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Dynamics of Rupee-Dollar Exchange Rates: Bayesian Analysis and Modelling

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Abstract:
Exchange rate movement is an important subject of macroeconomics analysis and market surveillance and affects decesions of foreign exchange investors, exporters, importers, bankers, businesses, financial institutions, policymakers and tourists in the developed as well as developing world. The objective of this paper is to find the points of structural shift in the exchange rate model under a Bayesian framework which incorporates the possibility of shift or no shift in both disturbances precision and regression parameter. The Bayesian analysis of a linear regression model has been carried out under the mixture of prior distributions for the parameters. In order to find the structural shift points, a test based on posterior odds ratio for testing the hypothesis of no structural shift against the alternative hypothesis of shift due to change in disturbances precision and regression parameters has been developed. Further, as a particular case to test the validity of proposed model, the effect of interest rate, growth rate of trade balance and GDP growth rate (market price) on Indian rupee-US dollar exchange rate has been considered. Finally, a numerical simulation of the proposed model indicates that the strongest structural shift points of exchange rate captured in the year 1997-98 and 2007-08. The stability in the foreign exchange market was disrupted due to intensification of East Asian crises during 1997-98 and the impact of global financial crisis in 2007-08.

Keywords: Exchange Rate, Bayesian Analysis, Linear Regression Model, Economic Framework, Financial Crisis

1. Introduction

Exchange rate movement is an important subject of macroeconomics analysis and market surveillance and affects decesions of foreign exchange investors,exporters, importers, bankers, businesses, financial institutions, policymakers and tourists in the developed as well as developing world. Schotman and Van Dijk (1991) proposed a posterior odds analysis of the hypothesis of unit root in real exchange rates and observed that French franc/ German mark is clearly stationary, while Japanese Yen/ US dollar is most likely a random walk. Chen and Tian (2010) proposed a procedure to monitor change points in linear regression models and applied it to IBM stock price data and Thailand/US foreign exchange rate data. Okafor (1999) applied empirical Bayes model to estimate exchange rate of currencies. Wright (2008) applied Bayesian model averaging for exchange rate forecasting. Exchange rate fluctuations may also affect the value of international investment portfolios, competitiveness of exports and imports, value of international reserves, currency value of debt payments, and the cost to tourists in terms of the value of their currency. Movements in exchange rates thus have important implications for the economy's business cycle, trade and capital flows and are therefore crucial for understanding financial developments and changes in economic policy. Basic foundation of Bayesian analysis for structural shift detection was laid during 1970s.Bromeling (1972) discussed Bayesian analysis of shifting normal sequence and switching regression model, and proposed a method of estimation and detection technique for the shift. Subsequently, Quandt (1972, 1974) introduced a technique to analyze switching regression model from the Bayesian point of view. Some other contribution to the Bayesian methods of analyzing structural change are Holbert (1973), Holbert and Broemeling (1977), Chin Choy (1977), Chi (1979), Salazar (1980, 1981), Smith (1975, 1977, 1980) and Tsurumi (1977, 1978). Assuming that the change point is known, Broemeling and Tsurumi (1987) considered a linear model with correlated error terms. Ng Vee Ming (1990) analysed a linear model in the presence of shift in mean and precision. Inclan and Tiao (1994) considered the problem of multiple change points in the variance of sequence of independent observations. Recently Wang and Zivot (2000) considered a Bayesian time series model of multiple structural changes in level, trend and variance. Jiahui Wang and Eric Zivot (2000) considered a deterministically trending dynamic time series model in which multiple structural changes in level trend and error variance are modeled explicitly.

In this paper, thelinear regression model involving structural changes in disturbance precision and regression parameters under the assumption of mixtureof prior distribution of the parametershasbeen considered. The motivation behind considering such a mixture of prior distributions is that it incorporates the possibility of structural change in parameters of the model. The main emphasis is to derive the posterior odds ratio test in such a way that it can capture the structural shift points in the model. Further numerical simulation has been carried out by considering the dependence of Indian rupee-US dollar exchange rate upon interest rate, growth rate for trade balance (Export-Import) and GDP growth rate (market price).

