THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Relative Efficiency Criteria for the Six Specific Calculus Optimum Values Second Order Rotatable Designs with a Practical Example on Twenty-Four Points

F.K. Koech
Part Time Lecturer, Department of Statistics and Computer Science,
School of Biological and Physical Sciences, Moi University, Eldoret, Kenya

Abstract:
In this study we focus on the existing six specific calculus optimum second order rotatable designs in three dimensions denoted by $M_1, M_2, M_3, M_4, M_5,$ and M_6 as illustrated in the text form. A design matrix X is developed from the designs and the information matrices $C_1, C_2, C_3, C_4, C_5,$ and C_6 are obtained and hence the relative efficienciesA-, D-, E-, T and I/IV- for the six specific Calculus optimum values designs are evaluated. From the results it is evident that the determinant criterion (D_{eff}) for the six designs has the highest relative efficiency average. M_1 is found to be the most efficient design as compared to the rest.

Keywords: *Response Surface Methodology, Design of experiments, A-Efficiency, D-Efficiency, E-Efficiency, T-Efficiency, Ieff/IVeff I-/ IV- Efficiency & Transpose of a matrix X*

1. Introduction

Box and Hunter (1957) gave the moment and non-singularity conditions for the second order rotatable designs and Draper (1960) developed six second order rotatable designs. Mutiso (1998) specified these six second order rotatable designs in his P.h.d thesis and in (2008) Mutiso gave a practical example of the twenty-four points second order rotatable designs in three dimensions. Kosgei (2002) determined their optimality criteria and Kosgei (2006) Published his paper on optimality criteria Cheruiyot (2015) gave the analysis of efficiencies and optimality criteria in his M.S.C thesis.

2. Specific Designs

2.1. The Twenty-Four Points Three-Dimensional Specific Rotatable Design of Order Two
We consider the design;
M_1=(S(1.1072569,1.1072569,0) +S(0.7829487,0,0)+S(1.2735263,0,0)
The design matrix X for M_1 is given as;

1.0000	1.1100	1.1100	0	1.2300	1.2300	0
1.0000	-1.1100	1.1100	0	1.2300	1.2300	0
1.0000	1.1100	-1.1100	0	1.2300	1.2300	0
1.0000	-1.1100	-1.1100	0	1.2300	1.2300	0
1.0000	0.7800	0	0	0.6100	0	0
1.0000	-0.7800	0	0	0.6100	0	0
1.0000	1.2700	0	0	1.6100	0	0
1.0000	-1.2700	0	0	1.6100	0	0
1.0000	1.1100	0	1.1100	1.2300	0	1.2300
1.0000	-1.1100	0	1.1100	1.2300	0	1.2300
1.0000	1.1100	0	-1.1100	1.2300	0	1.2300
1.0000	-1.1100	0	-1.1100	1.2300	0	1.2300
1.0000	0	0	0.7800	0	0	0.6100
1.0000	0	0	-0.7800	0	0	0.6100
1.0000	0	0	1.2700	0	0	1.6100
1.0000	0	0	-1.2700	0	0	1.6100
1.0000	0	1.1100	1.1100	0	1.2300	1.2300
1.0000	0	-1.1100	1.1100	0	1.2300	1.2300
1.0000	0	1.1100	-1.1100	0	1.2300	1.2300
1.0000	0	-1.1100	-1.1100	0	1.2300	1.2300
1.0000	0	0.7800	0	0	0.6100	0
1.0000	0	-0.7800	0	0	0.6100	0
1.0000	0	1.2700	0	0	1.6100	0
1.0000	0	-1.2700	0	0	1.6100	0

And the information matrix c_1 for the design M_1 is given by

c_1 =

1.0000	0.5949	0.5949	0.5949	0	0	0
0.5949	0.7516	0.2505	0.2505	0	0	0
0.5949	0.2505	0.7516	0.2505	0	0	0
0.5949	0.2505	0.2505	0.7516	0	0	0
0	0	0	0	0.5949	0	0
0	0	0	0	0	0.5949	0
0	0	0	0	0	0	0.5949

2.2. The Thirty-Two Points Three-Dimensional Specific Rotatable Design of Order Two

We consider the design;

M_2 = (S(1.3338955, 0.5360318) + S(0.2982681, 0.2982681, 0.2982681))

