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A New Virial Equilibrium for Galaxy Clusters Forming Finite Alignments

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Abstract:
Assuming only ellipticity it is argued that galaxy clusters can only be prolate, instead of spherical or oblate. With the help of this constraint, a lower limit for the cluster halo dark matter particle mass is derived depending on baryon particle mass. This takes the form of new 'scaling relations' between ellipticity, dark matter fraction, dark matter particle mass and the Hubble constant h and the cosmological parameter k. The usual intra cluster virial equilibrium and the virial mass fraction constant are challenged when galaxy clusters form finite alignments.

Keywords: *Cosmology, Galaxy Clusters, Ellipticity, Alignments*

1. Introduction: Why Ellipticity for Galaxy Clusters?

In the last thirty years several studies have discussed the problem of galaxy cluster shapes. Mostly one starts from a spherical model elaborating towards ellipticity, and finds that galaxy clusters can be described using prolate or oblate shapes (De Theije ea, 1995). Ellipticity is found to increase with redshift (Hopkins, 2005). Also, theories with rotating elliptical cluster shapes seem to fit better to data of X-ray observations of intercluster hot baryon gas leading to new and more accurate estimates of cluster masses. These in turn now correspond better to CBM microwave Sunyaev-Zel'Dovich mass estimates and gravitational lensing mass estimates, a matching problem that had been waiting for a solution for a long time (Wu and Fang, 1996, Girardi ea, 1997, Sadat, 1997, AMI Consortium, 2012).

In this study, I similarly start from a spherical cluster shape. Standard properties of spherical galaxy clusters are described in paragraph 2. Assumed is virial equilibrium between a spherically symmetric gravitational potential energy and thermal pressure as usual (Sadat, 1997). I introduce a geometric mass to radius relation adapted to elliptic shape, in paragraph 3. Using ellipticity instead of spherical symmetry when calculating the gravitational potential energy is possible at this point, however I will do this later on, in paragraph 4., when refining the results to include a constraint for cluster shape. In this way one finds results singularly due to ellipticity. Applying the constraint, I subsequently derive a criterium for the cluster dark matter particle mass, in paragraph 5. This last result includes 'scaling relations' between ellipticity, dark matter particle mass and fraction and the Hubble constant h and the cosmological parameter k. In section 6. finite alignments of galaxy clusters are discussed, challenging the usual three-dimensional virial equilibrium and the dark matter fraction constant: instead of intra cluster equilibrium proposed is inter cluster gravitation equilibrium, and baryon pressure balance between all neighboring clusters, in the alignment direction.

2. Properties of Galaxy Clusters Assuming Spherical Shape

Galaxy clusters are understood to exist of baryon gas, dark matter and galaxies, the last representing only a small percentage of the total cluster mass. For the baryon gas (the ICM intra cluster medium gas) mostly is used a power law temperature profile, sometimes extrapolated towards a flat temperature profile, i.e. with a constant cluster temperature independent of the distance R to the cluster centre. For the baryon density n, one often assumes a beta model profile: $n(R) = n_0 \,(1+ (R/R_c)^2)^{-3\beta/2}$ where R_c is the cluster core radius, n_0 a normalization constant equal to the central density and β a parameter to be decided by observation. The beta model was introduced in 1976 by Cavaliere and Fusco-Femiane. Cluster density profiles like these are described in detail in (Arnaud, 2009, Popesso, 2006) among others. The beta model seems to be a good fit observationally, for β = 2/3. For this value of β a simplified power law profile is acceptable outside the core: $n= n_0 \,(R/R_c)^{-2}$. Similar profiles, both for temperature and density, are applied in computer simulations of cluster distributions (De Theije ea, 1995, Zamboni, 2010).

The virial equilibrium, 2K + U = 0, for total kinematic energy K and total potential energy U, depends on observed velocity dispersion to estimate M_{vir}. The definition of the cluster matter halo (the matter within virial radius distance to the cluster center) follows from the related dynamic balance

$dP(R)/dR = -n(R)\, GM(R)/R^2$ at R_{vir} for a spherical cluster with mass M_{vir}. G is the gravitation constant, P(R) the baryon pressure and M(R) the total mass including dark matter and ICM baryons, where M(R) depends on R following spherical geometry. Outward ICM baryon pressure variation and inward gravitational pull cancel at the sphere surface with virial radius. Assuming the baryons to behave like an ideal gas, $P(R) = n\, k_bT/\mu m_p$ links P(R) to density n and temperature k_bT with μm_p the baryon particle mass, which can be measured observationally (Sadat, 1997 and references therein).