2. Model and Prior Distribution

Let us consider the linear regression model with structural shift in both regression parameter and disturbance term.

$$y_t = x_t'\beta + u_t \qquad ; t = 1,2,\dots,n_1$$
$$y_t = x_t'\beta + (1-\delta)x_t'\gamma + u_t \qquad ; t = n_1+1,\dots,n_1+n_2(=n) \qquad (2.1)$$

Where y_t is the t[th]observation of the dependent variable, β and γ,$k \times 1$ vector regression coefficients, $x_t$is a $k \times 1$ vector of observation on k explanatory variables, the disturbance term u_t's are the independent random variable and follow $N(0,\tau^{-1})$ for$t = 1,2,\dots,n_1$ and $N(0,(\delta\tau)^{-1})$ for $t = n_1+1,\dots,n_1+n_2(=n)$. Here n_1+1 is the shift point which is assumed to be known. Obviously as a particular case if y_t defines the exchange rate which is to be explained on the basis of the set of explanatory variables x_t which are interest rate, growth rate of trade balance and growth rate of GDP, for $\delta = 1$ there is no structural shift in the exchange rate model and for $\delta < 1$ structural change occurs in both regression parameter β and disturbances precisionτ which shift to $\beta + (1-\delta)\gamma$ and $\delta\tau$ respectively after the break. After deciding the sampling model which can capture the structural breaks, our main focus are on the selection of appropriate prior distribution for all the parameters. Further our belief on the parameters would be sufficient to define the prior distributions for the problem. In real applications there would be many parameters and we have a wide range of selecting the appropriate prior distributions of all the parameters. Therefore for our calculation convenience, we need to choose those prior assumptions of the parameters which can perform well and leads to us tractable posteriors densities. The prior distributions of unknown parametersβ, γ,τ and δ are given by $\beta \sim N(\beta_0,\tau^{-1}V)$, $\gamma \sim N(\gamma_0,\tau^{-1}W)$, $p(\tau) \propto \frac{1}{\tau}$ and $\delta \sim u(0,1)$. Let ϵ be the probability that shift has occurred in regression parameter and disturbance precision and $1-\epsilon$ be the probability that shift does not occur.

$$\text{Then}\begin{cases}\epsilon = P(\delta < 1) & ;0 < \delta < 1 \\ 1-\epsilon = P(\delta = 1) & \end{cases} \qquad (2.2)$$

The likelihood function is given by

$$p(y/X,\beta,\tau,\gamma,\delta) = \epsilon\, p(y/\delta < 1,\beta,\tau,\gamma) + (1-\epsilon)p(y/\delta = 1,\beta,\tau) \qquad (2.3)$$

Where $p(y|\delta < 1,\beta,\tau,\gamma)$ denotes the likelihood function under restriction $\delta < 1$ and $p(y|\delta = 1,\beta,\tau,)$ denotes the likelihood function under restriction $\delta = 1$. Thus likelihood function becomes

$$p(y|X,\beta,\tau,\gamma,\delta)$$
$$= \epsilon.\left(\frac{\tau}{2\pi}\right)^{\frac{n}{2}}\delta^{\frac{n_2}{2}}.exp\left[-\frac{\tau}{2}\left\{\sum_{t=1}^{n_1}(y_t - x'_t\beta)^2 + \delta\sum_{t=n_1+1}^{n}(y_t - x_t'\beta - (1-\delta)x_t'\gamma)^2\right\}\right]$$
$$+(1-\epsilon)\left(\frac{\tau}{2\pi}\right)^{\frac{n}{2}}exp\left[-\frac{\tau}{2}\left\{\sum_{t=1}^{n}(y_t - x'_t\beta)^2\right\}\right] \qquad (2.4)$$