The design matrix X is given by

1.0000	1.3300	0.5400	0.5400	1.7700	0.2900	0.2900
1.0000	-1.3300	0.5400	0.5400	1.7700	0.2900	0.2900
1.0000	1.3300	-0.5400	0.5400	1.7700	0.2900	0.2900
1.0000	1.3300	0.5400	-0.5400	1.7700	0.2900	0.2900
1.0000	-1.3300	-0.5400	0.5400	1.7700	0.2900	0.2900
1.0000	-1.3300	0.5400	-0.5400	1.7700	0.2900	0.2900
1.0000	1.3300	-0.5400	-0.5400	1.7700	0.2900	0.2900
1.0000	-1.3300	-0.5400	-0.5400	1.7700	0.2900	0.2900
1.0000	0.5400	0.5400	1.3300	0.2900	0.2900	1.7700
1.0000	-0.5400	0.5400	1.3300	0.2900	0.2900	1.7700
1.0000	0.5400	-0.5400	1.3300	0.2900	0.2900	1.7700
1.0000	0.5400	0.5400	-1.3300	0.2900	0.2900	1.7700
1.0000	-0.5400	-0.5400	1.3300	0.2900	0.2900	1.7700
1.0000	-0.5400	0.5400	-1.3300	0.2900	0.2900	1.7700
1.0000	0.5400	-0.5400	-1.3300	0.2900	0.2900	1.7700
1.0000	-0.5400	-0.5400	-1.3300	0.2900	0.2900	1.7700
1.0000	0.5400	1.3300	0.5400	0.2900	1.7700	0.2900
1.0000	-0.5400	1.3300	0.5400	0.2900	1.7700	0.2900
1.0000	0.5400	-1.3300	0.5400	0.2900	1.7700	0.2900
1.0000	0.5400	1.3300	-0.5400	0.2900	1.7700	0.2900
1.0000	-0.5400	-1.3300	0.5400	0.2900	1.7700	0.2900
1.0000	-0.5400	1.3300	-0.5400	0.2900	1.7700	0.2900
1.0000	0.5400	-1.3300	-0.5400	0.2900	1.7700	0.2900
1.0000	-0.5400	-1.3300	-0.5400	0.2900	1.7700	0.2900
1.0000	0.3000	0.3000	0.3000	0.0900	0.0900	0.0900
1.0000	-0.3000	0.3000	0.3000	0.0900	0.0900	0.0900
1.0000	0.3000	-0.3000	0.3000	0.0900	0.0900	0.0900
1.0000	0.3000	0.3000	-0.3000	0.0900	0.0900	0.0900
1.0000	-0.3000	-0.3000	0.3000	0.0900	0.0900	0.0900
1.0000	-0.3000	0.3000	-0.3000	0.0900	0.0900	0.0900
1.0000	0.3000	-0.3000	-0.3000	0.0900	0.0900	0.0900
1.0000	-0.3000	-0.3000	-0.3000	0.0900	0.0900	0.0900

The information matrix c_2 for the design M_2 is given by

$$c_2 = \begin{bmatrix} 1.0000 & 0.6107 & 0.6107 & 0.6107 & 0 & 0 & 0 \\ 0.6107 & 0.8347 & 0.2782 & 0.2782 & 0 & 0 & 0 \\ 0.6107 & 0.2782 & 0.8347 & 0.2782 & 0 & 0 & 0 \\ 0.6107 & 0.2782 & 0.2782 & 0.8347 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6107 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6107 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6107 \end{bmatrix}$$

2.3. The Twenty-Two Points Three-Dimensional Specific Rotatable Design of Order Two

We consider the design;

M_3 = (S(0.4899784, 0.4899784, 0.4899784) + S(0.9023011, 0.9023011, .9023011)+ S(1.5494481, 0, 0))

