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History and Philosophy of Quantum Mechanics I : The Ontological Interpretation

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Abstract:
The standard quantum mechanics provides a bare mathematical description. This positivistic attitude effectively replaces all forms of realism, causality and related determinism with mathematical recipes for calculating probable "expectation values" of the corresponding physical quantities. Thus, in the standard quantum mechanics, few illustrative and metaphoric devices represent the micro-physical processes of underlying quantum systems. On the other hand, the de Broglie-Bohm ontological interpretation of quantum mechanics transforms the Schrödinger's equation into a form of Newton's Second Law of Motion and re-constitutes particle ontology and dynamical determinism. Under the ontological interpretation, through particles' trajectories visualized in real space and time, the intuitive and cognitive representations on the dynamical motion can have some legitimate physical meanings in explaining quantum phenomena.

1. Standard Quantum Mechanics

The standard quantum mechanics, formulated in its present mathematical form in the early 1930s, has been one of the most successful physical theories in history of physical science. However, ever since its formulation, it has been a constant source of debates for its meanings and interpretations. One reason of the debate is that the standard quantum mechanics, or "the Copenhagen Interpretation," provides such a mere mathematical description that explanations for some possible underlying micro-physical processes involved in the descriptions are not provided.

This minimalist and positivistic attitude effectively eliminates all forms of realism, causality and related determinism of classical mechanics. They are considered unnecessary metaphysics for the theoretical structure of quantum mechanics and are thus replaced by mathematical recipes for calculating probable "expectation values" for the corresponding physical quantities. So, in the standard quantum mechanics, various forms of visual, intuitive and cognitive representations are taken to be primarily of ornamental purpose; they are not intended for explaining genuine underlying processes. (in other words, they are not functional.) Because of this, some philosophers of physics seriously doubt the status of the standard quantum mechanics in its present form as a legitimate physical theory of the descriptions of the quantum processes involved (i.e. Wyatt 2005).

In the quantum mechanics developed in the 1920s, the state of the physical system is represented by an abstract state vector in a linear vector space called a Hilbert space. We often denote the state of the system by the ket |Ψ>. This notation was first introduced by Dirac to indicate a (state) vector in a Hilbert space. Especially, if we use the state in the position representation, it is referred to as the wavefunction. In this linear vector space we introduce a set of axes |**x**>, one axis for each value of **x**. Then the wavefunction is the set of components of the state vector with respect to this basis: Ψ(**x**, t) =<**x**|Ψ(t)>. Also, the wavefunction is square-integrable, so that it is normalized in a Hilbert space as follows:

$$<\Psi|\Psi> = \int_{-\infty}^{\infty} |\Psi|^2 \, d^3\mathrm{x} = 1.$$

Then, if at time t a measurement is performed to determined the position of the system described by the function Ψ(**x**, t), the probability that the result lies in the element of space $d^3\mathrm{x}$ around the point **x** is given by

$$P(\mathrm{x},t)d^3x = |\Psi(\mathrm{x},\mathrm{t})|^2 \, d^3\mathrm{x}.$$

This P is also called the probability density distribution.

All observables in quantum mechanics are represented by linear Hermitian operators acting in the Hilbert space. For example, the classical canonical momentum **p** is replaced, in the position representation, as follows:

$$\mathrm{p} \rightarrow \hat{\mathrm{p}} = -i\frac{h}{2\pi}\nabla,$$

where $i = \sqrt{-1}$, h is Plank's constant and □ is the gradient operator.

The outcome of the measurement of an observable $\hat{A}$ is one of its eigenvalues, a, defined by the following equation,

$$\hat{A}\,|\,\mathrm{a}\rangle = \mathrm{a}\,|\,\mathrm{a}\rangle ,$$

where a is real and |a> is the corresponding eigenstate. In the position representation we shall write $\langle x\,|\,\mathrm{a}\rangle = \Psi_a(x)$. The eigenfunctions corresponding to distinct eigenvalues a, a' are orthonormal,

$$\int_{-\infty}^{\infty} \Psi^*_{a'}(x)\Psi_a(x)d^3x = \delta_{aa'} ,$$

and form a complete set so that an arbitrary wavefunction can be expanded in terms of them with some complex numbers as coefficients of the expansion :

$$\Psi(x) = \sum_a c_a \Psi_a(x) ,$$

where c_a are complex numbers. We have $c_a = \langle a\,|\,\Psi\rangle$ so that these numbers are the components of the vector |Ψ> with respect to the basis |a> in a Hilbert space.

