x_n$, that satisfied the given constrains simultaneously and also provides an optional (maximum or minimum) value of the given objective function. For LP problems that have only two variables, it is possible that the entire set of feasible solution can be displayed graphically by plotting linear constraints on a graph paper in order to locate the beat (optional) solution. The technique used to identify the optional solution is called the graphical solution method (approach or technique) for an LP problem with two variables. Since most real-world problems have more than two decision variables, such problems cannot be solved graphically. As we will later see in the course of this study, the graphical approach will not be ideal for solving the product mix problem in a bakery since we shall be dealing with more than two decision variables.

2. Literature Review

2.1. Review of Related Literatures

Linear Programming was developed as a discipline in 1940's, motivated initially by the need to solve complex planning problems in war time operations. This was possible by the recognition that most practical planning problems could be reformulated mathematically as finding a solution to a system of linear inequalities. The problem of solving a system of linear inequalities dates back at least as far as Fourier Joseph (1768 –1830) (Adams, 1969).

Ezema and Amaken (2012) highlighted the problem of low capacity utilization and consequently low outputs. Igwe et al (2011) reported that linear programming is a relevant technique in achieving efficiency.

The authors highlighted the maximization of gross return from semi-commercial agriculture in Ohafia zone in Abia state. A deterministic model has been used to check the optimum solutions. Here, numbers of hectares the farmer devoted to the production of crop and combination of crop or livestock capacity produced by the farmer has been used as the variables. Balogun et al (2012) highlighted the problem of production sectors on the basis of manpower, raw materials, capital etc. Simplex algorithm has been used to solve the problem. The study highlighted that only two (Fanta orange 50cl and Coke 50cl) contribute most to their profit maximization. The importance of linear programming in energy management has been highlighted by Snezanza and Milorad (2009). Veli(2010) reported that a mixed integer linear programming plays an important role in aggregate production planning (i.e. a macro production planning). This study helps to take decision how many employees the firm should retain and for manufacturing company. It can also estimate the quantity and mix of products to be produced. Fagoyinbo and Ajibode (2010) also highlighted the experience of an organization with linear programming for improving their power of decision making. Majeke (2013) highlighted the role of linear programming on commercial farmers. According Mula et al (2005), production planning is very helpful for decision making. This is known as sensitivity analysis. Waheed et al (2012) used linear programming for operation research and management sciences. The data analysis was carried out with R-statistical package, the result of the analysis showed that the company would obtain optimal monthly profit level of about N 271,296 if she concentrates mainly on the unit sales (one tablet per pack) of her medicated soap product ignoring other types of sales packages.

3. Research Methodology

3.1. Introduction

This section looks at the study design, data collection and methods used in carrying out analysis in this project. It gives the foundation of this study.

3.2. Method of Data Collection

There are several methods or types of data collection. In this study, we used a combination of primary data collection and interviews. Some data sets were collected over time while some were obtained from interviewing the Chief Executive Officer (CEO) of Crystal Bakery, Keffi.

3.3. Data Source and Description

The data has been collected from Crystal Bakery, Keffi, Nasarawa State for three sizes of bread namely; Big loaf, Medium Loaf and Small Loaf. Here, the major ingredients are: Flour, Sugar, Yeast, Salt, Soybean oil, wheat gluten and Baking time. Each of these raw materials has their functions as it relates to the production of bread where they are added in different proportions depending on the size of bread to be produced. The different dough sizes are baked in an oven at a temperature of between $150^0 C$ *and* $200^0 C$ for a fine crust of bread. The Manager and Chief Executive Officer (CEO) of the bakery stated that on a certain day, the production majorly depends on the sales and demand of previous days.

Now, the aim here is to determine the product mix, based on the data sets obtained during an interview and data collected over a period of two weeks (1st September, 2019 to 14th September, 2019), that will give the optimal profit as these products are combined. The raw data sets are presented in Chapter Four and in the appendix of this work.

Some assumptions were made during data collection, these include
- Demand for bread is constant, day in, day out
- Raw materials needed are scarce
- Quality of materials used are tested and ok
- Cost of labour and transportation are negligible
- Effective allocation of raw materials to variables will give optimal production and profit

Also, since the company majorly uses seven different raw materials for the manufacturing of her products, therefore, there are seven linear constraints for the LP model. The whole analysis was performed using R statistical package (www.cran.org). A self-contributed library package in R, the 'lpSolve' that implements the LPwas adopted and adapted in this study.