Further combining the likelihood function (2.4) with prior distributions of β,γ,δ and τ we get the joint posterior distribution of $(\beta,\gamma,\delta,\tau)$ as

$$p(\beta,\tau,\delta,\gamma|y)$$
$$\propto \epsilon\frac{\tau^{\frac{n+2k}{2}-1}\delta^{\frac{n_2}{2}}}{(2\pi)^{\frac{n+k}{2}}|V|^{\frac{1}{2}}|W|^{\frac{1}{2}}} \times$$
$$exp\left[-\frac{\tau}{2}\left\{\left(\gamma - \tilde{\gamma}(\delta)\right)'\Lambda(\delta)\left(\gamma - \tilde{\gamma}(\delta)\right) + \left(\beta - \beta_1(\delta)\right)'\Lambda^*(\delta)\left(\beta - \beta_1(\delta)\right) + \Psi_1(\delta)\right\}\right]$$

$$+(1-\epsilon)\frac{\tau^{\frac{n+k}{2}-1}}{(2\pi)^{\frac{n+k}{2}}|V|^{\frac{1}{2}}}exp\left[-\frac{\tau}{2}\left\{(\beta-\hat{\beta}^*)'\zeta(\beta-\hat{\beta}^*)+\Psi_2\right\}\right] \quad (2.5)$$

3. Posterior Distribution and Bayes Estimator

In order to develop the final posterior odds ratio test to capture the structural shift points in the above proposed exchange rate model, the following theorems are derived. For undestanding the detail derivation of the theorems, readers are referred to Broemeling (1985).

Theorem 3.1: The conditional posterior distribution of β given δ is given by

$$p(\beta|\delta)\propto\frac{\epsilon.\delta^{n_2/2}}{|W|^{\frac{1}{2}}|\Lambda(\delta)|^{1/2}}\left[\frac{1}{(\beta-\beta_1(\delta))'\Lambda^*(\delta)(\beta-\beta_1(\delta))+\Psi_1(\delta)}\right]^{\frac{n+k}{2}}$$

$$+(1-\epsilon)\left[\frac{1}{(\beta-\hat{\beta}^*)'\zeta(\beta-\hat{\beta}^*)+\Psi_2}\right]^{\frac{n+k}{2}} \quad (3.1)$$

Theorem 3.2: The posterior distribution of γ given δ is given by

$$p(\gamma|\delta)\propto\frac{\epsilon.\delta^{n_2/2}}{|V|^{\frac{1}{2}}|W|^{\frac{1}{2}}|\Gamma(\delta)|^{1/2}}\left[\frac{1}{(\gamma-\hat{\gamma}(\delta))'\Gamma_1(\delta)(\gamma-\hat{\gamma}(\delta))+\Phi_2(\delta)}\right]^{\frac{n+k}{2}} \quad (3.2)$$

Theorem 3.3: The posterior distribution of δ given β a mixture of discrete and continuous distribution and given by

$$p(\delta=1|\beta)=\frac{1}{N_r}\left[(1-\epsilon)\left\{\frac{1}{\sum_{t=1}^{n}(y_t-x_t'\beta)^2}\right\}^{\frac{n}{2}}\right] \quad (3.3)$$

and the pdf of δ given β in the range $0<\delta<1$

$$p(\delta|\beta)=\frac{1}{N_r}\left[\frac{\epsilon\delta^{n_2/2}}{|W|^{\frac{1}{2}}|\Omega(\delta)|^{\frac{1}{2}}}\left\{\frac{1}{(\beta-\beta_2(\delta))'\eta_1(\delta)(\beta-\beta_2(\delta))+\eta_3(\delta)}\right\}^{\frac{n}{2}}\right];0<\delta<1 \quad (3.4)$$