So-called scaling relations between characteristic properties follow from assuming cluster self-similarity, proposed by Kaiser in 1986 (Mathiesen, 2000, Popesso, 2006, Boulderstone, 2013, and references therein). Self-similarity implies that relations between properties do not depend on scale. They do depend on the cosmological model used. Mostly, scaling relations link observational quantities to mass. Used is the scaling relation between temperature and virial mass: k_bT scales with $M_{vir}^{2/3}$, which directly derives from expressing the virial mass in terms of the virial radius in an Einstein De Sitter cosmology.

3. Virial Mass Radius Relation for Elliptic Clusters

To describe an elliptic cluster shape, one can write an ellipse with the expression: $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, following (De Theije ea, 1995), while there are two independent directions x and z, with direction y similar to direction x, and a = b and c constants. When assuming a spherical geometry, the cluster virial mass radius relation includes the average baryon density <n> assumed equal to n(Rvir), the baryon density fraction f_b, and μm_p the baryon particle mass (μ is a certain constant near 0.6 and m_p the proton mass). In this widely used relation (Borgani and Kravtsov, 2011, Boulderstone, 2013, Weissman, 2013) one has only to add on the right-hand side the factor (c/a), to straightforwardly derive an elliptical geometry virial mass radius relation:

1) $M_{vir} = \mu m_p\, n(R_{vir})/f_b\, 4/3\pi\, R_{vir}^3\, (c/a)$

I will apply this relation to understand galaxy cluster ellipticity in terms of prolate and oblate, assuming intra cluster virial equilibrium, in paragraph 4.

Extensive discussion exists of real cluster shape as prolate and oblate ellipses. Some authors aim at finding observational proof of ellipticity for specific galaxy clusters (Plionis and Basilakos, 2004, Rembold and Pastoriza, 2012). Other authors use computer simulations for distributions of clusters, where one tries to find and adjust parameters to evaluate distributions to fit observations. Of the very diverse studies with computer simulations that favor prolate cluster ellipticity, I mention the following. When selecting distributions that result from groups of evenly balanced prolate and oblate clusters, Robotham ea study raw data and data corrected for cluster finity, of which the raw data are expressive of prolate abundance (Robotham ea, 2008). Hopkins (Hopkins ea, 2005) finds that cluster ellipticity and alignment increase with redshift. This suggests that prolate clusters, that are easier to form alignments than oblate clusters, at higher redshift are most abundant. Plionis (Plionis ea, 2013) finds that for poor clusters, that are more often present at high redshift, there is a preference for prolate shape. Sereno finds good fits to X-ray and SZ data for simulations with preferably prolate cluster distributions (Sereno ea, 2006).

In the next paragraph, I will demonstrate how one can deduce from the virial equilibrium a constraint for the ellipticity parameters a, b and c. It turns out that only prolate clusters fulfill virial equilibrium, in agreement with the above studies. To show this, first triaxiality is introduced in the standard way and then this is restricted to two independent directions.

4. From Triaxiality to Cluster Shape Constraint in Two Steps

4.1. Triaxiality Values and Oblates

It has become standard to use a triaxiality parameter $T = c^2\text{-}b^2/c^2\text{-}a^2$, defined by Franx ea, in 1991, to describe ellipticity with three independent directions x, y and z (De Theije ea, 1995, De Theije ea, 1998, and references therein). T and the parameter $\xi_1 = c/a$ can approximately be related as $T \approx 1 - a/c = 1 - 1/\xi_1$ when $c > b > a$ with $b^2 \approx ac$. These relations will be assumed true more generally, also for ellipticity in two directions. Distributions of mostly prolate or mostly oblate galaxy clusters are related to average T values 1 or 0, as shown by De Theije ea. It turns out in their study that the overall universe seems to prefer prolate groups of clusters (T = 1) in simulation calculations. This study was later refined by Robotham, who could only partly confirm the results. Nevertheless, it seems that, taking all evidence together, also Robotham ea find that at high redshift prolate clusters are most abundant. Also, the other authors mentioned above, whether studying observations or computer simulations, confirm this.

Using the above approximate relation between T values and cluster shape in terms of $\xi_1 = c/a$, it follows that only prolates and spheres are possible. In a next step, spheres are prohibited by assuming an elliptical gravitational potential leaving only prolates supported.

One writes an ellipse as before using a, b and c. For ellipticity in two directions there is b/a = 1 and $c/a = (A\, R_{vir} / R_{core} + B) = \xi_1$. A and B describe the large distance (A) and core (B) character of the shape to R relation (Zamboni, 2010). A positive value for A means that at large radius distance from the core the cluster shape is prolate, a negative value means similarly that this shape is oblate.

De Theije ea find that for our universe approximately T = 1, and thus from the definition $T = 1 - 1/\xi_1$ one concludes that $\xi_1 = c/a$ is (much) larger than 1. This suggests a non-spherical, that is, prolate cluster shape.