The design matrix X of M_3 is given by

$$\begin{bmatrix} 1.0000 & 0.4900 & 0.4900 & 0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & -0.4900 & 0.4900 & 0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & 0.4900 & -0.4900 & 0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & 0.4900 & 0.4900 & -0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & -0.4900 & -0.4900 & 0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & -0.4900 & 0.4900 & -0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & 0.4900 & -0.4900 & -0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & -0.4900 & -0.4900 & -0.4900 & 0.2400 & 0.2400 & 0.2400 \\ 1.0000 & 0.9000 & 0.9000 & 0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & -0.9000 & 0.9000 & 0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & 0.9000 & -0.9000 & 0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & 0.9000 & 0.9000 & -0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & -0.9000 & -0.9000 & 0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & -0.9000 & 0.9000 & -0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & 0.9000 & -0.9000 & -0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & -0.9000 & -0.9000 & -0.9000 & 0.8100 & 0.8100 & 0.8100 \\ 1.0000 & 1.5500 & 0 & 0 & 2.4000 & 0 & 0 \\ 1.0000 & -1.5500 & 0 & 0 & 2.4000 & 0 & 0 \\ 1.0000 & 0 & 1.5500 & 0 & 0 & 2.4000 & 0 \\ 1.0000 & 0 & -1.5500 & 0 & 0 & 2.4000 & 0 \\ 1.0000 & 0 & 0 & 1.5500 & 0 & 0 & 2.4000 \\ 1.0000 & 0 & 0 & -1.5500 & 0 & 0 & 2.4000 \end{bmatrix}$$

The information matrix c_3 for the design M_3 is given by

$$c_3 = \begin{bmatrix} 1.0000 & 0.6016 & 0.6016 & 0.6016 & 0 & 0 & 0 \\ 0.6016 & 0.7860 & 0.2620 & 0.2620 & 0 & 0 & 0 \\ 0.6016 & 0.2620 & 0.7860 & 0.2620 & 0 & 0 & 0 \\ 0.6016 & 0.2620 & 0.2620 & 0.7860 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6016 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6016 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6016 \end{bmatrix}$$

2.4. The Twenty Points Three-Dimensional Specific Rotatable Design of Order Two

We consider the design;

M_4=(s(0.6277576, .6277576, 0.6277576)+s(0.5614834, 0,0)+s(1.0339784,0,0)

The design matrix X of M_4 is given by

1.0000	0.5600	0	0	0.3100	0	0
1.0000	-0.5600	0	0	0.3100	0	0
1.0000	0	0	0.5600	0	0	0.3100
1.0000	0	0	-0.5600	0	0	0.3100
1.0000	0	0.5600	0	0	0.3100	0
1.0000	0	-0.5600	0	0	0.3100	0
1.0000	1.0300	0	0	1.0600	0	0
1.0000	-1.0300	0	0	1.0600	0	0
1.0000	0	0	1.0300	0	0	1.0600
1.0000	0	0	-1.0300	0	0	1.0600
1.0000	0	1.0300	0	0	1.0600	0
1.0000	0	-1.0300	0	0	1.0600	0
1.0000	0.6300	0.6300	0.6300	0.4000	0.4000	0.4000
1.0000	-0.6300	0.6300	0.6300	0.4000	0.4000	0.4000
1.0000	0.6300	-0.6300	0.6300	0.4000	0.4000	0.4000
1.0000	0.6300	0.6300	-0.6300	0.4000	0.4000	0.4000
1.0000	-0.6300	-0.6300	0.6300	0.4000	0.4000	0.4000
1.0000	-0.6300	0.6300	-0.6300	0.4000	0.4000	0.4000
1.0000	0.6300	-0.6300	-0.6300	0.4000	0.4000	0.4000
1.0000	-0.6300	-0.6300	-0.6300	0.4000	0.4000	0.4000

The information matrix c_4 for the design M$_4$ is given by

c_4 =

1.0000	0.2961	0.2961	0.2961	0	0	0
0.2961	0.1864	0.0621	0.0621	0	0	0
0.2961	0.0621	0.1864	0.0621	0	0	0
0.2961	0.0621	0.0621	0.1864	0	0	0
0	0	0	0	0.29610		0
0	0	0	0	0	0.2961	0
0	0	0	0	0	0	0.2961

2.5. Twenty-Six Points Three-Dimensional Specific Rotatable Design of Order Two

We consider the design;

M$_5$=(S(0.6703699, 0.6703699, 0) +S(0.9359294, 0.9359294, 0.9359294) + S(1.5993168, 0, 0))