If now a measurement of $\hat{A}$ is performed on a system in the general normalized state of $\Psi(x) = \sum_a c_a \Psi_a(x)$, the probability of the outcome a is given by a square of the absolute value of the coefficient of an eigenstate, $|\,\mathrm{c_a}\,|^2$. This is the probability of the measurement outcome for the eigenstate. Once a measurement is performed on a system, as a result of measurement, the system is said to have 'collapsed' into the eigenstate |a>. The expectation value of the operator $\hat{A}$ in the state |Ψ> is given by

$$\langle\hat{A}\rangle = \langle\Psi\,|\,\hat{\mathrm{A}}\,|\,\Psi\rangle = \sum_a a\,|\,\mathrm{c_a}\,|^2 .$$

Now, the classical Hamiltonian function, $H = p^2/2m + V(x)$, where V is the external potential energy and m is the mass of the system, is replaced by the Hermitian operator, $\hat{H} = \hat{p}^2/2m + \hat{V}(\hat{x})$. This determines the evolution of the quantum state via a dynamical equation called the Schrödinger equation:

$$i\frac{h}{2\pi}\frac{\partial\Psi}{\partial t} = \hat{H}\,|\,\Psi\rangle .$$

This equation gives the time evolution of the state vector under the influence of the Hamiltonian for the physical system.

The eigenvalues of the Hamiltonian operator are the possible energies of the system. In the position representation the Schrödinger equation becomes the partial differential equation

$$i\frac{h}{2\pi}\frac{\partial\Psi(x,t)}{\partial t} = -\frac{(h/2\pi)^2}{2m}\nabla^2\Psi(x,t) + V(x)\Psi(x,t) .$$

This is the time dependent Schrödinger equation. The solution Ψ describes a particle of mass m in the presence of a specified potential energy function V. This law of motion, the Schrödinger equation, is a linear equation of motion. In other words, in general, it admits a linear superposition of two solutions, Ψ_1 and Ψ_2 such that

$$\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t)$$

is also a solution, where two constants c_1 and c_2 are complex numbers.

In a general situation, to analytically solve the partial differential form of the Schrödinger equation for all time, physicists have some rules in applying the equation. They must specify the initial state of the wavefunction $\Psi(x)_0 = \Psi(x,0)$. In addition, the wave function and its derivative must satisfy normalization and certain boundary conditions. For a finite potential V, the wave function and its derivative must be bounded, continuous and single valued functions of **x**. For a discontinuous potential V along a surface, the wave function and the normal component of its derivative across the surface must be continuous. For an infinite potential step, the wave function along the surface and the normal component of its derivative across the surface are indeterminate.

One specific way to solve the Schrödinger equation can be the 'separation of variable.' This method looks for solutions that are products of a function of position and a function of time. Then, the wavefunction becomes

$$\Psi(x,y,z,t) = \Psi(x,y,z)e^{-iEt/(h/2\pi)} .$$

In the case of a spherical symmetry, physicists adopt the usual spherical coordinates (r, □, □). So, the time independent Schrödinger equation can now be solved by further separation of variables. Writing

□(r, □, □) = R(r)□(□)□(□),

we can then get three differential equations for R, □, and □. The solutions for □, and □□form a function, the so-called 'spherical harmonics.' It only shows the angular dependency carrying no reference to the specific potential. More interesting part is the 'radial Schrödinger equation', which has the potential term. After introducing a new function u(r) ≡ rR(r), the radial equation becomes

$$-\frac{(h/2\pi)^2}{2m}\frac{d^2u}{dr^2}+\left[V(r)+\frac{(h/2\pi)^2}{2m}\frac{l(l+1)}{r^2}\right]u=Eu\,,$$

where *l* is the separation constant introduced from the function R(r). At this point, we can put in the particular potential V(r) for the problem at hand. We will be then able to solve the radial equation for u(r) and combine the result with the appropriate spherical harmonics to get the full wave function