When, in rare instances, De Theije ea find oblate universes, with T = 0, then $\xi_1 = c/a$ equals 1. However, c/a = 1 geometrically means a spherical shape. Oblate shape cannot be reached within the T= 0 instances. Thus, a first inference is, just by analyzing the De Theije results, that oblate shapes are not supported by this description and only prolate or spherical shapes are expected.

4.2. Elliptical Gravitational Potential and Spheres

For $A \neq 0$ it is not possible that the cluster shape is spherical. Noticing that again $\xi_1 = (A\, R_{vir} /R_{core} + B)$, is it possible to find, with a zero value for A, a value for B for which the elliptic shape becomes straightforwardly spherical? This value for B should be B = 1 (the possibility A = 1 and B = 0 and $R_{vir} = R_{core}$ is exactly similar). Keeping A = 0 and leaving B undetermined, one now inserts in the virial equilibrium equation an elliptical gravitational potential. Using the shorthand definition $\xi_2 = (1 - 6/5\, a^2\, \xi_1/ R_{vir}^2)$, it follows:

2) $M_{vir} = (N (1+z) R_{vir} (a+b) / \mu m_p G \xi_2)^3$
Used are the temperature to virial mass scaling relation $k_bT = N (1+z) M_{vir}^{2/3}$ with N a normalization constant and z the redshift, and the elliptical virial equilibrium, derived as $M_{vir} = k_bT R_{vir} (a+b)/\mu m_p G \xi_2$. Combining 2) with the geometry relation 1) for M_{vir} one finds:

3) $\xi_1 = (N (1+z) (a+b) / \mu m_p G \xi_2)^3 (f_b / n) \mu m_p^{-1}$
Writing out ξ_1 in terms of A and B, the result is, leaving out cancelling terms:

4) $A = R_{core}/R_{vir} (\xi_2^{-3} - B)$
This means that $B = \xi_2^{-3}$ for A=0. In ξ_2 one can insert ξ_1= c/a. For a sphere, for which holds a = b = c = R_{vir} the result 4) reduces to:

5) $B \approx -125 \neq 1$
When B is not equal to 1, and indeed now B < 1 meaning oblate cluster shapes, the assumption A = 0, B = 1, meaning spherical shapes, is ruled out. Thus, combining with the earlier inference, the result is that oblate and spherical shapes are not supported, and only prolate shapes are expected in our universe.

5. From Prolate Cluster Shape to Dark Matter Particle Mass
Starting from the above results, assuming only clusters with prolate shape with c/a larger than 1, one can derive straightforwardly a lower limit on the ratio dark matter mass to baryon mass that is independent of other cosmological and matter parameters.
Used are the following three relations of which the first two are given as a priori and the last one is a result from paragraph 4. The first relation is similar to a distributive property for density fractions and the second one is understood to result from cosmological considerations that make the constant universal (Pen, 1997). This will be discussed further in paragraph 6. Used are abbreviations for properties including densities n, density fractions f, particle masses m and virial masses M, each in combination with matter type including baryon matter b, dark matter dm and all matter m. All matter is supposed to include only baryon matter or dark matter. Following tradition, the virial mass of all matter is written M_{vir} and $m_b = \mu m_p$.

6) $n_b = f_b n_m = f_b (n_b + n_{dm}) = f_b n_b + f_b n_{dm}$
7) f_b / f_{dm} = constant
8) $c/a = M_{vir} f_b / (1+z)^3 R_{vir}^3 \mu m_p <n_b>$
Now from $M_{vir} = M_b + M_{dm}$ and $<n>_{vir} = n_{vir}$ it follows from 6) and 8) that:

9) $c/a = f_b (1 + m_{dm} f_{dm} n_m / \mu m_p f_b n_m) = f_b + f_{dm} m_{dm} / \mu m_p =$
$= 1/n_m (n_b + n_{dm} + n_{dm} (-1 + m_{dm}/\mu m_p)) = 1 + (n_{dm} / n_m) (m_{dm} / \mu m_p – 1)$
$= 1 + \Omega_{dm}/\Omega_m (m_{dm} / \mu m_p – 1)$, with $\Omega_i = n_i / n_c$ (critical density)

One expects c/a > 1, and this constraint leads to the following criterium:

10) $m_{dm} /\mu m_p > 1$
At first sight, this criterium does not seem very spectacular. However, it rules out several candidates for the presently still un-identified dark matter particles and favors so called cold dark matter, including WIMP's, weakly interacting massive particles, such as Higgs particles.