The design matrix X of M$_5$ is given by

1.0000	0.6700	0.6700	0	0.4500	0.4500	0
1.0000	- 0.6700	0.6700	0	0.4500	0.4500	0
1.0000	0.6700	- 0.6700	0	0.4500	0.4500	0
1.0000	- 0.6700	- 0.6700	0	0.4500	0.4500	0
1.0000	0.9400	0.9400	0.9400	0.8800	0.8800	0.8800
1.0000	- 0.9400	0.9400	0.9400	0.8800	0.8800	0.8800
1.0000	0.9400	- 0.9400	0.9400	0.8800	0.8800	0.8800
1.0000	0.9400	0.9400	- 0.9400	0.8800	0.8800	0.8800
1.0000	0	1.6000	0	0	2.5600	0
1.0000	0.6700	0	0.6700	0.4500	0	0.4500
1.0000	- 0.6700	0	0.6700	0.4500	0	0.4500
1.0000	0.6700	0	- 0.6700	0.4500	0	0.4500
1.0000	- 0.6700	0	- 0.6700	0.4500	0	0.4500
1.0000	- 0.9400	- 0.9400	- 0.9400	0.8800	0.8800	0.8800
1.0000	- 0.9400	- 0.9400	0.9400	0.8800	0.8800	0.8800
1.0000	- 0.9400	0.9400	- 0.9400	0.8800	0.8800	0.8800
1.0000	0.9400	- 0.9400	- 0.9400	0.8800	0.8800	0.8800
1.0000	0	- 1.6700	0	0	2.5600	0
1.0000	0	0.6700	0.6700	0	0.4500	0.4500
1.0000	0	- 0.6700	0.6700	0	0.4500	0.4500
1.0000	0	0.6700	- 0.6700	0	0.4500	0.4500
1.0000	0	- 0.6700	- 0.6700	0	0.4500	0.4500
1.0000	1.6000	0	0	2.5600	0	0
1.0000	- 1.6000	0	0	2.5600	0	0
1.0000	0	0	1.6000	0	0	2.5600
1.0000	0	0	- 1.6000	0	0	2.5600

The information matrix c_5 for the design M$_5$ is given by

$$c_5 = \begin{bmatrix} 1.0000 & 0.6046 & 0.6046 & 0.6046 & 0 & 0 & 0 \\ 0.6046 & 0.8015 & 0.2672 & 0.2672 & 0 & 0 & 0 \\ 0.6046 & 0.2672 & 0.8015 & 0.2672 & 0 & 0 & 0 \\ 0.6046 & 0.2672 & 0.2672 & 0.8015 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6046 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6046 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6046 \end{bmatrix}$$

2.6. Thirty Points Three-Dimensional Specific Rotatable Design of Order Two

We consider the design;

M_6= (S(1.3003797, 0.5241245, 0.5241245) +S(0.3357566,0,0))

The design matrix X for the design M_6 is:

$$\begin{bmatrix} 1.0000 & 0.3400 & 0 & 0 & 0.1200 & 0 & 0 \\ 1.0000 & -0.3400 & 0 & 0 & 0.1200 & 0 & 0 \\ 1.0000 & 1.3000 & 0.5200 & 0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & -1.3000 & 0.5200 & 0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & 1.3000 & -0.5200 & 0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & 1.3000 & 0.5200 & -0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & -1.3000 & -0.5200 & 0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & -1.3000 & 0.5200 & -0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & 1.3000 & -0.5200 & -0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & -1.3000 & -0.5200 & -0.5200 & 1.6900 & 0.2700 & 0.2700 \\ 1.0000 & 0 & 0 & 0.3400 & 0 & 0 & 0.1200 \\ 1.0000 & 0 & 0 & -0.3400 & 0 & 0 & 0.1200 \\ 1.0000 & 0.5200 & 0.5200 & 1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & -0.5200 & 0.5200 & 1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & 0.5200 & -0.5200 & 1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & 0.5200 & 0.5200 & -1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & -0.5200 & -0.5200 & 1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & -0.5200 & 0.5200 & -1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & 0.5200 & -0.5200 & -1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & -0.5200 & -0.5200 & -1.3000 & 0.2700 & 0.2700 & 1.6900 \\ 1.0000 & 0 & 0.3400 & 0 & 0 & 0.1200 & 0 \\ 1.0000 & 0 & -0.3400 & 0 & 0 & 0.1200 & 0 \\ 1.0000 & 0.5200 & 1.3000 & 0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & -0.5200 & 1.3000 & 0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & 0.5200 & -1.3000 & 0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & 0.5200 & 1.3000 & -0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & -0.5200 & -1.3000 & 0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & -0.5200 & 1.3000 & -0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & 0.5200 & -1.3000 & -0.5200 & 0.2700 & 1.6900 & 0.2700 \\ 1.0000 & -0.5200 & -1.3000 & -0.5200 & 0.2700 & 1.6900 & 0.2700 \end{bmatrix}$$