The normalizing constant is defined as

$$N_r=\int_0^1\left[\frac{\epsilon\delta^{n_2/2}}{|W|^{\frac{1}{2}}|\Omega(\delta)|^{\frac{1}{2}}}\left[\frac{1}{(\beta-\beta_2(\delta))'\eta_1(\delta)(\beta-\beta_2(\delta))+\eta_3(\delta)}\right]^{\frac{n}{2}}\right]d\delta$$

$$+(1-\epsilon)\left[\frac{1}{\sum_{t=1}^{n}(y_t-x_t'\beta)^2}\right]^{\frac{n}{2}} \quad (3.5)$$

Further we can easily obtain the posterior expectation of δ given β as

$$E(\delta|\beta)=\int_0^1\delta\,p(\delta<1|\beta)d\delta+1.p(\delta=1|\beta)$$

Utilizing the result (3.4), (3.5) and (3.6) we obtain posterior expectation of δ given β as

$$E(\delta|\beta)=\frac{1}{N_r}\left[\int_0^1\frac{\epsilon\delta^{n_2+2/2}}{|W|^{\frac{1}{2}}|\Omega(\delta)|^{\frac{1}{2}}}\left\{\frac{1}{(\beta-\beta_2(\delta))'\eta_1(\delta)(\beta-\beta_2(\delta))+\eta_3(\delta)}\right\}^{\frac{n}{2}}d\delta\right.$$

$$\left.+(1-\epsilon)\left\{\frac{1}{\sum_{t=1}^{n}(y_t-x_t'\beta)^2}\right\}^{\frac{n}{2}}\right] \quad (3.6)$$

The integral involved in (3.6) can be evaluated numerically as it is not possible to simplify the above expression.

4. Posterior odds Ratio for Testing Structural Change

The comparison of two competing models can be done formally in a Bayesian framework using posterior odds ratio, which is the product of the prior odds and the Bayes factor. The Bayes factor between two competing models is the ratio of the likelihoods integrated out with the corresponding priors and summarizes how the data favors one model over another. The simulated value of posterior odds ratio greater than one indicates that the data prefers model assumptions taken in the numerator then assumptions taken in the denominator part of the Bayes factor.

For testing the null hypothesis $H_0 : \delta = 1$ against the alternative $H_1 : \delta < 1$, the posterior odds ratio in favor of H_0 is defined as

$$\lambda = \left(\frac{1-\epsilon}{\epsilon}\right)\frac{\iint p(H_0)p(\beta)p(\tau)\, d\beta\, d\tau}{\iiint\int p(H_1)p(\beta)p(\gamma)p(\tau)p(\delta)d\beta\, d\tau\, d\gamma\, d\delta} \quad (4.1)$$

Where $p(H_0)$ is likelihood function under $H_0 : \delta = 1$, $p(H_1)$ is likelihood function under $H_1 : \delta < 1$ and $p(\beta), p(\gamma), p(\tau)$and $p(\delta)$ are the prior pdf of $\beta, \gamma, \tau,$ and δ respectively. After simplification of numerator and denominator part of the equation (3.1) and utilizing the result (A.5) and (A.6) from the appendix, we have posterior odds ratio given by

$$\lambda = \left(\frac{1-\epsilon}{\epsilon}\right)|\zeta|^{-\frac{1}{2}}[\Psi_2]^{-\frac{n}{2}}|W|^{\frac{1}{2}}\left[\int_0^1 \frac{\delta^{\frac{n_2}{2}}}{|\Lambda(\delta)|^{\frac{1}{2}}|\Lambda^*(\delta)|^{\frac{1}{2}}}\left[\frac{1}{\Psi_1(\delta)}\right]^{\frac{n}{2}} d\delta\right]^{-1} \quad (4.2)$$

When the posterior odds ratio $\lambda < 1$ we reject H_0 and accept if $\lambda \geq 1$.