In the course of solving the radial equation above, physicists will allow only certain special values of E as acceptable results. For most values of E, they argue, the solution to the radial equation blows up at large r, and yields a non-normalizable wavefunction. Thus, the energy can take on only certain specific values, the so-called 'quantized energies' of the system. In this manner, when applied to the 'hydrogen atom', the radial equation is solved as follows. Physicists tacitly consider some kind of system consisting of an 'electron' and a 'proton', taking the proton essentially just sitting at the origin. They then take the wavefunction in question to be only that of the electron. It should be noted that the ontology of the electron or the proton is not part of the formalism. Under the Copenhagen formalism of quantum mechanics, "there are no 'electron' or 'atoms' in the sense of distinct localized entities beyond the act of observation. These are simply names attributed to the mathematical symbol Ψ to distinguish one functional form from another" (Holland 1993, p. 6). Physicists believe all they need is the mathematical function for the electron's potential energy to solve the equation. The electron's potential energy due to the electrical attraction of the proton is simply taken to be:

$$V(r)=-\frac{e^2}{r}\,,$$

where e is the charge of the electron and r is the distance between the electron and the proton. When this potential is put into the radial equation, it is found that normalizable solutions occur only when E assumes one of the special values

$$E_n=-\frac{me^4}{2(h/2\pi)^2n^2}=13.6eV/n^2 \quad (n=1, 2, 3, \ldots).$$

Then, finally the corresponding normalized wave function is taken to be that of the electron.

However, physicists say the wavefunction of the electron itself is not directly observable; it is not really what physicists are concerned with. The crucial thing is, they say, the allowed or quantized energies of the hydrogen atom. In practice, they do not even measure the energies directly, but rather the wavelength of the light emitted. When the electron makes a transition from a higher level to a lower one, the photon carries the difference in energy between the initial and final states. They again tacitly consider some kind of system consisting of an electron instantaneously moving up and down between the energy levels, taking the time and the motion in-between as essentially meaningless.

- Conclusion: The standard quantum mechanics provides a bare mathematical description. This positivistic attitude effectively replaces all forms of realism, causality and related determinism with mathematical recipes for calculating probable "expectation values" of the corresponding physical quantities. Thus, in the standard quantum mechanics, few illustrative and metaphoric devices represent the micro-physical processes of underlying quantum systems.

2. de Broglie-Bohm's Ontological Quantum Mechanics

In early 1950s, physicist David Bohm made himself clear in the position against the probability-based minimalism of the Copenhagen interpretation. The Bohmian interpretation was also known to be similar to the one already suggested by de Broglie in 1920s. Louis de Broglie originally came up with an idea of a guidance condition and a 'pilot wave' guiding the dynamical motion of a quantum particle. The idea was faced with unjust harsh criticism by Wolfgang Pauli and de Broglie himself moved on to other problems. However, since the revival of the similar idea independently by Bohm, it now comes to be known as the "de Broglie-Bohm ontological interpretation" of quantum mechanics.

This Bohmian ontological interpretation transforms the Schrödinger equation, into a form of "Newton's Second Law of Motion," re-constituting classical ontology and determinism back into the corresponding quantum systems. Bohmian particle behavior is then described by "Second Law of Motion," while its wave behavior is handled by the usual Schrödinger equation. This wave is also called the "pilot-wave" previously denoted by de Broglie and guides a particle through its dynamical motion, generating all kinds of quantum effects. However, as already mentioned in Chapter 1, there is a difference between de Broglie's approach and Bohm's. de Broglie used the guidance condition of equation (6) in this section 2.2 as an axiom, through which particles' velocity field is defined to generate such classical dynamical features as particle trajectories. From now on, whenever de Broglie-Bohm ontological interpretation is mentioned, it will specifically mean the particular ontological approach taken by Bohm.

Therefore, by visualizing particles' trajectories in real space and time, this ontological interpretation provides some intuitive and cognitive (rather than probabilistic, mathematical or formal as in the standard quantum mechanics) insights into particles' dynamical motion. However, physicists are suspicious about the legitimacy of this approach because the newly introduced term called the "quantum potential" in the mathematical transformation by Bohm seems to originate from no known physical source. This gives physicists a strong impression that it is only a mathematical entity, not a physical one. So, they doubt the ontological interpretation as an alternative form of quantum mechanics.