The information matrix c_6 for the design M_6 is given by

$$c_6 = \begin{bmatrix} 1.0000 & 0.6050 & 0.6050 & 0.6050 & 0 & 0 & 0 \\ 0.6050 & 0.8036 & 0.2679 & 0.2679 & 0 & 0 & 0 \\ 0.6050 & 0.2679 & 0.8036 & 0.2679 & 0 & 0 & 0 \\ 0.6050 & 0.2679 & 0.2679 & 0.8036 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6050 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6050 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6050 \end{bmatrix}$$

	A	D	E	T	I
M_1	0.00685555	0.518684	0.08817	0.719916756	1.8880
M_2	0.00826251	0.588535	0.119857	0.7623315	
M_3	0.36672210	0.540062	0.101498	0.737534	2.9124
M_4	0.13023463	0.536882	0.037407	0.3496119	33.2200
M_5	0.37913341	0.549356	0.107413	0.7454531	
M_6	0.380881157	0.550390	0.108214	0.746528	

Table 1: A Summary of the Particular Optimality Criteria for the Six Designs

3. The Relative Efficiency Criteria

3.1 A-efficiency

This measure is related to the A-optimality criterion:

A(ξ)= $\frac{tr(m^{-1}(\varepsilon_A^*))}{tr(m^{-1}(\varepsilon))}$

M_1	$= \frac{0.00685555}{0.00685555}$ = **100%**
M_2	$= \frac{0.00685555}{0.00826251}$ = **82.972%**
M_3	$= \frac{0.00685555}{0.36672210}$ = **1.869%**
M_4	$= \frac{0.00685555}{0.13023463}$ = **5.264%**
M_5	$= \frac{0.00685555}{0.37913341}$ = **1.808%**
M_6	$= \frac{0.00685555}{0.380881157}$ = **1.800%**

Table 2: Calculus Optimum ValuesA-Efficiencies

3.2. D-efficiency

This measure is related to the D-optimality criterion:

D (ξ) = [$\frac{M(\varepsilon)}{M^*(\varepsilon)}$]

M_1	$= \frac{0.51684}{0.51684}$ = 100%
M_2	$= \frac{0.51684}{0.568535}$ = 90.907%
M_3	$= \frac{0.51684}{0.540062}$ = 95.700%
M_4	$= \frac{0.51684}{0.536882}$ = 96.267%
M_5	$= \frac{0.51684}{0.549356}$ = 99.516%
M_6	$= \frac{0.51684}{0.550390}$ = 93.904%

Table 3: Calculus Optimum ValuesD-Efficiencies

3.3. T-efficiency

This measure is related to the T-optimality criterion:

T (ξ) = $\frac{\Delta_1(\varepsilon)}{\Delta_1(\varepsilon_T^*)}$

M_1	$= \frac{0.3496119}{0.719916756}$ = **47.563%**
M_2	$= \frac{0.3496119}{0.7623315}$ = **45.861%**
M_3	$= \frac{0.3496119}{0.737534}$ = **47.403%**
M_4	$= \frac{0.3496119}{0.3496119}$ = **100%**
M_5	$= \frac{0.3496119}{0.7454531}$ = **46.899%**
M_6	$= \frac{0.3496119}{0.746528}$ = **46.832%**

Table 4: Calculus Optimum T-efficiencies

3.4. E-Efficiency

This measure is related to the E-optimality criterion:

E(ξ) = $\frac{\lambda_{min}(\varepsilon)}{\lambda_{min}(\varepsilon^*)}$Where $\lambda_{min}(\varepsilon)$ is the Eigen value of the information matrix

M_1	= $\frac{0.037407}{0.08817}$ = **42.426%**
M_2	= $\frac{0.037407}{0.119857}$ = **31.210%**
M_3	= $\frac{0.037407}{0.101498}$ = **36.855%**
M_4	= $\frac{0.037407}{0.037407}$ = **100%**
M_5	= $\frac{0.037407}{0.107413}$ = **34.825%**
M_6	= $\frac{0.037407}{0.108214}$ = **34.568%**

Table 5: Calculus Optimum Values E-Efficiencies

3.5. I–/IV– efficiency

This measure is computed as:

I(ξ)= $\frac{tr(\,m\,m^{-1}(\xi_1^*))}{tr(m\,m^{-1}(\xi))}$

M_1	= $\frac{1.8880}{1.8880}$ = 100%
M_3	= $\frac{1.8880}{2.9124}$ = 64.826%
M_4	= $\frac{1.8880}{33.2200}$ = 25.876%

Table 6: Calculus Optimum ValuesI–/IV– Efficiencies

4. Results and Conclusions

The table below gives the summary of the relative efficiencies evaluated. From the table above it is clear that the most appropriate criterion for calculus optimum values is when we use D_{eff} to evaluate the efficiencies because it gives a high average as compared to the rest of the efficiency criteria, this criterion gives efficiencies of all the six designs close to 100. $M_1$is found to be the most efficient design as compared to the rest.

5. A Practical Hypothetical Example

We shall discuss hypothetical production of katumani hybrid maize to illustrate the use of specific optimum second order rotatable design of 24 points

$$D = \left\{ \frac{1}{2}G(1.1072569,1.1072569,0) + \frac{1}{4}G(0.7829487,0,0) + \frac{1}{4}G(1.2735263,0,0) \right\} \quad 5.1$$

Suppose that the three factors are potassium (x_{1u}), Sodium (x_{2u}), and calcium (x_{3u}) as a result of soil mapping investigations which indicated deficiencies of these mineral elements in the kibwezi loam soils. We wish to point out that out that the original letters f, C_1 and C_2 represent the variation in quantity application of a factor due to soil fertility gradient culminating in several radii manifestations of rotatability criterion. According to Box [1952] and Box and Wilson [1951] we can revert the natural levels of these elements denoted by $\psi_{iu}$where Bose and Draper [1959] scaling condition fixes a particular design when $\lambda_2 = 1$ whence

$$x_{iu} = \frac{\psi_{iu} - \psi_{i\bullet}}{S_i} \quad 5.2$$

$$\psi_{i\bullet} = \frac{\sum_{u=1}^{N} \psi_{iu}}{N}, \quad 5.3$$

$$S_i = \left[\frac{\sum_{u=1}^{N} (\psi_{iu} - \psi_{i\bullet})}{N} \right]^{0.5}, \quad 5.4$$

$$\psi_{iu} = x_{iu} S_i + \psi_{i\bullet}, \quad 5.5$$

For $\sum_{u=1}^{N} x_{iu}^{2} = N$ and $\sum_{u=1}^{N} x_{iu} = 0$. The design matrix can be then constituted but the evaluation of the inverse will be a major computational project to estimate the coefficients of the second order rotatable design model which give the optimum response or yield. This requires a separate discourse but the actual responses or yields can be obtained if a field experiment is conducted as explained. Let the scale parameter,

S_i, assume$S_1 = 0.5$, and$S_2 = 1$, and $S_3 = 1$

Suppose that

Potassium (K) :$\psi_{1\bullet} = 20 milligrams$,

Sodium (Na) :$\psi_{2\bullet} = 15 milligrams$,

Calcium (Ca) :$\psi_{3\bullet} = 30 milligrams$,

are the hoe hole average quantities of the levels of the mineral elements recommended by the soil mapping team. For D we have the following coded and natural levels respectively as treatments;

An example on how to obtain the coded levels from the natural levels we use the formula:

$$\psi_{iu} = x_{iu} S_i + \psi_{i\bullet}$$

For $\psi_{11} = x_{11} S_1 + \psi_{1\bullet}$

$= 1.1072569 \times 0.5 + 20 = 20.553628$

();	().
(1.1072569, 1.1072569, 0) ;	(20.553628, 15.332177, 30).
(1.1072569, 1.1072569, 0) ;	(19.446372, 15.332177, 30).
(1.1072569, 1.1072569, 0) ;	(20.553628, 14.667823, 30).
(1.1072569, 1.1072569, 0) ;	(19.446372, 14.667823, 30).
(1.1072569, 0, 1.1072569) ;	(20.553628,15, 31.107257).
(1.1072569, 0, 1.1072569) ;	(19.446372, 15, 31.107257).
(1.1072569, 0, 1.1072569) ;	(20.553628, 15, 28.892743).
(1.1072569, 0, 1.1072569) ;	(19.446372, 15, 28.892743).
(0, 1.1072569, 1.1072569) ;	(20, 15.332177, 31.107257).
(0, 1.1072569, 1.1072569) ;	(20, 14.667823, 31.107257).
(0, 1.1072569, 1.1072569) ;	(20, 15.332177, 28.892743).
(0, 1.1072569, 1.1072569) ;	(20, 14.667823, 28.892743).
(0.782948, 0, 0) ;	(20.391474, 15, 30).
0.78294, 0, 0) ;	(19.363237, 15, 30).
(0, 0, 0.782948) ;	(20, 15, 30.782949).
(0, 0, 0.782948) ;	(20, 15, 29.217051).
(0, 0.782948, 0);	(20, 15.234885, 30).
(0, 0.782948, 0);	(20, 14.765115, 30).
(1.2735263, 0, 0);	(20.391474, 15, 30).
(1.2735263, 0, 0);	(19.363237, 15, 30).
(0, 0, 1.2735263);	(20, 15, 31.273526).
(0, 0, 1.2735263);	(20, 15, 28.726474).
(0, 1.2735263, 0);	(20, 15.382058, 30).
(0, 1.2735263, 0);	(20, 14.617942, 30).

The values inform of a matrix above shows the natural values converted to coded values.

6. References

i. Box G.E.P and Wilson K.B. (1951). On the experimental attainment of optimum conditions. *Journal of Royal Statistical Society B13, 1-45*

ii. Bose, R.C and Draper, N.R. (1959). Second order rotatable designs in three dimensions. *Journal of Ann. of Mathematical Statistics 30, 1079-1112.*

iii. Box G.E.P and Hunter. J.S. (1957). Multifactor experimental designs for exploring response surfaces. *Ann. Mathematical Statistics 28, 195-214*

iv. Cheruiyot, A.K. (2015). Analysis of efficiencies and optimality criteria for the specific second order rotatable designs. M.S.C thesis, University of Eldoret.

v. Draper N.R. (1960). Third order rotatable designs in three dimensions *Journal of Ann. of Mathematical Statistics 31, 865-874.*

vi. Hezberg, A.M. (1967). A method for the construction of second order rotatable designs in k dimensions. *Journal of Ann. of Mathematical Statistics 38, 177-180*

vii. Kipkemoi, E.C.; Koske, J.K and Mutiso, J.M (2014). Construction of some new three incomplete block designs in two replicates. Inholep mathematical proceedings volume 1, Number 1.

viii. Kosgei, K.M.; koske, J.K. and Mutiso, J.M. (2006). On optimality of a second order rotatable designs in three dimensions. The East African Journal of Statistics Volume 1, Number2, pp. 123-128.

ix. Koske, J.K.; Kosgei, K.M. and Mutiso, J.M. (2011). A new third order rotatable designs in five dimensions through balanced incomplete block designs volume 13, No. 1, pp 157-163

x. Koske, J.K.; Mutiso, J.M. and Kosgei, K.M. (2014). A fourth order rotatable design in two dimensions. EAPAS Journal.

xi. Koske, J.K; Mutiso, J.M. and Kosgei, M.K. (2008). A specific optimum second order rotatable design of twenty four points with practical example. EAPAS vol.1, Issue 1, pp. 107-112

xii. Mutai, C.K.; Koske, J.K. and Mutiso, J.M. (2012). Construction of a four dimensional Third order rotatable designs through balanced incomplete block designs. Advances and applications in statistics, Volume 27, Number 1, pp 47-53.

xiii. Mutai, K.K.; Koske, J.K. and Mutiso, J.M. (2012). Optimal designs for mixture experiments and their application in agricultural research. JP Journal of Biostatistics volume 7, Number ‘, pp 77-90.

xiv. Mutiso J.M. (1998). Second order and third order specific and sequential rotatable designs in k dimensions. P.H.D thesis, Moi University

xv. Mutiso, J.M. and Koske, J.K. (2005). Some third order rotatable designs in five deimensions . The east African Journal of Statistics volume 1, Number 1, PP 117-122.

xvi. Mutiso, J.M. and Koske, J.K. (2007). Some third order rotatable designs in six dimensions. Journal of Agriculture Science and technology volume 9, Number 1, 78-87.

xvii. Mutiso, J.M. and Koske, J.K.(2014). Algebraic foundations of third order rotatability in two dimensions. Journal of mathematical sciences Vol. 25, No. 3, 93-101

xviii. Patel, M.S and Mutiso, J.m. (2010). Some third order rotatable designs in four dimensions. Kenya Journal of sciences.