5. Monitoring Change Points

Monitoring change points plays an important role to capture the break dates in the model. For the sequential detection of the first change point we need to use the larger number of observation than the parameters which we want to estimate on the basis of information available in data form. Bayesian monitoring procedure can be used which is not only more informative but also capable of deciding the true change point among all the temporary change points. In this paper we use the retrospective (sequential) procedure to the available data at every time instant. In this procedure we calculate the Bayes factor and consequently posterior odds ratio. The likelihoods under null hypothesis and alternative hypothesis has been calculated for all the data set taken sequentially and the sample size considered here must be greater than the parameters to be estimated. The monitoring procedure, when applied to real data, provides satisfactory solution to the problem of detecting the first change point because it is able to discriminate between isolated change points and the first true permanent change points.

The monitoring procedure is as follows.

$\lambda_1 = P(M_0|y_1)/\ P(M_1|y_1)$

$\lambda_2 = P(M_0|y_1, y_2)/P(M_1|y_1, y_2)$

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$\lambda_n = P(M_0|y_1, y_2 \ldots y_n)/P(M_{n-1}|y_1, y_2 \ldots y_n)$

Here M_0 and M_n represent the model having no breaks and breaks respectively.

6. Source of Data and Empirical Findings

The time series data set on four variables viz, interest rate, trade balance, Indian rupee -US dollar exchange rate and gross domestic product (GDP) at market price have been sourced from the “RBI-Handbook of Statistics on the Indian Economy 2010-11”, published by Reserve Bank of India. The Interest rate which has been considered here is the medium-term, 5-15 year Annual (Gross) Redemption Yield of Government of India Securities. The growth rate of trade balance has been taken, where trade balance is defined as export-import and amount of export and import are in terms of ‘ten millions’. Finally, the GDP (market price) at current price has been utilized to get the growth rate of GDP. Interest rate, growth rate of trade balance and growth rate of GDP are considered as explanatory variables and the exchange rate is a variable which is to be explained on the basis of explanatory variables.

Here we have assigned $t = 1, 2, \ldots, 21$ representing the year 1989-90 to 2009-10. Order of y_t is 1×1 and it explains the value of exchange rate for the year t, x_t is of order 3×1 which contains interest rate, growth rate of trade balance and GDP growth rate as elements. The coefficients vector β is a 3×1 vector. Fixing the value n_1 and $\epsilon = 0.1, 0.2, \ldots, 0.99$ thevalues of posterior odds ratio λ have been numerically evaluated sequentially using MATLAB application software. Instead of single break point, the above proposed model captures the multiple breaks in the model which are at $n_1 = 4,\ 8, 9, 10, 13,$ and 18, where $n_1 + 1$ defines the structural break dates. The model remains unchanged structurally for other time points. Table 1shows the posterior odds ratio λ for different assigned value of $n_1 + 1$ (Break dates) and probabilityϵ. From Table 1**,** it may be observed that asϵ increases, λ decreases exponentially for all value of $n_1 + 1$. Therefore, posterior odds ratio is non-increasing function of probability ϵ. Further figures 1(a), 1(b) and 1(c) are the graphical presentation of the output of posterior odds ratio of Table 2. Figure 2 depicts the movement in rupee-dollar exchange rate, Interest rate (medium-term, 5-15 year-Annual (Gross) Redemption Yield of Government of India), growth rate of trade balance and GDP growth rate at market price during the years under consideration.