Quantum mechanics, as sketched in the previous section 2.1, provides a rather sparse description of the world. It prohibits our "customary intuition" and the "benefit of direct visualizability" from ordinary mechanics (see below for further discussions). In a sense, quantum mechanics tries to demonstrate that the mathematical formulation itself can totally replace our perceptual metaphors and illustrations in describing the quantum world. Thus, our intuitive pictures are not particularly "functional" in understanding the quantum world and thus even considered to be "trash" according to Werner Heisenberg.

Citing MacKinnon (1980, chapter 2), Cushing (1994, p.20) however emphasizes a desire for visualizable models or causal explanations as an essential part of any scientific endeavor. Cushing (1994, p.21) continues, "we attempt to generate theories that make the world comprehensible in terms of our inherent patterns of thought. Understandability in a successful scientific theory is an important pragmatic virtue, both for visualizing fundamental phenomena for new research and for allowing a wider audience to comprehend that theory." Cushing then criticizes that the standard quantum mechanics does not provides a comprehensible and understandable account in the quantum world through direct visualization of fundamental phenomena. In accordance with Cushing's view, John Hendry (1985, p.392) sees the loss of visualization as a radical and contingent shift, i.e., a major change that did not logically have to take place in the development of Copenhagen hegemony. Arthur Miller (1984) has further discussed the key role that visualization played in the *early* history of quantum mechanics, although not in the final formulation by the Copenhagen school.

Here, let's follow Miller(1984)'s account on the role of visualization in the history of quantum mechanics. In 1913, Niels Bohr proposed his model of the solar system atom with the help of what Werner Heisenberg later called our "customary intuition." Most physicists including Max Born welcomed Bohr's atomic theory as laws of the micro-cosmos reflecting the terrestrial world. However, by 1925, the empirical failure of this imagery atom of solar system became so obvious that Heisenberg finally announced "the failure of our customary intuition" and he "recalled the necessity to liberate oneself from intuitive picture" (p.250). Heisenberg then further declared that the "benefit of direct visualizability" was faced with internal contradictions.

So, since 1925, mathematics had taken the place of visualization and become the guide when Heisenberg started his new quantum mechanics based on the observable spectral lines of the atoms, instead of the unobservable electron orbits. However, this situation in which there was no mental image involved for the atomic world provoked a direct response from Erwin Schrödinger. Schrödinger in 1926 proposed a wave mechanics based on the "customary intuition" of waves envisaged with the orbital electrons. Schrödinger "was "repelled" by the quantum mechanics' "lack of visualizability" and by its mathematics" (p.251). Nevertheless, "Heisenberg referred to Schrödinger's wave pictures as "trash," and to the wave mechanics as useful only for calculational purposes" (p.251). At this point, in 1926, Bohr and Heisenberg met to begin their intense discussions on the interpretation of the wave and quantum mechanics. In early 1927, Bohr apparently arrived at a resolution by emphasizing "the restrictions imposed by language on our capacity to form images for scientific theories" (p.252), which eventually led to the principle of complementarity described by Miller as follows (p. 252):

- In the atomic domain an essential difference lies between pictures and the actual development of atomic systems. For in this domain physical laws require a "departure from visualization in the usual sense."
- The perceived mode of an atomic entity depends on the experimental arrangement in use. For example, light and electrons display their wave mode. […] But wave and particle modes cannot be exhibited in a single experiment because they are mutually exclusive. Yet, both modes are required to characterize fully an atomic entity.

Emphasizing the complementarity principle as the "restriction on perceptual metaphors" in the atomic domain, Miller then concludes that "according to complementarity, mental images are connected only loosely to the theory's mathematical, that is, logical, apparatus and are not completely functional" (p.260).

The Copenhagen interpretation thus founded by Niels Bohr, Werner Heisenberg, Max Born and other physicists was the first general attempt to understand the common formal features represented by quantum mechanics. Some of the main ideas behind the Copenhagen interpretation are indeterminism, the correspondence principle, the statistical interpretation of the wave function, and the complementarity of quantum phenomena (e.g. Faye (1991) for further discussion). (However, the measurement problem still remains the single most debated topic of all.)