3.4. Method

3.4.1. Structure of Liner Programming Model

The general structure of an LP model consists of the following three basic components.
- Decision variable (activities) the evaluation of various courses of action 9alternativves) and select the best of arrive at the optimal value of objective function, is guide by the nature of objective function and availability of resources. For this, certain activities 9 also called decision variables) usually denoted by $x_1, x_2, \ldots \ldots \ldots x_n$ are conducted. The value of these variables (activities) represents the extent to which each of these is performed. For example, in a product-mix manufacturing problem, an LP model may be used to determine units of each of the products to be manufactured by using limited resources such as personnel, machinery, money, material, etc.
- The value of certain variables may or may not be under the decision-makers control if value are under the control of the decision-maker, then such variables are said to be controllable, otherwise they are said to be uncontrollable, these decision variable, usually interrelated in terms of consumption of resources, require simultaneous solutions. In an LP model all decision variables are continuous, controllable and non-negative. That is $x_1 \geq 0, x_2 \geq 0, \ldots \ldots \ldots x_n \geq 0$.
- The objective function the objective function of each LP problem is expressed in terms of decision variables to optimize the criterion of optimality (also called measure of performance) such as profit, cost, revenue, distance etc. in its general form, it is represented as:
- Optimize (maximize or minimize) Z = $c_1 x_1 + c_1 x_2 + \cdots + c_n x_n$
- Where Z is the measure of performance variable, which is a function of $x_1, x_2 \ldots \ldots \ldots x_n$ quantities $c_1, c_2 \ldots \ldots \ldots c_n$ are parameters that represented the contribution of a unit of the respective variable $x_1, x_2 \ldots \ldots \ldots x_n$ to the measure of performance Z the optimal vale of the given objective function is obtain by the graphical method or simplex method.
- The constraints there are always certain limitations (or constraints) on the use of resources, such as labour, machine, raw material, space, money, etc. that limit the degree to which an objective can be achieved such constraints must be expressed as linear equation in terms of decision variables. The solution of an LP model must satisfy three constraints.

3.4.2. The Simplex Algorithm

The method adopted is the Simplex (Tableau) Algorithm. Simplex algorithm is an iterative procedure that examines the vertices of the feasible region to determine the optimal value of the objective function. This method is the principal algorithm used in solving LPP consisting of two or more decision variables. It involves a sequence of exchange so that the trial solution proceeds systematically from one vertex to another in k, each step produces a feasible solution. This procedure is stopped when the volume of $c^T X$ is no longer increased as a result of the exchange. Listed below are the procedures required in the fore mentioned:
- Step 1: Setting up the Initial Simplex Tableau

In developing the initial simplex tableau, convert the constraints into equations by introducing slack variables to the inequalities. So that the problem can be re-written in standard form as a maximization problem. Such that, we can find the initial basic feasible solution by setting the decision variables $x_1, x_2, \ldots \ldots \ldots, x_n$ to zero in the constraints we get the basic feasible solution and the objective function becomes Z = 0.
- Step 2: Optimality Process

Having setup, the initial simplex tableau, determine the entering variable (key column) and the departing variable (key row). From the $C_j - Z_j$ row we locate the column that contains the largest

Positive number and this become the Pivot Column. In each row divide the value in the R.H.S by the positive entry in the pivot column (ignoring all zero or negative entries) and the smallest one of these ratios gives the pivot row. The number at the intersection of the pivot column and the pivot row is called the pivot. Now, divide the entries of that row in the matrix by the pivot and use row operation to reduce all other entries in the pivot column, apart from the pivot, to zero.
- Step 3: The Stopping Criterion

The simplex method will always terminate in a finite number of steps when the necessary condition for optimality is reached. The optimal solution to a maximum linear program problem is reached when all the entries in the net evaluation row, that is $C_j - Z_j$, are all negative or zero. Here, simplex method has been used.