Shift point $(n_1 + 1)$→ Probability (ε) ↓	**1993-94**	**1997-98**	**1998-99**	**1999-00**	**2002-03**	**2007-08**
0.1	3.425600	0.116012	0.059537	0.000001172694	0.024424076542	0.000000017000
0.2	1.522500	0.051561	0.026461	0.000000521197	0.010855145130	0.000000001107
0.3	0.888100	0.030077	0.015436	0.000000304032	0.006332167992	0.000000000492
0.4	0.570900	0.019335	0.009923	0.000000195449	0.004070679424	0.000000000287
0.5	0.380600	0.012890	0.006615	0.000000130299	0.002713786282	0.000000000185
0.6	0.253700	0.008593	0.004410	0.000000086866	0.001809190855	0.000000000123
0.7	0.163100	0.005524	0.002835	0.000000055843	0.001163051264	0.000000000082
0.8	0.095200	0.003223	0.001654	0.000000032575	0.000678446571	0.000000000053
0.9	0.042300	0.001432	0.000735	0.000000014478	0.000301531809	0.000000000031
0.99	0.003800	0.000130	0.000067	0.000000001316	0.000027411983	0.000000000014

Table 1: Test for Structural Changes

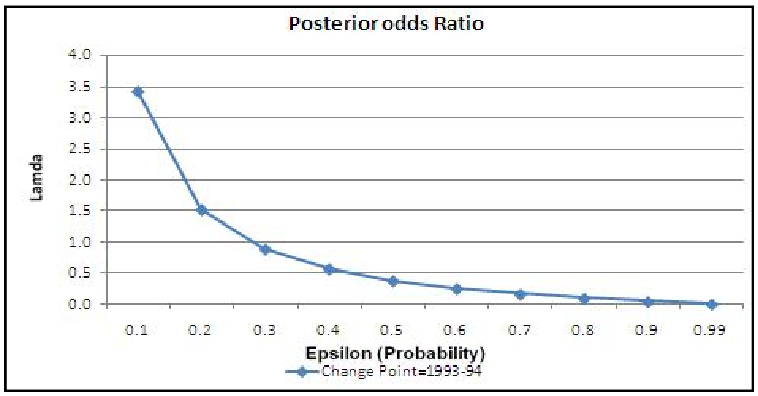

Figure 1(a)

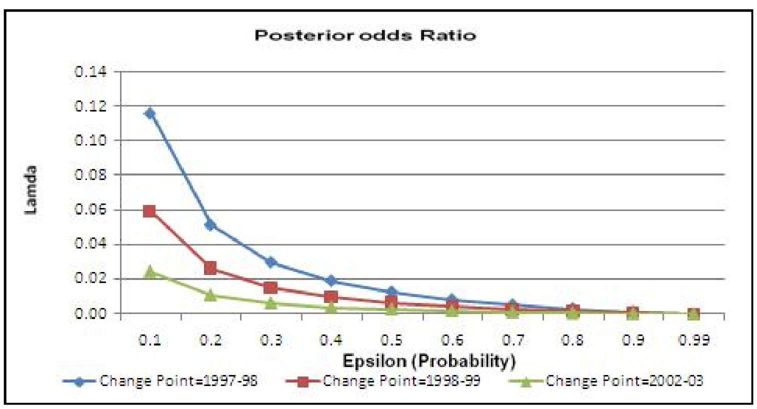

Figure 1(b)

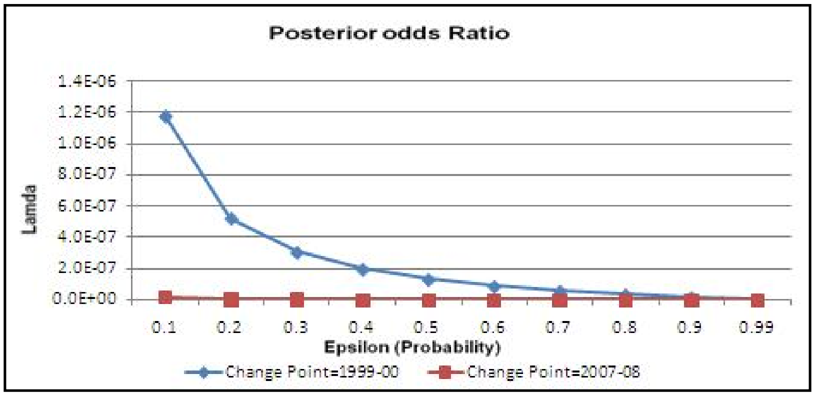

Figure 1(c)