Therefore, the familiar physical properties of particles having a definite position, velocity or energy etc. are all denied in the Copenhagen interpretation of quantum mechanics. "This [standard] interpretation requires complementarity (e.g., wave-particle duality), inherent indeterminism at the most fundamental level of quantum phenomena, and the impossibility of an event-by-event causal representation" (Cushing 1994, p.24). Due to the uncertainty postulated as an intrinsic feature in quantum systems, it is only possible to obtain statistical results about observational measurements; one is forbidden to think about physical processes as deterministic events composed of causally-connected visual sequences in space and time. So, particles do not follow any definite trajectories in their dynamical motion since causality and determinism have no place in quantum mechanics. In turn, whereas classical particles can be studied both individually and statistically depending on the number of particles, quantum systems only allow a statistical description of their observational measurements, even for a single particle. This widely accepted statistical interpretation of the wavefunction was given originally by Born.

In addition, the mathematical formalism of quantum mechanics itself is not entirely well defined and closed in its structure; it frequently requires additional interpretations for each given problem setting, often with no established general consensus. In a sense, the formalism itself does not provide much information and thus demands a case-by-case way of understanding the mathematical equations in application to given problems. The primary reason of this ambiguity arises from, as discussed before, the lack of our "customary intuition" and "benefit of direct visualizability." The mathematical formulation itself cannot fully replace the perceptual metaphors and illustrations formed from our intuitive pictures about the world. That is, they cannot be simply abandoned as "trash."

Holland agrees that "it is not clear that one can apply the formalism to concrete problems without at least some mental image of the system studied. […] Thus, in applying the theory physicists tacitly consider, and perhaps have to consider, quantum mechanics to be something more than just a means of correlating experimental results, and attribute to it the ability to describe some kind of reality beyond the phenomena"(Holland 1993, p.6). Holland also points out, "quantum mechanics appears essentially as a set of working rules for computing the likely outcomes of certain as yet undefined processes called 'measurement'" (Holland 1993, p.6). This is what they call 'the measurement problem' of the standard quantum mechanics (more discussed in Chapter 6). The linear Schrödinger equation naturally suggests as a general solution a linear combination (superposition) of various quantum states. However, upon measurement on a quantum system, superposition disappears and only one of the states corresponding to a value of the measurement is definitely realized. In the standard quantum mechanics, the quantum state is then said to collapse or to be projected to a particular state of the measurement (i.e. the projection postulation). Apparently, this suggests that something happens in a process called measurement (thus, a.k.a. the measurement problem).

Ever since the Copenhagen interpretation, philosophers and physicists have tried to introduce different interpretations of quantum mechanics. What these scholars are usually trying to do is not to question the general validity of the mathematical formalism of current quantum mechanics, but rather to rearrange some of the mathematical formalisms so that new physical insights into it can be achieved.[1] For example, the modal interpretation of quantum theory was born by Bas van Fraassen (e.g. van Fraassen 1991). He proposed a distinction between the 'value state' of a system, and the 'dynamical state' of a system to eliminate the measurement problem or the projection postulation. Hugh Everett(1957)'s relative-state formulation of quantum mechanics was another attempt to solve the measurement problem by explaining why observers get determinate measurement records. Several attempts to reconstruct Everett's quantum mechanics have led to such formulations as the many-worlds interpretation of quantum mechanics (e.g. Barrett 1999). In this interpretation, many parallel worlds coexists with ours at the same space and time to have determinate measurement records by removing randomness in the quantum measurement process. On the other hand, the GRW theory (Ghirardi, Rimini and Weber 1985, 1986) is a rather mathematical attempt to change the formal structure of quantum mechanics in solving the measurement problem by introducing an additional stochastic 'noise' term to the Schrödinger equation.