4. Data Presentation, Analysis and Results

4.1. Introduction
This Chapter gives a more detailed presentation of the datasets, analysis performed in this study and results obtained.

4.2. Data Presentation
The data for this project was collected from Crystal Bakery in Keffi, Nasarawa State as stated earlier. Seven different raw materials are used here and this includes (Flour, Sugar, Salt, Yeast, Wheat gluten, Soybean oil and Baking Time). The quantity of each variable required for production of each bread size is also given, viz a viz the available resources and profit per each unit size of bread produced are also made available. R version 3.6.1 was used for all data analysis. Content of each bread size in terms of raw materials is given below:

4.2.1. Flour (Measured in kg)
Available amount of flour is 450kg
Every unit of Big loaf needs 0.42kg of flour
Every unit of Medium loaf needs 0.35kg of flour
Every unit of Small loaf needs 0.27kg of flour

4.2.2. Sugar (Measured in kg)
Available amount of Sugar is 75kg
Every unit of Big loaf needs 0.23kg of Sugar
Every unit of Medium loaf needs 0.20kg of Sugar
Every unit of Small loaf needs 0.18kg of Sugar

4.2.3. Salt (Measured in Grams)
Available amount of salt is 15g
Every unit of Big loaf needs 0.003g of salt
Every unit of Medium loaf needs 0.0025g of salt
Every unit of Small loaf needs 0.002g of salt

4.2.4. Yeast (Measured in Grams)
Available amount of yeast is 250g
Every unit of Big loaf needs 0.03g of yeast
Every unit of Medium loaf needs 0.03g of yeast
Every unit of Big Small needs 0.03g of yeast

4.2.5. Wheat Gluten (Measured in Grams)
Available amount of wheat gluten is 15g
Every unit of Big loaf needs 0.0024g of Wheat gluten
Every unit of Medium loaf needs 0.002g of Wheat gluten
Every unit of Small loaf needs 0.0015g of Wheat gluten

4.2.6. Soybean Oil (Measured in Liters)
Available amount of Soybean oil is 35l
Every unit of Big loaf needs 0.024l of oil
Every unit of Medium loaf needs 0.019l of oil
Every unit of Small loaf needs 0.016l of oil

4.2.7. Baking Time (Measured in Hours)
Available amount of baking time is 30 hours
Every unit of Big loaf needs 0.17hrs of time
Every unit of Medium loaf needs 0.13hrs of time
Every unit of Small loaf needs 0.08hrs of time

4.2.8. The Profit per Unit Size of Bread
A unit of Big loaf contributes N120 to profit
A unit of Medium loaf contributes N99 to profit
A unit of Small loaf contributes N30 to profit

A detailed summary of the above information is presented below:

Raw Materials	Bread Size			Available Resources
	Big Loaf	Medium Loaf	Small Loaf	
Flour (kg)	0.42	0.35	0.27	500
Sugar (kg)	0.23	0.20	0.18	75
Salt (g)	0.003	0.0025	0.002	15
Yeast (g)	0.03	0.03	0.03	250
Wheat Gluten(g)	0.0024	0.002	0.0015	15
Soybean Oil (l)	0.024	0.019	0.016	35
Baking Time (hrs)	0.17	0.13	0.08	30
Unit Profit (N)	120	99	30	

Table 1

4.3. Data Analysis

The above data is analyzed by first transforming the data sets to Linear Programming problem as done below. The decision variables were defined clearly, the Objective function identified and the constraints stated.

4.3.1. Model Formulation

Before we formulate our linear programming problem model, we shall define our variables below:

Let

$x_1 = amount\ of\ Big\ loaf$

$x_2 = amount\ of\ Medium\ loaf$

$x_3 = amount\ of\ Small\ loaf$

$P = Objective\ function$

From the data above, our linear programming problem (model) is formulated below as