5.1. Scaling relations with Hubble constant h, cosmological parameter k and dark matter fraction
The fractions f_b and f_{dm} can be rewritten depending on cosmological expansion factors, with h the Hubble constant and the Ω's the densities divided by the critical density (Pen, 1997): $f_b = h^{3/2} \Omega_b / \Omega_m$ and a similar expression for $f_{dm} = h^{3/2} \Omega_{dm} / \Omega_m$. There is $\Omega_m = \Omega_b + \Omega_{dm}$ because of 6). In the following, fraction f_{dm} and constants h and k are considered to be undetermined, in alteration. In this way scaling relations between c/a, f_{dm}, h, k, and m_{dm} are derived. Scaling relations cannot easily be inverted, therefore the notation * is used.
While assuming a determined Hubble constant h while f_{dm} undetermined, m_{dm} will turn out accordingly due to equation 9), however still fulfilling criterium 10). In this way f_{dm} and m_{dm} scale similarly:

11) c/a scales with $f_{dm} * m_{dm}$

When f_{dm} is assumed determined, and we allow a higher value for a yet undetermined Hubble constant h, also m_{dm} would turn out to be higher, again due to equation 9). The other way around, a lower estimate for m_{dm} will predict a lower estimate for the Hubble constant h: $h^{3/2}$ scales inversely compared to m_{dm}:

12) c/a scales with $h^{-3/2} * m_{dm}$
A less expressive prolate shape then correlates with a higher Hubble constant h. This agrees with relative density Ω_{dm}^{-1} and $h^{3/2}$ as expansion parameters, assuming f_{dm} constant. In a highly expanding universe with low Ω and high h expectation, clusters will still be

prolate, however with lower values for c/a, like experiencing higher shear. While 1/1+z, z redshift, is a scaling parameter, fluctuations and c/a values, in a small-scale universe will be relatively larger than fluctuations and c/a values in a large-scale universe, applying equation 8).
When the Hubble constant h is undetermined this involves also the cosmological parameter k, the integration constant in Friedman's equation: c^2 k = R^2 h^2 - 8π/3 R^2 G n_c with n_c the critical density and R a measure for the universe radius. In this way scaling relation 12) can directly be translated from h to k. Assuming the gravitational constant G and the velocity of light *c* unchanged, one finds: while the critical density is understood to depend only on a certain value $(c/a)_0$, then with deviating c/a, k scales with R^2 h^2, which cancels to $(dR/dt)^2$ for a Hubble constant h equal to (dR/dt / R) and 12) rewrites to:

13) c/a scales with $R^{3/2} * k^{-3/4} * m_{dm}$

It is possible to resume all scaling factors in a diagram.

fixed		mdm	fdm	h	k	R
h	c/a	1	1			
fdm	c/a	1		-3/2		
fdm	c/a	1			-3/4	3/2

Table 1

6. Discussion: A New Virial Equilibrium for Clusters Forming Finite Alignments

Imagine a new fluctuation of the background density emerging as a prolate cluster, between existing similar prolate clusters on a finite rod-like alignment in the z direction. As Hopkins finds (Hopkins ea, 2005), ellipticity and alignments correlate. The gravitational pull at the virial surface of the new cluster cancels out, due to the gravitational presence of the already existing clusters on the alignment. The fluctuation will relax towards a certain virial surface baryon pressure value similar to that of its neighbors that on both sides balance it. Every cluster moves on a little, to make place for the new cluster till no pressure imbalance is left between them. One expects the baryon pressures, after some confusion, to stabilize just above the original virial surface pressures since now one more cluster is included in the, assumed finite, rod universe.
This pressure excess will cause all clusters on the alignment to interact with their neighbors at least in the z direction while in the x and y directions the intra cluster equilibrium still holds. If intra cluster gravitational pull and surface pressure balance each other in ordinary 3D virial equilibrium no heating or expansion would occur and clusters would remain non-interacting.
In this description, when finite filaments of clusters are real in the z direction, the virial equilibrium holds when accounting for all the energies of all interacting clusters. Because of pressure increase at the connections, velocity dispersion measures, in this z direction independent of gravitational pull, will tend to increase. The dark matter virial mass due to an average velocity dispersion measurement will tend to be overestimated likewise. In conclusion, the 3D intra cluster virial equilibrium does not hold, at least for prolate clusters forming finite alignments, and should be replaced by an inter cluster equilibrium. The difference is in the z direction treatment, leaving the two other directions unaltered. As a consequence of a lower real dark matter virial mass estimate, the constant of the baryon to dark matter density fraction, in equation 7), is challenged too and expected to be too low.
Even when at least a partial inter cluster virial equilibrium is valid, the above criterium 10) for dark matter particle mass is assumed to remain valid, although it is reached starting from intra cluster equilibrium. It can easily be checked that this criterium is independent of the dimension of the equilibrium space. Also, while the constant in equation 7) for f_g / f_{dm} is questioned, the derived results still seem true, since their derivation itself implies an undetermined f_{dm}.

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