In this respect, the quantum theory of motion, developed previously by Louis de Broglie and later independently by David Bohm, is a theoretical formalism combining accurate quantum mechanical predictions and a causal interpretation with "functional" (as discussed by Miller) visual illustrations and metaphors on the physical phenomena involved (Holland 1993). At a first look, this Bohmian formalism looks simply as cosmetic or ornamental embellishment from a rearrangement of the original framework of quantum mechanics. However, through this arrangement, a new visual insight on quantum dynamical processes can be achieved. In this alternate formalism each particle always has a definite momentum and position throughout the entire motion with complete determinism. This allows to trace the entire path of a particle' dynamical motion, showing what happens during the process, not just the final outcomes (i.e. expectation values) of it. Similar to (and distinct from) Erwin Madelung's 'hydrodynamical' formulation of quantum mechanics (Madelung 1926), it first introduces a polar form for the wavefunction into the Schrödinger equation, and de Broglie concept of the 'pilot wave' for the wavefunction (de Broglie 1925a; 1925b). Particles are guided by this surrounding pilot wave, which is a solution of the Schrödinger equation, such that the space-time orbits of an ensemble of particles influenced by the pilot waves (and the quantum potential) reproduce the statistical quantum predictions.

Bohm initially began with the non-relativistic Schrödinger equation and, by means of a mathematical transformation, rewrote the basic dynamics of quantum mechanics in a 'Newtonian' form of the second law of motion with an additional, new term called the 'quantum potential' (Bohm 1952). The result was exact; no approximation was made in passing from the initial Schrödinger equation to Bohm's Newtonian form. Consequently, the transformation generates a set of new equations which are numerically equivalent to all aspects of traditional non-relativistic quantum mechanics. Thus, through this transformation, the ontological interpretation of Bohmian quantum mechanics has the same mathematical consistency and observational outcomes as those of the Copenhagen.[2] It is a 'genuine' quantum mechanics equipped with a classical picture to describe the quantum domain.

In this ontological picture, we can still enjoy such classical concepts as particle ontology and trajectory with complete determinism in describing its motion, given the 'hidden' position variables as initial conditions of a particle. Based on this classical insight, causality is preserved such that the initial and final states of a quantum process are causally connected by particles' dynamical trajectories, presenting significant conceptual differences from the standard Copenhagen interpretation. With well-defined particle trajectories of motion as a function of time, this theory provides a clear visual insight into the quantum domain. Cushing wrote, "we are able to construct a less incomprehensible, more nearly picturable representation of the physical universe with Bohm than with Copenhagen" (Cushing 1994, p.21).

For a more formal discussion, let us now define a quantum mechanical system as constituted essentially by a particle having a precisely defined position, varying continuously as a function of time, and a wave expanding in space and time, and guiding the motion of the particle. In this way, a particle guided by a (pilot) wave moves in classical trajectories in real space and time. Then, to obtain the equations of motion for such a particle, the quantum wavefunction is written in polar form:

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[1] The GRW theory may be an exception here. This theory actually tries to modify the Schrödinger equation itself by adding a correctional term of some background noise to it.

[2] In particular, the initial objections by Pauli to Bohm's formulation turned out to be groundless (Cushing, 1994). The following forms of all Bohmian ontological interpretations of quantum mechanics, rooted in the previous work of Louis de Broglie in the 1920s (de Broglie 1960) and independently reformulated and extended by David Bohm (1952) and his colleagues (Madelung(1926), Bohm and Vigier(1954), Nelson(1966), Bohm and Hiley(1982, 1984, 1985, 1989, 1993), Valentini (1991a,b), Durr, Goldstein and Zanghi (1992a,b,c, 1993)), all explain the observational data equally well.

$$\Psi(r,t) = R(r,t)e^{iS(r,t)/\hbar} \qquad (1)$$

with the real-valued function $R^2 = \Psi^*\Psi$ and $S = (\hbar/2i)(\ln\Psi - \ln\Psi^{*)}$ are the amplitude (or 'probability density') and phase (or 'action') of , respectively. When the wavefunction in this polar form is introduced into the time-dependent Schrödinger equation as given below,

$$i\frac{h}{2\pi}\frac{\partial\Psi}{\partial t} = \left[-\frac{(h/2\pi)^2}{2m}\nabla^2 + V\right]\Psi \qquad (2)$$

where *V(r,t)* is the potential due to the external classical field, two real coupled equations are then obtained as follows,

$$\frac{\partial R^2}{\partial t} + \nabla\cdot\left[R^2\frac{\nabla S}{m}\right] = 0 \qquad (3)$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{(h/2\pi)^2}{2m}\frac{\nabla^2 R}{R} = 0 \qquad (4)$$

with corresponding imaginary and real parts respectively.