Maximize

$$P = 120x_1 + 99x_2 + 30x_3$$

Subject to the constraints

$Flour\quad 0.42x_1 + 0.35x_2 + 0.27x_3 \leq 500$

$Sugar\quad 0.23x_1 + 0.20x_2 + 0.18x_3 \leq 75$

$Salt\quad 0.003x_1 + 0.0025x_2 + 0.002x_3 \leq 15$

$Yeast\quad 0.03x_1 + 0.03x_2 + 0.03x_3 \leq 250$

$Wheat\ Gluten\ 0.0024x_1 + 0.002x_2 + 0.0015x_3 \leq 15$

$Soybean\ Oil\quad 0.0024x_1 + 0.0019x_2 + 0.0016x_3 \leq 35$

$Baking\ Time\quad 0.17x_1 + 0.13x_2 + 0.08x_3 \leq 30$

$Non-negative\ Constraints\quad x_1, x_2, x_3 \geq 0$

4.3.2. Simple Procedure Implemented In R

In solving any linear programming problem, we are exposed to two methods: the graphical method and the Simplex method. The graphical method can only be used when we are trying to solve a problem involving only two decision variables. In this study, we have three decision variables to worry about, as such, we must use the simplex algorithm. The initial simplex tableau is given below:

Basis	x_1	x_2	x_3	w_1	w_2	w_3	w_4	w_5	w_6	w_7	B	Check
w_1	0.42	0.35	0.27	1	0	0	0	0	0	0	500	502.04
w_2	0.23	0.20	0.18	0	1	0	0	0	0	0	75	76.61
w_3	0.003	0.0025	0.002	0	0	1	0	0	0	0	15	16.0075
w_4	0.03	0.03	0.03	0	0	0	1	0	0	0	250	251.09
w_5	0.0024	0.002	0.0015	0	0	0	0	1	0	0	15	16.028
w_6	0.0024	0.0019	0.0016	0	0	0	0	0	1	0	35	36.006
w_7	0.17	0.13	0.08	0	0	0	0	0	0	1	30	31.38
P	-120	-99	-30	0	0	0	0	0	0	0	0	-249

Table 2

To solve this problem using R, we followed the procedures and commands below:
We installed and loaded the 'lpSolve' package in the R console by using the code

```
>install.packages('lpSolve')
>library(lpSolve)
```

The Objective function, Constraints, inequality direction and Right-hand side (RHS) were defined as shown below

```
>Objective=c (120,99,30)
>Constraint=matrix(c(0.42,0.35,0.27,0.23,0.20,0.18,0.03,0.03,0.03,0.003,0.0025,0.002,0.024,0.019,0.016,1.7,1.3,0.8,0.0024,0.002,0.0015), ncol=3, byrow=TRUE)
>ConstraintIQ=c('<=','<=','<=','<=','<=','<=','<=')
>RHS=c(500,75,250,15,35,30,15)
```

To carry out the optimization, the command below was deployed

```
>Prob.Sol=lp('max',Objective,Constraint,ConstraintIQ,RHS,compute.sens=TRUE)
>Prob.Sol$solution
>Prob.Sol$objval
```

4.4. Results

- The Prob.Sol$solution command above returns the values of the decision variables x_1, x_2 and x_3 as

```
>Prob.Sol$solution
[1]  0.0000 230.7692  0.0000
```

That is $x_1 = 0$, $x_2 = 230.77$, $x_3 = 0$. This implies that, big loaf and Small loaf contributes almost nothing to the profit margin of the bakery, while Medium loaf is said to contribute more to profit with a quantity of approximately 231 units of medium loaf produced daily.

- The Prob.Sol$objval command gives us the result below

```
>Prob.Sol$objval
[1] 23076.92
```

This implies that, producing zero (0) units of Small and Big loafs and about 231 units of medium loaf of bread will give as a profit of around twenty-three thousand naira (N23,000.00) only.

5. Summary, Conclusion and Recommendations

5.1. Summary

This study focuses on usage of optimal allocation of raw materials in the production of breads. R has been used for implementation. The company used here is Crystal bakery limited for different sizes of breads. The result highlighted the quantity of raw materials. Data obtained was transformed to a linear programming problem model and the initial simplex tableau generated. The result indicates the optimal product mix. Results obtained shows that about 231 units of medium loaf, 0 unit of Big loaf and 0 Unit of Small loaf should be produced to give a maximum profit of about N23,000.00.

5.2. Conclusion

Based on the analysis carried out, Crystal bakery Ltd. Should produce the three sizes of the bread just to satisfy their consumers, but more attention be given to the production of the medium loaf since it contributes more to profit.

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