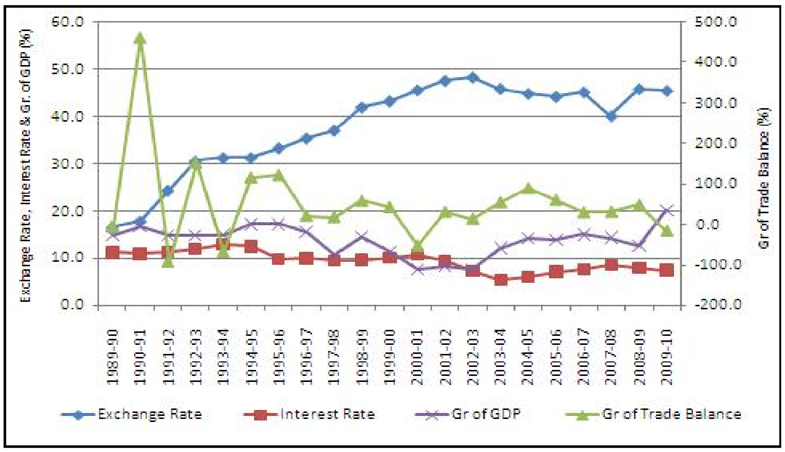

Figure 2: Movement in Exchange Rate, Interest Rate, Growth Rate of Trade Balance and GDP Growth Rate
Source of data: *RBI*-Handbook of Statistics *on the Indian Economy 2010-11*

7. Concluding Remarks

The main focus of the paper has been on establishing a test procedure for capturing the structural break points and its application to rupee-dollar exchange rates in India during 1989-90 to 2009-10. Further, as a particular case to test the validity of proposed model, the effect of interest rate, growth rate of trade balance and GDP growth rate (market price) on Indian rupee-US dollar exchange rate has been considered.

The posterior odds ratio test clearly indicates that there exists mainly six structural shift points in the model. These are 1993-94, 1997-98, 1998-99, 1999-00, 2002-03, 2007-08. There is strong evidence of structural shift in the years2007-08, 1999-00, 2002-03, 1998-99 and 1997-98, a weaker evidence of structural shift in the year 1993-94. The strongest structural shift is observed in the year 2007-08 because it rejects the null hypothesis (there is no structural change) more strongly against the alternative (change exists due to shift in regression parameter and disturbances precision) as compared to the year 1999-00. The post reform period 1993-94 observed considerable improvement in the FOREX market turnover with monthly turnover increased from about US $ 15 billion in 1987-88 to US $ 50 billion in 1993-94 and in the post-1993 period, the foreign institutional investors (FIIs) have also emerged as a major player in the foreign exchange market. The stability in the foreign exchange market was disrupted due to intensification of East Asian crises and uncertainties in the domestic development in the period 1997-98 to 1998-99. During the period 1999-00, there was a sharp increase in the imports bills. This sharp increase in the import bill was mainly attributed to increase in the oil bill payment compared to previous year. There was also sharp increase (5.5 billion US-dollars) in the foreign exchange reserve in year 1999-00 compare to 3.1 billion US-dollars in the previous year. The considerable changes in the composition of our trade basket with the dominance of services rising post 2000 constituted break in exchange rate model during the period 2002-03. The model also detects the effect of global financial crises on the exchange rate as a structural breaks in 2007-08 which is commonly believed to have begun in July 2007 with the credit crunch, when a loss of confidence by US investors in the value of sub-prime mortgages caused a liquidity crisis.

As a future agenda, it will be interesting to explore the break dates at a unknown change point, forecasting exchange rate in Bayesian frame work when the break dates have been established, and study the above proposed idea on the other economic variables, like inflation, interest rate etc.

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