Especially from the second equation (4) above, it is common to use a Hamilton-Jacobi type of formalism in the de Broglie-Bohm theory. This formalism provides a general theory of waves and rays which goes beyond classical mechanics. The two equations (3) and (4) can be interpreted, respectively, as the conservation of probability density (Bohm 1953), and a quantum Hamilton-Jacobi equation, similar to its classical counterpart except for an extra term:

$$Q = -\frac{(h/2\pi)^2}{2m}\frac{\nabla^2 R}{R} \qquad (5)$$

This term is called the 'quantum potential' and vanishes very slowly over distance because it is only determined by the curvature (second derivative) of the wavefunction amplitude, R. Thus, some effects, such as diffraction patterns, happen well beyond the region of classical action of forces. Moreover, the quantum potential encodes information on the whole problem, exhibiting key inherent features of quantum mechanics such as 'context dependence', 'wholeness', 'non-separability', or 'non-locality' etc. In other words, this quantum potential keeps the two parts connected (e.g. Sanz, Borondo, and Miret-Artés 2002); two very distant parts of a system described by the same quantum state maintain a strong correlation, as happens for example with entangled states. Consequently, the quantum-potential term alone produces all the quantum mechanical effects on the motion of the particle. Features of the quantum potential are summarized as follows by Wyatt (2005, p.55);

- Introduces all quantum effects into the hydrodynamic equations
- Gives the shape contribution to the total kinetic energy.
- Measures the curvature-dependent internal stress.
- Influences the quantum trajectories through the quantum force, which is defined to be a negative gradient of the quantum potential.
- Introduces contextuality (dependence on initial wavefunction).
- Is the source of nonlocality in the dynamics.
- For non-stationary states, diverges on a nodal surface where R=0.

However, at the same time, the quantum potential has so many notorious unphysical properties involving non-local, unbound, singular, non-linear, and time dependent features that it is hard for physicists to take it as something real and physical, governing all fundamental features of quantum mechanics. In particular, Michael Dickson also points out, in a private communication, that the quantum potential itself is defined in a 3N configuration space for N particle system. In this 3N configuration, there exists a strong non-local correlation of all kinds, for example, between the x coordinate of the first particle and the z coordinate of the third etc. Thus, it is not considered to be a real physical space and any physical meanings out of it are very hard to obtain. Furthermore, the quantum potential has no known physical source, whereas the gravitational field and its potential have a mass as a source, and the electric field and its potential have a charge as a source. Having no physical origin, the quantum potential seems to be only a spurious mathematical object, generated only after a mathematically-unnecessary transformation from the Schrödinger equation. (In 3.4, more difficulties in the Bohmian interpretations will be discussed.)

The role of the quantum potential Q is more clearly seen if we rearrange equation (4) by defining a velocity field *v* (as is done in the classical Hamilton-Jacobi theory) as

$$v = \dot{r} = \frac{\nabla S}{m} \qquad (6)$$

This expression is called the guidance condition and implies that each particle trajectory is orthogonal to the S=constant manifold. By applying the operator to equation (4) and substituting (6) in the resulting expression, one obtains

$$m\frac{\partial v}{\partial t} + m(v\cdot\nabla)v = -\nabla(Q+V) \qquad (7)$$

from which a generalized Newton's second law

$$m\frac{dv}{dt} = -\nabla(Q+V) \qquad (8)$$

can be obtained by using the Lagrangian time derivative operator

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v\cdot\nabla \qquad (9)$$

In (8) it is observed that a quantum force, $-\nabla Q$, acts on the particle, in addition to the classical force given by $-\nabla V$, and then effects other than those derived from the classical potential, V, are expected. Note that V vanishes much earlier than Q does. Thus, Q has much longer influence than V.

- Conclusion: The de Broglie-Bohm ontological interpretation of quantum mechanics transforms the Schrödinger's equation into a form of Newton's Second Law of Motion and re-constitutes particle ontology and dynamical determinism. Under the ontological interpretation, through particles' trajectories visualized in real space and time, the intuitive and cognitive representations on the dynamical motion can have some legitimate physical meanings in explaining quantum phenomena.

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